Mathematics Notes for Class 9th

KPK All Boards

Mathematics

Chapter # 1

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Chapter # 1
Matrices

Exercise# 1.1

Matrix

A matrix is a rectangular array (arrangements) of real numbers enclosed in square brackets. Each number in a matrix is called an element or entry of the matrix. Matrices are mostly denoted by capital letters.

Examples

$$A=\begin{bmatrix}2 & 3\\0 & 5\end{bmatrix},\ C=\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}$$

Rows and Columns of a Matrix

The rows of a matrix run horizontally, and the columns of a matrix run vertically.

Order or Dimension of a Matrix

The number of rows and columns that a matrix has is called order of a matrix.

Order of a matrix is represented by:

Order of matrix $= m\times n$

OR

Order of matrix = m-by-n

Here "m" represents number of Rows

And "n" represents number of columns

Note

Order of a matrix is also called dimension or size of a matrix.

Examples

$$D=\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}$$

Examples

$$D=\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}$$

In this example

2, 5,1,3 all are the elements of a matrix D.

2, 5 and 1, 3 are the rows of a matrix D.

2, 1 and 5, 3 are the columns of matrix D.

As No. of Rows= 2

And No. of Columns= 2

So order is $2-by-2$ (OR) 2×2

Equal Matrix

When two matrices of the same order and the corresponding elements are same.

Exercise # 1.1
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Q1: Which of the following are square and which are rectangular matrices?

(i) $A=\begin{bmatrix}2 & 3\\0 & 5\end{bmatrix}$

As No. of Rows $=$ No. of Columns

So it is Square matrix.

(ii) $B=\begin{bmatrix}6 & 3 & -1\\1 & 5 & 2\end{bmatrix}$

As No. of Rows $\neq$ No. of Columns

So it is Rectangular matrix.

(iii) $C=\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}$

As No. of Rows $=$ No. of Columns

So it is Square matrix.

(iv) $D=\begin{bmatrix}-5\end{bmatrix}$

As No. of Rows $=$ No. of Columns
So it is Square matrix.

(v) $E=\begin{bmatrix}-3 & 4\end{bmatrix}$

As No. of Rows $\neq$ No. of Columns
So it is Rectangular matrix.

(vi) $F=\begin{bmatrix}-1\\7\end{bmatrix}$

As No. of Rows $\neq$ No. of Columns
So it is Rectangular matrix.

Q2: List the order of the following matrices.

(i) $A=\begin{bmatrix}1 & 2 & -1\\3 & 4 & 2\end{bmatrix}$

As No. of Rows$=2$
And No. of Columns$=3$
So order is $2-by-3$ (OR) 2×3

(ii) $B=\begin{bmatrix}-4\end{bmatrix}$

As No. of Rows$=1$
And No. of Columns$=1$
So order is $1-by-1$ (OR) 1×1

(iii) $C=\begin{bmatrix}2 & 3 & -1\\1 & 2 & 5\end{bmatrix}$

As No. of Rows$=2$
And No. of Columns$=3$
So order is $2-by-3$ (OR) 2×3

(iv) $F=\begin{bmatrix}2 & 1\\3 & 2\\4 & -1\end{bmatrix}$

As No. of Rows$=3$
And No. of Columns$=2$
So order is $3-by-2$(OR) 3×2

(v) $E=\begin{bmatrix}3 & 2\end{bmatrix}$

As No. of Rows$=1$
And No. of Columns$=2$
So order is $1-by-2$ (OR) 1×2

(vi) $D=\begin{bmatrix}1 & 2 & 3\\6 & 5 & 9\\0 & 0 & 0\end{bmatrix}$

As No. of Rows$=3$
And No. of Columns$=3$
So order is $3-by-3$ (OR) 3×3

Q3: If $A=\begin{bmatrix}3 & 2 & -4\\-2 & 5 & 0\\2 & 1 & 5\\-3 & 4 & 6\end{bmatrix}$, **give the following elements.**

Solution

$$A=\begin{bmatrix}3 & 2 & -4\\-2 & 5 & 0\\2 & 1 & 5\\-3 & 4 & 6\end{bmatrix}$$

As $\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\\a_{41} & a_{42} & a_{43}\end{bmatrix}$

Answers:

(i) $a_{12}=2$
(ii) $a_{23}=0$
(iii) $a_{32}=1$
(iv) $a_{43}=6$
(v) $a_{13}=-4$
(vi) $a_{43}=6$

Q4: Which of the following matrices are equal?

$$A=\begin{bmatrix}2&5\\1&3\end{bmatrix},\qquad B=\begin{bmatrix}2&5\\4&3\end{bmatrix},$$

$$C=\begin{bmatrix}1+1&3+2\\4&2+1\end{bmatrix},\quad D=\begin{bmatrix}2&4+1\\1&3\end{bmatrix}$$

Solution:

As $C=\begin{bmatrix}2&5\\4&3\end{bmatrix}$ and $D=\begin{bmatrix}2&5\\1&3\end{bmatrix}$

So A and D are equal i.e. $A = D$

And B and C are equal i.e. $B = C$

Q5: Let $A=\begin{bmatrix}2&-3\\u&0\end{bmatrix}$ and $B=\begin{bmatrix}v&-3\\5&w\end{bmatrix}$, for what values of $u, v, and\ w$ are when A and B equal.

Solution

$$A=\begin{bmatrix}2&-3\\u&0\end{bmatrix},\ B=\begin{bmatrix}v&-3\\5&w\end{bmatrix}$$

As A and B are equal. So

$$\begin{bmatrix}2&-3\\u&0\end{bmatrix}=\begin{bmatrix}v&-3\\5&w\end{bmatrix}$$

Now compare the corresponding elements

$2 = v$

Or $\boldsymbol{v = 2}$

$\boldsymbol{u = 5}$

$0 = w$

Or $\boldsymbol{w = 0}$

Q6: If

$$\begin{bmatrix}x+3&z+4&2y-7\\-6&a-1&0\\b-3&-21&0\end{bmatrix}=\begin{bmatrix}0&6&3y-2\\-6&-3&2c+2\\2b+4&-21&0\end{bmatrix}$$

find the values of a, b, c, x, y and z.

Solution:

As

$$\begin{bmatrix}x+3&z+4&2y-7\\-6&a-1&0\\b-3&-21&0\end{bmatrix}=\begin{bmatrix}0&6&3y-2\\-6&-3&2c+2\\2b+4&-21&0\end{bmatrix}$$

Now compare the corresponding elements

$x + 3 = 0$

$x = -3$

Now

$z + 4 = 6$

$z = 6 - 4$

$z = 2$

Now

$2y - 7 = 3y - 2$

$-7 + 2 = 3y - 2y$

$-5 = y$

$y = -5$

Now

$a - 1 = -3$

$a = -3 + 1$

$a = -2$

Now

$0 = 2c + 2$

$0 - 2 = 2c$

$-2 = 2c$

$\frac{-2}{2} = c$

$-1 = c$

$c = -1$

Now

$b - 3 = 2b + 4$

$-3 - 4 = 2b - b$

$-7 = b$

$b = -7$

Answers:

$a = -2$

$b = -7$

$c = -1$

$x = -3$

$y = -5$

$z = 2$

Q7: **Solve the following equation for a, b, c, d.**

$$\begin{bmatrix} a+b & b+2c \\ 2c+d & 2a-d \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 8 & 0 \end{bmatrix}$$

Solution

$$\begin{bmatrix} a+b & b+2c \\ 2c+d & 2a-d \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 8 & 0 \end{bmatrix}$$

Now compare the corresponding elements

$a + b = -1$ ------- equ(i)

$b + 2c = 4$ ------- equ(ii)

$2c + d = 8$ ------- equ(iii)

$2a - d = 0$ ------- equ(iv)

Subtract equ(ii) from equ(i)

$(a + b) - (b + 2c) = -1 - 4$

$a + b - b - 2c = -5$

$a - 2c = -5$ ------- equ(v)

Now Add equ(iii) and equ(v)

$2c + d + (a - 2c = 8 + (-5)$

$2c + d + a - 2c = 8 - 5$

$d + a = 3$ ------- equ(vi)

Now add equ(iv) and equ(vi)

$2a - d + d + a = 0 + 3$

$2a + a = 3$

$3a = 3$

$a = \frac{3}{3}$

$a = 1$

Put $a = 1$ in equ(i)

$1 + b = -1$

$b = -1 - 1$

$b = -2$

Put $b = -2$ in equ(ii)

$-2 + 2c = 4$

$2c = 4 + 2$

$2c = 6$

$c = \frac{6}{2}$

$c = 3$

Put $c = 3$ in equ(iii)

$2(3) + d = 8$

$6 + d = 8$

$d = 8 - 6$

$d = 2$

Answers:

$a = 1$

$b = -2$

$c = 3$

$d = 2$

Ex 1.1 End

Exercise # 1.2

Types of matrices

Row matrix

A matrix having just one row is called row matrix.

$$A = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix}, \ B = \begin{bmatrix} 5 \end{bmatrix}$$

Column matrix

A matrix having just one column is called column matrix.

$$A = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \end{bmatrix}$$

Square matrix

A matrix in which number of rows and columns are equal is called square matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}, \ B = \begin{bmatrix} 5 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

Rectangular matrix

A matrix in which number of rows and columns are not equal is called rectangular matrix.

$$A = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 9 & 8 \end{bmatrix}$$

Zero matrix or Null matrix

A matrix in which all the elements are zero is called Zero or Null matrix. A null matrix is generally denoted by O.

$$O=\begin{bmatrix}0\\0\end{bmatrix},\quad O=\begin{bmatrix}0&0\\0&0\end{bmatrix},\quad O=\begin{bmatrix}0&0&0\end{bmatrix}$$

Diagonal matrix

A square matrix on which all elements are zero except diagonal elements is known as diagonal matrix.

$$A=\begin{bmatrix}1&0\\0&3\end{bmatrix},\quad B=\begin{bmatrix}-2&0\\0&-2\end{bmatrix},\quad I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Scalar matrix

A matrix in which diagonal elements are same is called scalar matrix.

$$I=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad A=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix},\quad C=\begin{bmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{bmatrix}$$

Note

Every scalar matrix is a diagonal matrix but every diagonal matrix is not necessarily a scalar matrix.

Identity or Unit matrix

A matrix in which the diagonal elements are equal to “1” is called identity matrix. It is generally denoted by “I”.

$$I=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Transpose of a matrix

A matrix obtained by interchanging all rows and columns with each other is called transpose of a matrix. The transpose of a matrix B is written as B^t.

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$$

$$A^t=\begin{bmatrix}a&c\\b&d\end{bmatrix}$$

Symmetric matrix

In a square matrix, when $A^t=A$, then A is said to be symmetric matrix.

$$A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$$

$$A^t=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$$

$$A^t=A$$

Skew-Symmetric matrix

In a square matrix, when $A^t=-A$, then A is said to be skew-symmetric matrix.

$$A=\begin{bmatrix}0&4\\-4&0\end{bmatrix}$$

$$A^t=\begin{bmatrix}0&-4\\4&0\end{bmatrix}$$

$$A^t=-\begin{bmatrix}0&4\\-4&0\end{bmatrix}$$

$$A^t=-A$$

Exercise # 1.2

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Q1: Write the transpose of the following matrices.

(i) $P=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}$

Solution:

$P=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}$

Taking transpose on both sides

$P^t=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}^t$

$P^t=\begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}$

(ii) $Q=\begin{bmatrix}l & m\\ n & p\end{bmatrix}$

Solution:

$Q=\begin{bmatrix}l & m\\ n & p\end{bmatrix}$

Taking transpose on both sides

$Q^t=\begin{bmatrix}l & m\\ n & p\end{bmatrix}^t$

$Q^t=\begin{bmatrix}l & n\\ m & p\end{bmatrix}$

(iii) $R=[6]$

Solution

$R=[6]$

Taking transpose on both sides

$R^t=[6]^t$

$R^t=[6]$

(iv) $S=\begin{bmatrix}-5 & 1\\ -2 & 1\\ 4 & 4\end{bmatrix}$

Solution

$S=\begin{bmatrix}-5 & 1\\ -2 & 1\\ 4 & 4\end{bmatrix}$

Taking transpose on both sides

$S^t=\begin{bmatrix}-5 & 1\\ -2 & 1\\ 4 & 4\end{bmatrix}^t$

$S^t=\begin{bmatrix}-5 & -2 & 4\\ 1 & 1 & 4\end{bmatrix}$

(v) $T=\begin{bmatrix}6 & 7 & 8\\ 13 & 1 & 3\\ 2 & 4 & 5\end{bmatrix}$

Solution

$T=\begin{bmatrix}6 & 7 & 8\\ 13 & 1 & 3\\ 2 & 4 & 5\end{bmatrix}$

Taking transpose on both sides

$T^t=\begin{bmatrix}6 & 7 & 8\\ 13 & 1 & 3\\ 2 & 4 & 5\end{bmatrix}^t$

$T^t=\begin{bmatrix}6 & 13 & 2\\ 7 & 1 & 4\\ 8 & 3 & 5\end{bmatrix}$

Ex # 1.2

Q2: **Which of the following matrices are transpose of the each other?**

$$A=\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix},\ B=\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$C=\begin{bmatrix} -3 & 1 & -1 \\ 4 & 2 & 7 \end{bmatrix},\ D=\begin{bmatrix} -3 & 4 \\ 1 & 2 \\ -1 & 7 \end{bmatrix}$$

Solution:

As $A^t=\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}=B$

And $B^t=\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}=A$

Thus A and B are the transpose of each other.

As $C^t=\begin{bmatrix} -3 & 4 \\ 1 & 2 \\ -1 & 7 \end{bmatrix}=D$

And $D^t=\begin{bmatrix} -3 & 1 & -1 \\ 4 & 2 & 7 \end{bmatrix}=C$

Thus C and D are the transpose of each other.

Q3: **Which of the following are symmetric?**

(i) $A=\begin{bmatrix} 5 & -7 \\ -1 & 5 \end{bmatrix}$

Solution:

$$A=\begin{bmatrix} 5 & -7 \\ -1 & 5 \end{bmatrix}$$

By taking transpose, we get

$$A^t=\begin{bmatrix} 5 & -1 \\ -7 & 5 \end{bmatrix}$$

$\boldsymbol{A^t \neq A}$

Thus A is not symmetric matrix

Ex # 1.2

(ii) $B=\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$

Solution:

$$B=\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

By taking transpose, we get

$$B^t=\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

$\boldsymbol{B^t = B}$

Thus B is symmetric matrix

(iii) $C=\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

Solution

$$C=\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

By taking transpose, we get

$$C^t=\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$\boldsymbol{C^t \neq C}$

Thus C is not symmetric matrix

(iv) $D=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Solution

$$D=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

By taking transpose, we get

$$D^t=\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$\boldsymbol{D^t \neq D}$

Thus D is not symmetric matrix

Ex # 1.2

Q4: Which of the following matrices are skew-symmetric?

(i) $A=\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$

Solution

$A=\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$

By taking transpose, we get

$A^t=\begin{bmatrix}0 & -4\\ 4 & 0\end{bmatrix}$

$A^t=-\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$

$\boldsymbol{A^t=-A}$

Thus A is a skew-symmetric matrix

(ii) $B=\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix}$

Solution

$B=\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix}$

By taking transpose, we get

$B^t=\begin{bmatrix}0 & 5\\ -5 & 0\end{bmatrix}$

$B^t=-\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix}$

$\boldsymbol{B^t=-B}$

Thus B is a skew-symmetric matrix

(iii) $C=\begin{bmatrix}0 & 7\\ 7 & 0\end{bmatrix}$

Solution

$C=\begin{bmatrix}0 & 7\\ 7 & 0\end{bmatrix}$

By taking transpose, we get

$C^t=\begin{bmatrix}0 & 7\\ 7 & 0\end{bmatrix}$

Ex # 1.2

$C^t=-\begin{bmatrix}0 & -7\\ -7 & 0\end{bmatrix}$

$\boldsymbol{C^t\neq -C}$

Thus C is not a skew-symmetric matrix

(iv) $D=\begin{bmatrix}0 & 3 & 2\\ -3 & 0 & 1\\ -2 & -1 & 0\end{bmatrix}$

Solution

$D=\begin{bmatrix}0 & 3 & 2\\ -3 & 0 & 1\\ -2 & -1 & 0\end{bmatrix}$

By taking transpose, we get

$D^t=\begin{bmatrix}0 & -3 & -2\\ 3 & 0 & -1\\ 2 & 1 & 0\end{bmatrix}$

$D^t=-\begin{bmatrix}0 & 3 & 2\\ -3 & 0 & 1\\ -2 & -1 & 0\end{bmatrix}$

$\boldsymbol{D^t=-D}$

Thus D is a skew-symmetric matrix

Exercise # 1.3

Conformability for Addition or Subtraction

When two matrices have the same order, then they are conformability for Addition and Subtraction.

Adding and Subtracting of matrices

(a) Addition can be obtained by adding the corresponding elements of the matrices.

(b) Subtraction can be obtained by subtracting the corresponding elements of the matrices.

Example

$$A=\begin{bmatrix}3 & 8\\ 4 & 6\end{bmatrix},\ B=\begin{bmatrix}4 & 0\\ 1 & -9\end{bmatrix}$$

$$A+B=\begin{bmatrix}3 & 8\\ 4 & 6\end{bmatrix}+\begin{bmatrix}4 & 0\\ 1 & -9\end{bmatrix}$$

$$A+B=\begin{bmatrix}3+4 & 8+0\\ 4+1 & 6-9\end{bmatrix}$$

$$A+B=\begin{bmatrix}7 & 8\\ 5 & -3\end{bmatrix}$$

$$A-B=\begin{bmatrix}3 & 8\\ 4 & 6\end{bmatrix}-\begin{bmatrix}4 & 0\\ 1 & -9\end{bmatrix}$$

$$A-B=\begin{bmatrix}3-4 & 8-0\\ 4-1 & 6+9\end{bmatrix}$$

$$A-B=\begin{bmatrix}-1 & 8\\ 3 & 15\end{bmatrix}$$

Multiplication of a matrix by a real number

The real number is multiplying to each element of the matrix. The real number is called the scalar multiplication of that matrix i.e. 3 is scalar multiplication in the following matrix.

Multiply $A=\begin{bmatrix}6 & 2\\ -3 & 1\end{bmatrix}$ by 3

$$3A=3\begin{bmatrix}6 & 2\\ -3 & 1\end{bmatrix}$$

$$3A=\begin{bmatrix}18 & 6\\ -9 & 3\end{bmatrix}$$

Commutative Property w.r.t Addition

If two matrices of same order then $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$ is called the Commutative law under addition.

Associative Property w.r.t Addition

If three matrices of same order, then $A+(B+C)=(A+B)+C$ is called the Associative law under addition.

Additive Identity of matrices

Normally zero (0) is called additive identity. Thus Zero or Null matrix is additive identity matrix.

Additive Inverse of a matrix

When the sum of two matrices is zero (0), then these matrices are called inverse of each other. $A=-B$ or $B=-A$

Exercise # 1.3

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Q1: Let A & B be $2-by-3$ matrices and let C & D be 2 –square matrices. Which of the following matrix operation are defined? For those which are defined, give the dimension of the resulting matrix.

(i) $\boldsymbol{A}+\boldsymbol{B}$

As the order of A is $2-by-3$

And the order of B is $2-by-3$

So $\boldsymbol{A}+\boldsymbol{B}$ are conformable

(ii) $\boldsymbol{B}+\boldsymbol{D}$

As the order of B is $2-by-3$

And the order of D is $2-by-2$

So $\boldsymbol{B}+\boldsymbol{D}$ are <u>not</u> conformable

(iii) $\boldsymbol{3A}-\boldsymbol{2C}$

As the order of A is $2-by-3$

And the order of C is $2-by-2$

So $3\boldsymbol{A}-2\boldsymbol{C}$ are <u>not</u> conformable

(iv) $\boldsymbol{7C}-\boldsymbol{2D}$

As the order of C is $2-by-2$

And the order of D is $2-by-2$

So $7\boldsymbol{C}-2\boldsymbol{D}$ are conformable

Ex # 1.3

Q2: Multiply the following matrices by real numbers as indicated.

(i) Multiply $A=\begin{bmatrix}1\\2\\3\end{bmatrix}$ **by 2**

Solution:

$$A=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

Multiply B.S by 2

$$2A=2\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$2A=\begin{bmatrix}2\\4\\6\end{bmatrix}$$

(ii) Multiply $B=\begin{bmatrix}a&b&c\\d&e&f\end{bmatrix}$ **by p ∈ R**

Solution:

$$B=\begin{bmatrix}a&b&c\\d&e&f\end{bmatrix}$$

Multiply B.S by p

$$pB=p\begin{bmatrix}a&b&c\\d&e&f\end{bmatrix}$$

$$pB=\begin{bmatrix}pa&pb&pc\\pd&pe&pf\end{bmatrix}$$

Q3: Find a matrix X such that $4X=\begin{bmatrix}1&2&1\\4&2&3\\-1&9&7\end{bmatrix}$

Solution:

$$4X=\begin{bmatrix}1&2&1\\4&2&3\\-1&9&7\end{bmatrix}$$

Ex # 1.3

Multiply B.S by $\frac{1}{4}$

$$\frac{1}{4}\times 4X=\frac{1}{4}\times\begin{bmatrix}1&2&1\\4&2&3\\-1&9&7\end{bmatrix}$$

$$X=\begin{bmatrix}1\times\frac{1}{4}&2\times\frac{1}{4}&1\times\frac{1}{4}\\4\times\frac{1}{4}&2\times\frac{1}{4}&3\times\frac{1}{4}\\-1\times\frac{1}{4}&9\times\frac{1}{4}&7\times\frac{1}{4}\end{bmatrix}$$

$$X=\begin{bmatrix}\frac{1}{4}&\frac{1}{2}&\frac{1}{4}\\1&\frac{1}{2}&\frac{3}{4}\\\frac{-1}{4}&\frac{9}{4}&\frac{7}{4}\end{bmatrix}$$

Q4: If $A=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$ **and** $B=\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$**, then find** $\boldsymbol{3A-B}$**.**

Solution:

$$A=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix},\ B=\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$$

Now

$$3A-B=3\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}-\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$$

$$3A-B=\begin{bmatrix}3&6\\9&12\\15&18\end{bmatrix}-\begin{bmatrix}-3&-2\\1&-5\\4&3\end{bmatrix}$$

$$3A-B=\begin{bmatrix}3+3 & 6+2\\ 9-1 & 12+5\\ 15-4 & 18-3\end{bmatrix}$$

$$3A-B=\begin{bmatrix}6 & 8\\ 8 & 17\\ 11 & 15\end{bmatrix}$$

Q5: **Given** $A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 3 & 0\end{bmatrix}$**,**

find the matrix C such that $A+2B=C$

Solution:

$$A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix},\ B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 3 & 0\end{bmatrix}$$

As $A+2B=C$

Or $C=A+2B$

$$C=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+2\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 3 & 0\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}6 & -2 & 4\\ 8 & 4 & 10\\ 4 & 6 & 0\end{bmatrix}$$

$$C=\begin{bmatrix}1+6 & 2-2 & -3+4\\ 5+8 & 0+4 & 2+10\\ 1+4 & -1+6 & 1+0\end{bmatrix}$$

$$C=\begin{bmatrix}7 & 0 & 1\\ 13 & 4 & 12\\ 5 & 5 & 1\end{bmatrix}$$

Q6: **If** $A=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$**, then find**

the matrix X such that $2A+3X=5B$

Ex # 1.3

Solution:

$$A=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix},\ B=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$$

As we have

$$2A+3X=5B$$

$$2\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}+3X=5\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$$

$$\begin{bmatrix}4 & -4\\ 8 & 4\\ -10 & 2\end{bmatrix}+3X=\begin{bmatrix}40 & 0\\ 20 & -10\\ 15 & 30\end{bmatrix}$$

$$3X=\begin{bmatrix}40 & 0\\ 20 & -10\\ 15 & 30\end{bmatrix}-\begin{bmatrix}4 & -4\\ 8 & 4\\ -10 & 2\end{bmatrix}$$

$$3X=\begin{bmatrix}40-4 & 0+4\\ 20-8 & -10-4\\ 15+10 & 30-2\end{bmatrix}$$

$$3X=\begin{bmatrix}36 & 4\\ 12 & -14\\ 25 & 28\end{bmatrix}$$

Multiply B.S by $\frac{1}{3}$**, we get**

$$\frac{1}{3}\times 3X=\begin{bmatrix}36\times\frac{1}{3} & 4\times\frac{1}{3}\\ 12\times\frac{1}{3} & -14\times\frac{1}{3}\\ 25\times\frac{1}{3} & 28\times\frac{1}{3}\end{bmatrix}$$

$$X=\begin{bmatrix}12 & \frac{4}{3}\\ 4 & \frac{-14}{3}\\ \frac{25}{3} & \frac{28}{3}\end{bmatrix}$$

Ex # 1.3

Q7: **Find x, y, z and w if**

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 3+w & 3 \end{bmatrix}$$

Solution:

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 3+w & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+3+w & 2w+3 \end{bmatrix}$$

By comparing their corresponding elements

$3x = x + 4$

$3x - x = 4$

$2x = 4$

$x = \frac{4}{2}$

$\boldsymbol{x = 2}$

Now

$3y = 6 + x + y$

$3y - y = 6 + 2$ **Putting $x = 2$**

$2y = 8$

$y = \frac{8}{2}$

$\boldsymbol{y = 4}$

Now

$3w = 2w + 3$

$3w - 2w = 3$

$\boldsymbol{w = 3}$

Now

$3z = -1 + 3 + w$

$3z = 2 + 3$ **Putting $w = 3$**

$3z = 5$

$\boldsymbol{z = \frac{5}{3}}$

Answers:

$\boldsymbol{x = 2,\ y = 4,\ z = \frac{5}{3}}$ **and $w = 3$**

Q8: **Find X and Y if** $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$,

$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

Solution:

$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ **………… equ (i)**

$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ **………… equ (ii)**

Add equ (i) and equ (ii)

$$X + Y + X - Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$X + X + Y - Y = \begin{bmatrix} 5+3 & 2+6 \\ 0+0 & 9-1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\frac{1}{2} \times 2X = \frac{1}{2} \times \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 8\times\frac{1}{2} & 8\times\frac{1}{2} \\ 0\times\frac{1}{2} & 8\times\frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

Put the values of X in equ (i)

$$\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5-4 & 2-4 \\ 0-0 & 9-4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Thus

$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$ **and** $Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

Q9: **Let** $A=\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 5\\ -1 & 3\end{bmatrix}$ **and** $C=\begin{bmatrix}3 & -1\\ 0 & 4\end{bmatrix}$.

If $c=2$ and $d=-4$ then verify that:

$$A=\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}, B=\begin{bmatrix}2 & 5\\ -1 & 3\end{bmatrix} \text{ and } C=\begin{bmatrix}3 & -1\\ 0 & 4\end{bmatrix}$$

$c=2,\ d=-4$

(i) $(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}=\boldsymbol{cA}+\boldsymbol{dA}$

Solution:

L.H.S:

$(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}$

$$(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}=(2+(-4))\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}=(2-4)\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}=-2\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$(\boldsymbol{c}+\boldsymbol{d})\boldsymbol{A}=\begin{bmatrix}-4 & 6\\ -8 & -10\end{bmatrix}$$

RHS

$\boldsymbol{cA}+\boldsymbol{dA}$

$$\boldsymbol{cA}+\boldsymbol{dA}=2\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}+(-4)\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$\boldsymbol{cA}+\boldsymbol{dA}=\begin{bmatrix}4 & -6\\ 8 & 10\end{bmatrix}+\begin{bmatrix}-8 & 12\\ -16 & -20\end{bmatrix}$$

$$\boldsymbol{cA}+\boldsymbol{dA}=\begin{bmatrix}4-8 & -6+12\\ 8-16 & 10-20\end{bmatrix}$$

$$\boldsymbol{cA}+\boldsymbol{dA}=\begin{bmatrix}-4 & 6\\ -8 & -10\end{bmatrix}$$

Hence $(c+d)A=cA+dA$

(ii) $\boldsymbol{c}(\boldsymbol{A}+\boldsymbol{B})=\boldsymbol{cA}+\boldsymbol{cB}$

Solution:

LHS

$c(A+B)$

$$c(A+B)=2\left(\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}+\begin{bmatrix}2 & 5\\ -1 & 3\end{bmatrix}\right)$$

$$c(A+B)=2\begin{bmatrix}2+2 & -3+5\\ 4-1 & 5+3\end{bmatrix}$$

$$c(A+B)=2\begin{bmatrix}4 & 2\\ 3 & 8\end{bmatrix}$$

$$c(A+B)=\begin{bmatrix}8 & 4\\ 6 & 16\end{bmatrix}$$

RHS

$$cA+cB=2\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}+2\begin{bmatrix}2 & 5\\ -1 & 3\end{bmatrix}$$

$$cA+cB=\begin{bmatrix}4 & -6\\ 8 & 10\end{bmatrix}+\begin{bmatrix}4 & 10\\ -2 & 6\end{bmatrix}$$

$$cA+cB=\begin{bmatrix}4+4 & -6+10\\ 8-2 & 10+6\end{bmatrix}$$

$$cA+cB=\begin{bmatrix}8 & 4\\ 6 & 16\end{bmatrix}$$

Hence $\boldsymbol{c}(\boldsymbol{A}+\boldsymbol{B})=\boldsymbol{cA}+\boldsymbol{cB}$ **Proved**

(iii) $\boldsymbol{cd}(\boldsymbol{A})=\boldsymbol{c}(\boldsymbol{dA})$

Solution:

$cd(A)=c(dA)$

LHS

$cd(A)$

$$cd(A)=(2)(-4)\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$cd(A)=-8\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}$$

$$cd(A)=\begin{bmatrix}-16 & 24\\ -32 & -40\end{bmatrix}$$

LHS

$c(dA)$

$$c(dA)=2\left(-4\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}\right)$$

$$c(dA)=2\begin{bmatrix}-8 & 12\\ -16 & -20\end{bmatrix}$$

$$c(dA)=\begin{bmatrix}-16 & 24\\ -32 & -40\end{bmatrix}$$ **Hence** $cd(A)=c(dA)$

Q10 **Let** $A=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}$, $B=\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}$ **and** $C=\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$. **Compute the following if possible.**

(i) $\boldsymbol{A+2B}$

Solution:

$$A+2B=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+2\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}$$

$$A+2B=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}6&-2&4\\-10&6&8\\-6&-8&0\end{bmatrix}$$

$$A+2B=\begin{bmatrix}-1+6&2-2&3+4\\4-10&2+6&0+8\\-3-6&2-8&5+0\end{bmatrix}$$

$$A+2B=\begin{bmatrix}5&0&7\\-6&8&8\\-9&-6&5\end{bmatrix}$$

(ii) $\boldsymbol{3A-4B}$

Solution:

$$3A-4B=3\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}-4\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}-3&6&9\\12&6&0\\-9&6&15\end{bmatrix}-\begin{bmatrix}12&-4&8\\-20&12&16\\-12&-16&0\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}-3-12&6+4&9-8\\12+20&6-12&0-16\\-9+12&6+16&15-0\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}-15&10&1\\32&-6&-16\\3&22&15\end{bmatrix}$$

(iii) $\boldsymbol{(A+B)-C}$

Solution:

$$(A+B)-C=\left(\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}\right)-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}-1+3&2-1&3+2\\4-5&2+3&0+4\\-3-3&2-4&5+0\end{bmatrix}-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}2&1&5\\-1&5&4\\-6&-2&5\end{bmatrix}-\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}2-2&1+3&5-6\\-1-0&5-4&4+1\\-6+5&-2-1&5-3\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}0&4&-1\\-1&1&5\\-1&-3&2\end{bmatrix}$$

(iv) $\boldsymbol{A+(B+C)}$

Solution:

$$A+(B+C)=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\left(\begin{bmatrix}3&-1&2\\-5&3&4\\-3&-4&0\end{bmatrix}+\begin{bmatrix}2&-3&6\\0&4&-1\\-5&1&3\end{bmatrix}\right)$$

$$A+(B+C)=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}3+2&-1-3&2+6\\-5+0&3+4&4-1\\-3-5&-4+1&0+3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}-1&2&3\\4&2&0\\-3&2&5\end{bmatrix}+\begin{bmatrix}5&-4&8\\-5&7&3\\-8&-3&3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}-1+5&2-4&3+8\\4-5&2+7&0+3\\-3-8&2-3&5+3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}4&-2&11\\-1&9&3\\-11&-1&8\end{bmatrix}$$

Q11 **Prove that the following matrices commutative law of addition holds.**

$$A=\begin{bmatrix}7 & 1\\ 2 & 4\end{bmatrix},\quad B=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$$

Solution:

(i) $$A=\begin{bmatrix}7 & 1\\ 2 & 4\end{bmatrix},\quad B=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$$

$\boldsymbol{A+B=B+A}$

LHS

$$A+B=\begin{bmatrix}7 & 1\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$$

$$A+B=\begin{bmatrix}7+1 & 1+1\\ 2+2 & 4+2\end{bmatrix}$$

$$A+B=\begin{bmatrix}8 & 2\\ 4 & 6\end{bmatrix}$$

RHS

$$B+A=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}+\begin{bmatrix}7 & 1\\ 2 & 4\end{bmatrix}$$

$$B+A=\begin{bmatrix}1+7 & 1+1\\ 2+2 & 2+4\end{bmatrix}$$

$$B+A=\begin{bmatrix}8 & 2\\ 4 & 6\end{bmatrix}$$ **Hence $\boldsymbol{A+B=B+A}$ Proved**

(ii) $$C=\begin{bmatrix}-3 & 4 & -5\\ 2 & 3 & 1\end{bmatrix},\quad D=\begin{bmatrix}-3 & -4 & 5\\ 1 & 2 & 3\end{bmatrix}$$

Solution:

$$C=\begin{bmatrix}-3 & 4 & -5\\ 2 & 3 & 1\end{bmatrix},\quad D=\begin{bmatrix}-3 & -4 & 5\\ 1 & 2 & 3\end{bmatrix}$$

$\boldsymbol{C+D=D+C}$

LHS:

$$C+D=\begin{bmatrix}-3 & 4 & -5\\ 2 & 3 & 1\end{bmatrix}+\begin{bmatrix}-3 & -4 & 5\\ 1 & 2 & 3\end{bmatrix}$$

$$C+D=\begin{bmatrix}-3-3 & 4-4 & -5+5\\ 2+1 & 3+2 & 1+3\end{bmatrix}$$

$$C+D=\begin{bmatrix}-6 & 0 & 0\\ 3 & 5 & 4\end{bmatrix}$$

RHS

$$D+C=\begin{bmatrix}-3 & -4 & 5\\ 1 & 2 & 3\end{bmatrix}+\begin{bmatrix}-3 & 4 & -5\\ 2 & 3 & 1\end{bmatrix}$$

$$D+C=\begin{bmatrix}-3-3 & -4+4 & 5-5\\ 1+2 & 2+3 & 3+1\end{bmatrix}$$

$$D+C=\begin{bmatrix}-6 & 0 & 0\\ 3 & 5 & 4\end{bmatrix}$$

Hence $C+D=D+C$ **Proved**

Q12: **Verify $\boldsymbol{A+(B+C)=(A+B)+C}$ for the following matrices.**

(i) $$A=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix},\ B=\begin{bmatrix}5 & -2\\ 3 & 6\end{bmatrix},\ C=\begin{bmatrix}1 & 7\\ -6 & -3\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix},\ B=\begin{bmatrix}5 & -2\\ 3 & 6\end{bmatrix},\ C=\begin{bmatrix}1 & 7\\ -6 & -3\end{bmatrix}$$

$\boldsymbol{A+(B+C)=(A+B)+C}$

LHS: $A+(B+C)$

$$B+C=\begin{bmatrix}5 & -2\\ 3 & 6\end{bmatrix}+\begin{bmatrix}1 & 7\\ -6 & -3\end{bmatrix}$$

$$B+C=\begin{bmatrix}5+1 & -2+7\\ 3-6 & 6-3\end{bmatrix}$$

$$B+C=\begin{bmatrix}6 & 5\\ -3 & 3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}+\begin{bmatrix}6 & 5\\ -3 & 3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2+6 & -3+5\\ 4-3 & 1+3\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}8 & 2\\ 1 & 4\end{bmatrix}$$

RHS: $(A+B)+C$

$$A+B=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}+\begin{bmatrix}5 & -2\\ 3 & 6\end{bmatrix}$$

$$A+B=\begin{bmatrix}2+5 & -3-2\\ 4+3 & 1+6\end{bmatrix}$$

$$A+B=\begin{bmatrix}7 & -5\\ 7 & 7\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}7 & -5\\ 7 & 7\end{bmatrix}+\begin{bmatrix}1 & 7\\ -6 & -3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}7+1 & -5+7\\ 7-6 & 7-3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}8 & 2\\ 1 & 4\end{bmatrix}$$

Hence $A+(B+C)=(A+B)+C$ **Proved**

(ii) $A=\begin{bmatrix}a & b & c\\ 3 & 4 & 5\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 3\\ -2 & 1 & 4\end{bmatrix},$

$$C=\begin{bmatrix}2 & 1 & -1\\ 3 & 1 & -2\end{bmatrix}$$

Solution

$$A=\begin{bmatrix}a & b & c\\ 3 & 4 & 5\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 3\\ -2 & 1 & 4\end{bmatrix},$$

$$C=\begin{bmatrix}2 & 1 & -1\\ 3 & 1 & -2\end{bmatrix}$$

$\boldsymbol{A+(B+C)=(A+B)+C}$

LHS: $A+(B+C)$

$$B+C=\begin{bmatrix}1 & 2 & 3\\ -2 & 1 & 4\end{bmatrix}+\begin{bmatrix}2 & 1 & -1\\ 3 & 1 & -2\end{bmatrix}$$

$$B+C=\begin{bmatrix}1+2 & 2+1 & 3-1\\ -2+3 & 1+1 & 4-2\end{bmatrix}$$

$$B+C=\begin{bmatrix}3 & 3 & 2\\ 1 & 2 & 2\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a & b & c\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}3 & 3 & 2\\ 1 & 2 & 2\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a+3 & b+3 & c+2\\ 3+1 & 4+2 & 5+2\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}a+3 & b+3 & c+2\\ 4 & 6 & 7\end{bmatrix}$$

RHS: $(A+B)+C$

$$A+B=\begin{bmatrix}a & b & c\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ -2 & 1 & 4\end{bmatrix}$$

$$A+B=\begin{bmatrix}a+1 & b+2 & c+3\\ 3-2 & 4+1 & 5+4\end{bmatrix}$$

$$A+B=\begin{bmatrix}a+1 & b+2 & c+3\\ 1 & 5 & 9\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+1 & b+2 & c+3\\ 1 & 5 & 9\end{bmatrix}+\begin{bmatrix}2 & 1 & -1\\ 3 & 1 & -2\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+1+2 & b+2+1 & c+3-1\\ 1+3 & 5+1 & 9-2\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}a+3 & b+3 & c+2\\ 4 & 6 & 7\end{bmatrix}$$

Hence $\boldsymbol{A+(B+C)=(A+B)+C}$ **Proved**

Q13: Find the additive inverse of the following matrices.

(i) $A=\begin{bmatrix}3 & 4\\ 6 & 2\end{bmatrix}$

Additive Inverse:

$$-A=\begin{bmatrix}-3 & -4\\ -6 & -2\end{bmatrix}$$

(ii) $B=\begin{bmatrix}a & -a & b\\ -c & a & -b\\ l & m & n\end{bmatrix}$

Additive Inverse:

$$-B=\begin{bmatrix}-a & a & -b\\ c & -a & b\\ -l & -m & -c\end{bmatrix}$$

Q14: Show that the following matrices are additive inverse of the each other.

(i) $A=\begin{bmatrix}1 & -2 & 3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & -3\end{bmatrix}$

$$A+B=\begin{bmatrix}1 & -2 & 3\end{bmatrix}+\begin{bmatrix}-1 & 2 & -3\end{bmatrix}$$

$$A+B=\begin{bmatrix}1-1 & -2+2 & 3-3\end{bmatrix}$$

$$A+B=\begin{bmatrix}0 & 0 & 0\end{bmatrix}$$

Ex # 1.3

(ii) $C=\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}, D=\begin{bmatrix} -a & b \\ c & -d \end{bmatrix}$

$$C+D=\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}+\begin{bmatrix} -a & b \\ c & -d \end{bmatrix}$$

$$C+D=\begin{bmatrix} a-a & -b+b \\ -c+c & d-d \end{bmatrix}$$

$$C+D=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(iii) $E=\begin{bmatrix} 1 & -2 & -4 \\ 2 & 1 & 3 \\ -3 & 4 & -2 \end{bmatrix}, F=\begin{bmatrix} -1 & 2 & 4 \\ -2 & -1 & -3 \\ 3 & -4 & 2 \end{bmatrix}$

$$E+F=\begin{bmatrix} 1 & -2 & -4 \\ 2 & 1 & 3 \\ -3 & 4 & -2 \end{bmatrix}+\begin{bmatrix} -1 & 2 & 4 \\ -2 & -1 & -3 \\ 3 & -4 & 2 \end{bmatrix}$$

$$E+F=\begin{bmatrix} 1-1 & -2+2 & -4+4 \\ 2-2 & 1-1 & 3-3 \\ -3+3 & 4-4 & -2+2 \end{bmatrix}$$

$$E+F=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Exercise # 1.4

Conformability for multiplication of matrices

Two matrices are conformable for multiplication, when number of columns of first matrix is equal to number of rows of second matrix.

Multiplication of Matrices

For multiplication, multiply each element of a row of first matrix by the corresponding element of column of second matrix and then add these products.

OR

Multiply first row of the matrix A with each corresponding elements of the first column of the matrix B and then add these products.

Commutative Law of Multiplication

Commutative law of multiplication of matrices may or may not be holds.

(i) $AB \neq BA$ (Mostly

(ii) $AB = BA$

Associative Law under Multiplication

$A(BC) = (AB)C$ is called Associative law of matrices under multiplication

Distributive Las of Multiplication over Addition

$$A(B+C) = AB + AC$$

$$(A+B)C = AC + BC$$

Multiplicative Identity of a Matrix

Any matrix multiplied with Identity matrix will be the same matrix. e.g. $A.I = I.A = A$

Transpose of a Matrix

A matrix obtained by interchanging all rows and column with each other is called transpose of a matrix. The transpose of a matrix B is written as B^t.

Note:

$$(AB)^t = B^t A^t$$

Exercise # 1.4

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Q1: Show that which of the following matrices are conformable for multiplication.

$$A=\begin{bmatrix} a \\ b \end{bmatrix}, B=\begin{bmatrix} p & q \end{bmatrix}, C=\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} D=\begin{bmatrix} p & r & s \end{bmatrix}$$

(i) AB

As number of Columns in matrix A =1
And number of Rows in matrix $B = 1$
Thus AB is conformable for multiplication.

(ii) AC

As number of Columns in matrix A =1
And number of Rows in matrix $C = 2$
Thus AC is not conformable for multiplication.

(iii) AD

As number of Columns in matrix A =1
And number of Rows in matrix $D = 1$
Thus AD is conformable for multiplication.

(iv) BA

As number of Columns in matrix $B = 2$
And number of Rows in matrix $A = 2$
Thus BA is conformable for multiplication.

(vi) BC

As number of Columns in matrix $B = 2$
And number of Rows in matrix $C = 2$
Thus BC is conformable for multiplication.

(vii) BD

As number of Columns in matrix $B = 2$
And number of Rows in matrix D= 1
Thus BD is not conformable for multiplication.

(viii) CA

As number of Columns in matrix $C = 2$
And number of Rows in matrix $A = 2$
Thus CA is conformable for multiplication.

(ix) CB

As number of Columns in matrix C =2
And number of Rows in matrix $B = 1$
Thus CB is not conformable for multiplication.

(x) CD

As number of Columns in matrix $C = 2$
And number of Rows in matrix $D = 1$
Thus CD is not conformable for multiplication.

(xi) DA

As number of Columns in matrix $D = 3$
And number of Rows in matrix $A = 2$
Thus DA is not conformable for multiplication.

xii) DB

As number of Columns in matrix $D = 3$
And number of Rows in matrix $B = 1$
Thus DB is not conformable for multiplication.

xiii) DC

As number of Columns in matrix $D = 3$
And number of Rows in matrix $C = 2$
Thus DC is not conformable for multiplication.

Q2: If $A=\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(i) Is it possible to find AB?
(ii) Is it possible to find BA?
(iii) Find the possible product/ products.

Solution:

$$A=\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

(i) AB

As number of Columns in matrix $A = 2$
And number of Rows in matrix $B = 2$
Thus AB is possible for multiplication.

(ii) BA

As number of Columns in matrix $B = 1$
And number of Rows in matrix $A = 2$
Thus BA is not possible for multiplication.

Ex # 1.4

(iii) Now

$$AB=\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}3\\ -2\end{bmatrix}$$

$$AB=\begin{bmatrix}(-1)(3)+(0)(-2)\\ (2)(3)+(1)(-2)\end{bmatrix}$$

$$AB=\begin{bmatrix}-3+0\\ 6+(-2)\end{bmatrix}$$

$$AB=\begin{bmatrix}-3\\ 6-2\end{bmatrix}$$

$$AB=\begin{bmatrix}-3\\ 4\end{bmatrix}$$

Q3: $A=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$, $C=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$ **and**

$D=\begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{2}{3}\end{bmatrix}$ **Find (i) AB and (ii) CD**

Solution:

(i $$AB=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$$

$$AB=\begin{bmatrix}(4)(1)+(1)(-3) & (4)(-1)+(1)(4)\\ (3)(1)+(1)(-3) & (3)(-1)+(1)(4)\end{bmatrix}$$

$$AB=\begin{bmatrix}4+(-3) & -4+4\\ 3+(-3) & -3+4\end{bmatrix}$$

$$AB=\begin{bmatrix}4-3 & 0\\ 3-3 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

(ii) $$CD=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{2}{3}\end{bmatrix}$$

$$CD=\begin{bmatrix}(3)(1)+(4)\left(-\frac{1}{2}\right) & (3)(-2)+(4)\left(\frac{2}{3}\right)\\ (1)(1)+(2)\left(-\frac{1}{2}\right) & (1)(-2)+(2)\left(\frac{2}{3}\right)\end{bmatrix}$$

$$CD=\begin{bmatrix}3+(2)(-1) & -6+\frac{8}{3}\\ 1+(1)(-1) & -2+\frac{4}{3}\end{bmatrix}$$

$$CD=\begin{bmatrix}3-2 & \frac{-18+8}{3}\\ 1-1 & \frac{-6+4}{3}\end{bmatrix}$$

$$CD=\begin{bmatrix}1 & \frac{-10}{3}\\ 0 & \frac{-2}{3}\end{bmatrix}$$

Q4: **Given that** $A=\begin{bmatrix}2 & 1\\ 3 & 0\\ -1 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$ **(i) Find AB**

(ii) Does BA exist?

Solution:

(i) $$AB=\begin{bmatrix}2 & 1\\ 3 & 0\\ -1 & 4\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}(2)(1)+(1)(2) & (2)(0)+(1)(1)\\ (3)(1)+(0)(2) & (3)(0)+(0)(1)\\ (-1)(1)+(4)(2) & (-1)(0)+(4)(1)\end{bmatrix}$$

$$AB=\begin{bmatrix}2+2 & 0+1\\ 3+0 & 0+0\\ -1+8 & 0+4\end{bmatrix} \qquad \rightarrow AB=\begin{bmatrix}4 & 1\\ 3 & 0\\ 7 & 4\end{bmatrix}$$

(ii) Does BA exists?

BA

As number of Columns in matrix $B = 2$

And number of Rows in matrix $A = 3$

Thus BA is not possible for multiplication.

Q5: If $A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$**,** $B=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ **then show that**

AB$\neq BA$

Solution:

$$A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$

AB$\neq BA$

LHS:

$$AB=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}(1)(0)+(1)(0) & (1)(1)+(1)(0)\\(0)(0)+(0)(0) & (0)(1)+(0)(0)\end{bmatrix}$$

$$AB=\begin{bmatrix}0+0 & 1+0\\0+0 & 0+0\end{bmatrix}$$

$$AB=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$

RHS

$$BA=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$$

$$BA=\begin{bmatrix}0+0 & 0+0\\0+0 & 0+0\end{bmatrix}$$

$$BA=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Hence

AB$\neq BA$ **Proved:**

Ex # 1.4

Q6: If $A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$, then find $A \times A$

Solution:

$$A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$$

$$A\times A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$$

$$A\times A=\begin{bmatrix}(1)(1)+(1)(0) & (1)(1)+(1)(0)\\(0)(1)+(0)(0) & (0)(1)+(0)(0)\end{bmatrix}$$

$$A\times A=\begin{bmatrix}1+0 & 1+0\\0+0 & 0+0\end{bmatrix}$$

$$A\times A=\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$$

Q7: **If** $A=\begin{bmatrix}-2 & 3\\2 & -1\end{bmatrix}$**,** $B=\begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$**. Is** $AB = BA$

Solution:

$$AB=\begin{bmatrix}-2 & 3\\2 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$$

$$AB=\begin{bmatrix}(-2)(1)+(3)(2) & (-2)(-1)+(3)(4)\\(2)(1)+(-1)(2) & (2)(-1)+(-1)(4)\end{bmatrix}$$

$$AB=\begin{bmatrix}-2+6 & 2+12\\2-2 & -2-4\end{bmatrix}$$

$$AB=\begin{bmatrix}4 & 14\\0 & -6\end{bmatrix}$$

Now

$$BA=\begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}\begin{bmatrix}-2 & 3\\2 & -1\end{bmatrix}$$

$$BA=\begin{bmatrix}(1)(-2)+(-1)(2) & (1)(3)+(-1)(-1)\\(2)(-2)+(4)(2) & (2)(3)+(4)(-1)\end{bmatrix}$$

$$BA=\begin{bmatrix}-2-2 & 3+1\\-4+8 & 6-4\end{bmatrix}$$

$$BA=\begin{bmatrix}-4 & 4\\4 & 2\end{bmatrix}$$

Hence AB is not equal to BA

Q8: **If** $A=\begin{bmatrix}-1\\1\end{bmatrix}, B=\begin{bmatrix}1&-2\end{bmatrix}, C=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$, **then**

(i) **find (AB)C and A(BC)**

Solution:

$$A=\begin{bmatrix}-1\\1\end{bmatrix}, B=\begin{bmatrix}1&-2\end{bmatrix}, C=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$$

<u>(AB)C:</u>

$$AB=\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}1&-2\end{bmatrix}$$

$$AB=\begin{bmatrix}(-1)(1)&(-1)(-2)\\(1)(1)&(1)(-2)\end{bmatrix}$$

$$AB=\begin{bmatrix}-1&2\\1&-2\end{bmatrix}$$

Now

$$(AB)C=\begin{bmatrix}-1&2\\1&-2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}(-1)(3)+(2)(-1)&(-1)(1)+(2)(2)\\(1)(3)+(-2)(-1)&(1)(1)+(-2)(2)\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-3-2&-1+4\\3+2&1-4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-5&3\\5&-3\end{bmatrix}$$

<u>A(BC):</u>

$$BC=\begin{bmatrix}1&-2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}$$

$$BC=\begin{bmatrix}(1)(3)+(-2)(-1)&(1)(1)+(-2)(2)\end{bmatrix}$$

$$BC=\begin{bmatrix}3+2&1-4\end{bmatrix}$$

$$BC=\begin{bmatrix}5&-3\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-1\\1\end{bmatrix}\begin{bmatrix}5&-3\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}(-1)(5)&(-1)(-3)\\(1)(5)&(1)(-3)\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-5&3\\5&-3\end{bmatrix}$$

(ii) **Determine whether $(\boldsymbol{AB})\boldsymbol{C}=\boldsymbol{A}(\boldsymbol{BC})$**

Yes

$$(AB)C=\begin{bmatrix}-5&3\\5&-3\end{bmatrix}=A(BC)=\begin{bmatrix}-5&3\\5&-3\end{bmatrix}$$

(iii) **Interpret which law of multiplication this result shows?**

This shows Associative Property of Multiplication

Q9: **Verify that $(\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{AB}+\boldsymbol{AC}$ for the following matrices.**

(i) $A=\begin{bmatrix}1&2\\3&-1\end{bmatrix}, B=\begin{bmatrix}1&0\\0&2\end{bmatrix}, C=\begin{bmatrix}3&-1\\0&2\end{bmatrix}$

Solution:

$$A=\begin{bmatrix}1&2\\3&-1\end{bmatrix}, B=\begin{bmatrix}1&0\\0&2\end{bmatrix}, C=\begin{bmatrix}3&-1\\0&2\end{bmatrix}$$

$$A(B+C)=AB+AC$$

LHS: $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})$

Now

$$B+C=\begin{bmatrix}1&0\\0&2\end{bmatrix}+\begin{bmatrix}3&-1\\0&2\end{bmatrix}$$

$$B+C=\begin{bmatrix}1+3&0-1\\0+0&2+2\end{bmatrix}$$

$$B+C=\begin{bmatrix}4&-1\\0&4\end{bmatrix}$$

Now

$$A(B+C)=\begin{bmatrix}1&2\\3&-1\end{bmatrix}\begin{bmatrix}4&-1\\0&4\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}(1)(4)+(2)(0)&(1)(-1)+(2)(4)\\(3)(4)+(-1)(0)&(3)(-1)+(-1)(4)\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}4+0&-1+8\\12+0&-3-4\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}4&7\\12&-7\end{bmatrix}$$

Ex # 1.4

RHS: $AB + AC$

Now

$$AB=\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$

$$AB=\begin{bmatrix}(1)(1)+(2)(0) & (1)(0)+(2)(2)\\(3)(1)+(-1)(0) & (3)(0)+(-1)(2)\end{bmatrix}$$

$$AB=\begin{bmatrix}1+0 & 0+4\\3+0 & 0-2\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 4\\3 & -2\end{bmatrix}$$

Now

$$AC=\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}$$

$$AC=\begin{bmatrix}(1)(3)+(2)(0) & (1)(-1)+(2)(2)\\(3)(3)+(-1)(0) & (3)(-1)+(-1)(2)\end{bmatrix}$$

$$AC=\begin{bmatrix}3+0 & -1+4\\9+0 & -3-2\end{bmatrix}$$

$$AC=\begin{bmatrix}3 & 3\\9 & -5\end{bmatrix}$$

Now

$$AB+AC=\begin{bmatrix}1 & 4\\3 & -2\end{bmatrix}+\begin{bmatrix}3 & 3\\9 & -5\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1+3 & 4+3\\3+9 & -2-5\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}4 & 7\\12 & -7\end{bmatrix}$$

Hence $A(B+C)=AB+AC$ **Proved:**

(ii) $A=\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}, B=\begin{bmatrix}1\\2\end{bmatrix}, C=\begin{bmatrix}-1\\1\end{bmatrix}$

Solution

$$A=\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}, B=\begin{bmatrix}1\\2\end{bmatrix}, C=\begin{bmatrix}-1\\1\end{bmatrix}$$

$$\boldsymbol{A(B+C)=AB+AC}$$

LHS: $A(B+C)$

Now

$$B+C=\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}-1\\1\end{bmatrix}$$

$$B+C=\begin{bmatrix}1-1\\2+1\end{bmatrix}$$

$$B+C=\begin{bmatrix}0\\3\end{bmatrix}$$

Now

$$A(B+C)=\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}0\\3\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}(3)(0)+(-1)(3)\\(0)(0)+(2)(3)\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}0-3\\0+6\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}-3\\6\end{bmatrix}$$

RHS: $AB + AC$

$$AB=\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$AB=\begin{bmatrix}(3)(1)+(-1)(2)\\(0)(1)+(2)(2)\end{bmatrix}$$

$$AB=\begin{bmatrix}3-2\\0+4\end{bmatrix}$$

$$AB=\begin{bmatrix}1\\4\end{bmatrix}$$

$$AC=\begin{bmatrix}3 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}$$

$$AC=\begin{bmatrix}(3)(-1)+(-1)(1)\\(0)(-1)+(2)(1)\end{bmatrix}$$

$$AC=\begin{bmatrix}-3-1\\0+2\end{bmatrix}$$

$$AC=\begin{bmatrix}-4\\2\end{bmatrix}$$

Now

$$AB+AC=\begin{bmatrix}1\\4\end{bmatrix}+\begin{bmatrix}-4\\2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1-4\\4+2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}-3\\6\end{bmatrix}$$

Hence $A(B+C)=AB+AC$ **Proved:**

Q10: Let $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, A=\begin{bmatrix}5 & -3\\4 & 6\end{bmatrix}, B=\begin{bmatrix}-7 & 3\\2 & 8\end{bmatrix}$

(i) AI

Solution:

$$AI=\begin{bmatrix}5 & -3\\4 & 6\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$AI=\begin{bmatrix}(5)(1)+(-3)(0) & (5)(0)+(-3)(1)\\(4)(1)+(6)(0) & (4)(0)+(6)(1)\end{bmatrix}$$

$$AI=\begin{bmatrix}5+0 & 0-3\\4+0 & 0+6\end{bmatrix}$$

$$AI=\begin{bmatrix}5 & -3\\4 & 6\end{bmatrix}=A$$

(ii) BI

Solution:

$$BI=\begin{bmatrix}-7 & 3\\2 & 8\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$BI=\begin{bmatrix}(-7)(1)+(3)(0) & (-7)(0)+(3)(1)\\(2)(1)+(8)(0) & (2)(0)+(8)(1)\end{bmatrix}$$

$$BI=\begin{bmatrix}-7+0 & 0+3\\2+0 & 0+8\end{bmatrix}$$

$$BI=\begin{bmatrix}-7 & 3\\2 & 8\end{bmatrix}=\boldsymbol{B}$$

Q11: Let $A=\begin{bmatrix}3 & 2 & 1\end{bmatrix}, B=\begin{bmatrix}-3 & 4 & 2\end{bmatrix}$, then prove that

(i) $(\boldsymbol{A}+\boldsymbol{B})^t=\boldsymbol{A}^t+\boldsymbol{B}^t$

LHS: $(\boldsymbol{A}+\boldsymbol{B})^t$

$$A+B=\begin{bmatrix}3 & 2 & 1\end{bmatrix}+\begin{bmatrix}-3 & 4 & 2\end{bmatrix}$$

$$A+B=\begin{bmatrix}3-3 & 2+4 & 1+2\end{bmatrix}$$

$$A+B=\begin{bmatrix}0 & 6 & 3\end{bmatrix}$$

Now

$$(A+B)^t=\begin{bmatrix}0\\6\\3\end{bmatrix}$$

RHS: $\boldsymbol{A}^t+\boldsymbol{B}^t$

As $A^t=\begin{bmatrix}3\\2\\1\end{bmatrix}$

And $B^t=\begin{bmatrix}-3\\4\\2\end{bmatrix}$

Now

$$A^t+B^t=\begin{bmatrix}3\\2\\1\end{bmatrix}+\begin{bmatrix}-3\\4\\2\end{bmatrix}$$

$$A^t+B^t=\begin{bmatrix}3-3\\2+4\\1+2\end{bmatrix}$$

$$A^t+B^t=\begin{bmatrix}0\\6\\3\end{bmatrix}$$

Hence $(\boldsymbol{A}+\boldsymbol{B})^t=\boldsymbol{A}^t+\boldsymbol{B}^t$ **Proved:**

$(\boldsymbol{A}-\boldsymbol{B})^t=\boldsymbol{A}^t-\boldsymbol{B}^t$

LHS: $(\boldsymbol{A}-\boldsymbol{B})^t$

$$A-B=\begin{bmatrix}3 & 2 & 1\end{bmatrix}-\begin{bmatrix}-3 & 4 & 2\end{bmatrix}$$

$$A-B=\begin{bmatrix}3+3 & 2-4 & 1-2\end{bmatrix}$$

$$A-B=\begin{bmatrix}6 & -2 & -1\end{bmatrix}$$

Now

$$(A-B)^t=\begin{bmatrix}6\\-2\\-1\end{bmatrix}$$

RHS: A^t-B^t

AS $A^t=\begin{bmatrix}3\\2\\1\end{bmatrix}$

And $B^t=\begin{bmatrix}-3\\4\\2\end{bmatrix}$

Now

$$A^t-B^t=\begin{bmatrix}3\\2\\1\end{bmatrix}-\begin{bmatrix}-3\\4\\2\end{bmatrix}$$

$$A^t-B^t=\begin{bmatrix}3+3\\2-4\\1-2\end{bmatrix}$$

$$A^t-B^t=\begin{bmatrix}6\\-2\\-1\end{bmatrix}$$

Hence $(A-B)^t=A^t-B^t$ **Proved:**

(ii) **If** $C=\begin{bmatrix}7&-3\\2&-1\end{bmatrix}$**,** $D=\begin{bmatrix}1&1\\2&2\end{bmatrix}$ **then prove that**

$(C+D)^t=C^t+D^t$

Solution:

LHS: $(C+D)^t$

$$C+D=\begin{bmatrix}7&-3\\2&-1\end{bmatrix}+\begin{bmatrix}1&1\\2&2\end{bmatrix}$$

$$C+D=\begin{bmatrix}7+1&-3+1\\2+2&-1+2\end{bmatrix}$$

$$C+D=\begin{bmatrix}8&-2\\4&1\end{bmatrix}$$

Now

$$(C+D)^t=\begin{bmatrix}8&4\\-2&1\end{bmatrix}$$

RHS: C^t+D^t

As $C^t=\begin{bmatrix}7&2\\-3&-1\end{bmatrix}$

And $D^t=\begin{bmatrix}1&2\\1&2\end{bmatrix}$

Now

$$C^t+D^t=\begin{bmatrix}7&2\\-3&-1\end{bmatrix}+\begin{bmatrix}1&2\\1&2\end{bmatrix}$$

$$C^t+D^t=\begin{bmatrix}7+1&2+2\\-3+1&-1+2\end{bmatrix}$$

$$C^t+D^t=\begin{bmatrix}8&4\\-2&1\end{bmatrix}$$

Hence $(C+D)^t=C^t+D^t$ **Proved:**

$(C-D)^t=C^t-D^t$

Solution:

LHS: $(C-D)^t$

$$C-D=\begin{bmatrix}7&-3\\2&-1\end{bmatrix}-\begin{bmatrix}1&1\\2&2\end{bmatrix}$$

$$C-D=\begin{bmatrix}7-1&-3-1\\2-2&-1-2\end{bmatrix}$$

$$C-D=\begin{bmatrix}6&-4\\0&-3\end{bmatrix}$$

Now

$$(C-D)^t=\begin{bmatrix}6&0\\-4&-3\end{bmatrix}$$

RHS: C^t+D^t

As $C^t=\begin{bmatrix}7&2\\-3&-1\end{bmatrix}$

And $D^t=\begin{bmatrix}1&2\\1&2\end{bmatrix}$

Now

$$C^t-D^t=\begin{bmatrix}7&2\\-3&-1\end{bmatrix}-\begin{bmatrix}1&2\\1&2\end{bmatrix}$$

$$C^t - D^t = \begin{bmatrix} 7-1 & 2-2 \\ -3-1 & -1-2 \end{bmatrix}$$

$$C^t - D^t = \begin{bmatrix} 6 & 0 \\ -4 & -3 \end{bmatrix}$$

Hence $(C-D)^t = C^t - D^t$ **Proved:**

Q12: (i) **If** $A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}$**,** $B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ **show that** $(AB)^t = B^t A^t$

Solution:

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$(AB)^t = B^t A^t$$

LHS: $(AB)^t$

$$AB = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (2)(-1)+(5)(2) & (2)(1)+(5)(3) \\ (-3)(-1)+(4)(2) & (-3)(1)+(4)(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+10 & 2+15 \\ 3+8 & -3+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 17 \\ 11 & 9 \end{bmatrix}$$

Now

$$(AB)^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix}$$

RHS: $B^t A^t$

As $A^t = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$

And $B^t = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

Now

$$B^t A^t = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} (-1)(2)+(2)(5) & (-1)(-3)+(2)(4) \\ (1)(2)+(3)(5) & (1)(-3)+(3)(4) \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} -2+10 & 3+8 \\ 2+15 & -3+12 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix}$$

Hence $(AB)^t = B^t A^t$ **Proved:**

(ii) **If** $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, show that** $\left(C^t\right)^t = C$

Solution:

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

By taking transpose, we get

$$C^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Now again take transpose, so we get

$$\left(C^t\right)^t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(iii) **If** $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}$**,** $B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$**, show that** $(AB)^t = A^t B^t$

Solution:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$$

$$(AB)^t = A^t B^t$$

LHS: $(AB)^t$

$$AB = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(1)+(0)(-8)+(-1)(0) & (1)(7)+(0)(4)+(-1)(1) \\ (2)(1)+(0)(-8)+(6)(0) & (2)(7)+(0)(4)+(6)(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+0+0 & 7+0-1 \\ 2+0+0 & 14+0+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 6 \\ 2 & 20 \end{bmatrix}$$

Now

$$(AB)^t=\begin{bmatrix}1 & 2\\ 6 & 20\end{bmatrix}$$

RHS: A^tB^t

As $A^t=\begin{bmatrix}1 & 2\\ 0 & 0\\ -1 & 6\end{bmatrix}$

And $B^t=\begin{bmatrix}1 & -8 & 0\\ 7 & 4 & 1\end{bmatrix}$

Now

$$A^tB^t=\begin{bmatrix}1 & 2\\ 0 & 0\\ -1 & 6\end{bmatrix}\begin{bmatrix}1 & -8 & 0\\ 7 & 4 & 1\end{bmatrix}$$

$$A^tB^t=\begin{bmatrix}(1)(1)+(2)(7) & (1)(-8)+(2)(4) & (1)(0)+(2)(1)\\ (0)(1)+(0)(7) & (0)(-8)+(0)(4) & (0)(0)+(0)(1)\\ (-1)(1)+(6)(7) & (-1)(-8)+(6)(4) & (-1)(0)+(6)(1)\end{bmatrix}$$

$$A^tB^t=\begin{bmatrix}1+14 & -8+8 & 0+2\\ 0+0 & 0+0 & 0+0\\ -1+42 & 8+24 & 0+6\end{bmatrix}$$

$$A^tB^t=\begin{bmatrix}15 & 0 & 2\\ 0 & 0 & 0\\ 41 & 32 & 6\end{bmatrix}$$

Hence $(AB)^t \neq A^tB^t$

Exercise # 1.5

Determinant of a Square Matrix

Determinant of A denoted by $|A|$ or det A.

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$$|A|=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$$

$$|A|=ad-cb$$

(i) Singular

If $|A|=0$ then A is Singular Matrix.

(ii) Non-Singular Matrix

If $|A|\neq 0$ then A is Non-Singular Matrix.

Adjoint of Square Matrix

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

As change the places of a and d with each other and change the size of b and c. So

$$AdjA=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

Multiplicative Inverse

If $AB = BA = I$ then A is the multiplicative inverse of B

For Non-Singular matrix,

$$A^{-1}=\frac{1}{|A|}\text{Adj A}$$

Let $A=\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}, B=\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$

Now

$$AB=\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}\begin{bmatrix}3 & -2\\ -4 & 3\end{bmatrix}$$

$$AB=\begin{bmatrix}(3)(3)+(2)(-4) & (3)(-2)+(2)(3)\\ (4)(3)+(3)(-4) & (4)(-2)+(3)(3)\end{bmatrix}$$

$$AB=\begin{bmatrix}9-8 & -6+6\\ 12-12 & -8+9\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Since $AB = I = BA$

Therefore, A is the inverse of B.

Verification of $AA^{-1} = I = A^{-1}A$

$$A=\begin{bmatrix}-2 & -1\\ 3 & 4\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\frac{-4}{5} & \frac{-1}{5}\\ \frac{3}{5} & \frac{2}{5}\end{bmatrix}$$

$$AA^{-1}=\begin{bmatrix}-2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}\frac{-4}{5} & \frac{-1}{5}\\ \frac{3}{5} & \frac{2}{5}\end{bmatrix}$$

$$AA^{-1}=\begin{bmatrix}(-2)\left(\frac{-4}{5}\right)+(-1)\left(\frac{3}{5}\right) & (-2)\left(\frac{-1}{5}\right)+(-1)\left(\frac{2}{5}\right)\\ (3)\left(\frac{-4}{5}\right)+(4)\left(\frac{3}{5}\right) & (3)\left(\frac{-1}{5}\right)+(4)\left(\frac{2}{5}\right)\end{bmatrix}$$

$$AA^{-1}=\begin{bmatrix}\frac{-8}{5}-\frac{3}{5} & \frac{2}{5}-\frac{2}{5}\\ \frac{-12}{5}+\frac{12}{5} & \frac{-3}{5}+\frac{8}{5}\end{bmatrix}$$

$$AA^{-1}=\begin{bmatrix}\frac{5}{5} & 0\\ 0 & \frac{5}{5}\end{bmatrix}$$

$$AA^{-1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Now

$$A^{-1}A=\begin{bmatrix}\frac{-4}{5} & \frac{-1}{5}\\ \frac{3}{5} & \frac{2}{5}\end{bmatrix}\begin{bmatrix}-2 & -1\\ 3 & 4\end{bmatrix}$$

$$A^{-1}A=\begin{bmatrix}\left(\frac{-4}{5}\right)(-2)+\left(\frac{-1}{5}\right)(3) & \left(\frac{-4}{5}\right)(-1)+\left(\frac{-1}{5}\right)(4)\\ \left(\frac{3}{5}\right)(-2)+\left(\frac{2}{5}\right)(3) & \left(\frac{3}{5}\right)(-1)+\left(\frac{2}{5}\right)(4)\end{bmatrix}$$

$$A^{-1}A=\begin{bmatrix}\frac{-8}{5}-\frac{3}{5} & \frac{4}{5}-\frac{4}{5}\\ \frac{-6}{5}+\frac{6}{5} & \frac{-3}{5}+\frac{8}{5}\end{bmatrix}$$

$$A^{-1}A=\begin{bmatrix}\frac{5}{5} & 0\\ 0 & \frac{5}{5}\end{bmatrix}$$

$$A^{-1}A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Thus $AA^{-1} = I = A^{-1}A$

Exercise # 1.5

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Q1: Find the determinant of the following matrices and evaluate them.

(i) $A=\begin{bmatrix}5 & 6\\ -4 & 1\end{bmatrix}$

Solution:

$$A=\begin{bmatrix}5 & 6\\ -4 & 1\end{bmatrix}$$

$$|A|=\begin{vmatrix}5 & 6\\ -4 & 1\end{vmatrix}$$

$$|A|=(5)(1)-(-4)(6)$$

$$|A|=5-(-24)$$

$$|A|=5+24$$

$$|A|=29$$

(ii) $B=\begin{bmatrix}4 & -2\\ 5 & 13\end{bmatrix}$

Solution:

$$B=\begin{bmatrix}4 & -2\\ 5 & 13\end{bmatrix}$$

$$|B|=\begin{vmatrix}4 & -2\\ 5 & 13\end{vmatrix} \longrightarrow$$

$$\boxed{\begin{aligned}|B|&=(4)(13)-(5)(-2)\\ |B|&=52-(-10)\\ |B|&=52+10\\ |B|&=62\end{aligned}}$$

(iii) $C=\begin{bmatrix}11 & 7\\ -6 & 5\end{bmatrix}$

Solution:

$$C=\begin{bmatrix}11 & 7\\ -6 & 5\end{bmatrix}$$

$$|C|=\begin{vmatrix}11 & 7\\ -6 & 5\end{vmatrix}$$

$$|C|=(11)(5)-(-6)(7)$$

$$|C|=55-(-42)$$

$$|C|=55+42$$

$$|C|=97$$

(iv) $D=\begin{bmatrix}5 & 6\\ -8 & -9\end{bmatrix}$

Solution:

$$D=\begin{bmatrix}5 & 6\\ -8 & -9\end{bmatrix}$$

$$|D|=\begin{vmatrix}5 & 6\\ -8 & -9\end{vmatrix}$$

$$|D|=(5)(-9)-(-8)(6)$$

$$|D|=-45-(-48)$$

$$|D|=-45+48$$

$$|D|=3$$

(V) $E=\begin{bmatrix}2p & -3q\\ r & -s\end{bmatrix}$

Solution:

$$E=\begin{bmatrix}2p & -3q\\ r & -s\end{bmatrix}$$

$$|E|=\begin{vmatrix}2p & -3q\\ r & -s\end{vmatrix}$$

$$|E|=(2p)(-s)-(r)(-3q)$$

$$|E|=-2ps-(-3qr)$$

$$|E|=-2ps+3qr$$

$$|E|=3qr-2ps$$

(vi) $F=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Solution:

$$F=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$|F|=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

Expand by Row 1:

$$|F|=1\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}-0\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix}+0\begin{vmatrix}0 & 1\\ 0 & 0\end{vmatrix}$$

$$|F|=1(1-0)-0+0$$

$$|F|=1(1)$$

$$|F|=1$$

(vii) $G=\begin{bmatrix}1 & 2 & 2\\ 3 & 2 & 3\\ -2 & -3 & 4\end{bmatrix}$

Solution:

$$G=\begin{bmatrix}1 & 2 & 2\\ 3 & 2 & 3\\ -2 & -3 & 4\end{bmatrix}$$

$$|G|=\begin{vmatrix}1 & 2 & 2\\ 3 & 2 & 3\\ -2 & -3 & 4\end{vmatrix}$$

$$|G|=1\begin{vmatrix}2 & 3\\ -3 & 4\end{vmatrix}-2\begin{vmatrix}3 & 3\\ -2 & 4\end{vmatrix}+2\begin{vmatrix}3 & 2\\ -2 & -3\end{vmatrix}$$

$$|G|=1(8-(-9))-2(12-(-6))+2(-9-(-4))$$

$$|G|=1(8+9)-2(12+6)+2(-9+4)$$

$$|G|=1(17)-2(18)+2(-5)$$

$$|G|=17-36-10$$

$$|G|=-29$$

(Viii) $H = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

Solution:

$$H = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$|H| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$|H| = a\begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & c \end{vmatrix} + 0\begin{vmatrix} 0 & b \\ 0 & 0 \end{vmatrix}$$

$$|H| = a(bc-0)-0+0$$

$$|H| = a(bc)$$

$$|H| = abc$$

Q2: Find which of the following matrices are singular ad which are non-singular.

(i) $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 5-6$$

$$|A| = -1 \neq 0$$

Thus A is a non-singular matrix.

(ii) $B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3 & -6 \\ -2 & 4 \end{vmatrix}$$

$$|B| = 12-12$$

$$|B| = 0$$

Thus B is a singular matrix.

(Iii) $C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$

Solution:

$$C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$$

$$|C| = \begin{vmatrix} 3a & -2b \\ 2a & b \end{vmatrix}$$

$$|C| = 3ab-(-4ab)$$

$$|C| = 3ab+4ab$$

$$|C| = 7ab \neq 0$$

Thus C is a non-singular matrix.

(iV) $D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$$

$$|D| = \begin{vmatrix} -3 & 6 \\ 2 & -4 \end{vmatrix}$$

$$|D| = 12-12$$

$$|D| = 0$$

Thus D is a singular matrix.

Q3: Find the adjoint of the following matrices.

(i) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$Adj\ \text{A} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

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(ii) $B=\begin{bmatrix}-3 & -1\\ 2 & 3\end{bmatrix}$

Solution:

$$B=\begin{bmatrix}-3 & -1\\ 2 & 3\end{bmatrix}$$

$$Adj\ \text{B}=\begin{bmatrix}3 & 1\\ -2 & -3\end{bmatrix}$$

(iii) $C=\begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$

Solution:

$$C=\begin{bmatrix}2 & -4\\ 3 & 1\end{bmatrix}$$

$$\textbf{Adj } \boldsymbol{C}=\begin{bmatrix}1 & 4\\ -3 & 2\end{bmatrix}$$

(iv) $D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$

Solution:

$$D=\begin{bmatrix}-3 & 6\\ 2 & -4\end{bmatrix}$$

$$\boldsymbol{Adj\ D}=\begin{bmatrix}-4 & -6\\ -2 & -3\end{bmatrix}$$

Q4: Find the multiplicative inverse of the following matrices if they exist.

(i) $A=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$

Solution:

$$A=\begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$$

$A^{-1}=\dfrac{1}{|A|}\text{Adj } A$ ……. **Equ (i)**

$$|A|=\begin{vmatrix}4 & 1\\ 3 & 1\end{vmatrix}$$

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$$|A|=4-3$$

$$|A|=-1\neq 0$$

$$\text{Adj A}=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$$

Put the values in equ (i)

$$A^{-1}=\frac{1}{1}\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}$$

(ii) $B=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$

Solution:

$$B=\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}$$

$B^{-1}=\dfrac{1}{|B|}\text{Adj } B$ ……. **Equ (i)**

$$|B|=\begin{vmatrix}3 & 4\\ 1 & 2\end{vmatrix}$$

$$|B|=6-4$$

$$|B|=2\neq 0$$

$$\text{Adj B}=\begin{bmatrix}2 & -4\\ -1 & 3\end{bmatrix}$$

Put the values in equ (i)

$$B^{-1}=\frac{1}{2}\begin{bmatrix}2 & -4\\ -1 & 3\end{bmatrix}$$

(iii) $C=\begin{bmatrix}4 & -3\\ -1 & 2\end{bmatrix}$

Solution:

$$C=\begin{bmatrix}4 & -3\\ -1 & 2\end{bmatrix}$$

$C^{-1}=\dfrac{1}{|C|}\text{adj } C$ ……. **Equ (i)**

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$$|C| = \begin{vmatrix} 4 & -3 \\ -1 & 2 \end{vmatrix}$$

$$|C| = 8 - 3$$

$$|C| = 5 \neq 0$$

$$\text{Adj C} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

Put the values in equ (i)

$$C^{-1} = \frac{1}{5}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

(iv) $D = \begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix}$$

$D^{-1} = \frac{1}{|D|} \text{adj } D$ ……. **Equ (i)**

$$|D| = \begin{vmatrix} 0 & -3 \\ 2 & 4 \end{vmatrix}$$

$$|D| = 0 - (-6)$$

$$|D| = 0 + 6$$

$$|D| = 6 \neq 0$$

$$\text{Adj D} = \begin{bmatrix} 4 & 3 \\ -2 & 0 \end{bmatrix}$$

Put the values in equ (i)

$$D^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 3 \\ -2 & 0 \end{bmatrix}$$

(v) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$I^{-1} = \frac{1}{|I|} \text{adj I}$ ……. **Equ (i)**

Ex # 1.5

$$|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$|I| = 1 - 0$$

$$|I| = 1 \neq 0$$

$$\text{Adj I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Put the values in equ (i)

$$I^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q5: If $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$, find

(i) **AB**

Solution:

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

Now

$$AB = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (2)(1)+(0)(-1) & (2)(-1)+(0)(3) \\ (-3)(1)+(1)(-1) & (-3)(-1)+(1)(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+0 & -2+0 \\ -3-1 & 3+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -2 \\ -4 & 6 \end{bmatrix}$$

(ii) BA

Solution:

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

Now

$$BA = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

$$BA=\begin{bmatrix}(1)(2)+(-1)(-3) & (1)(0)+(-1)(1)\\(-1)(2)+(3)(-3) & (-1)(0)+(3)(1)\end{bmatrix}$$

$$BA=\begin{bmatrix}2+3 & 0-1\\-2-9 & 0+3\end{bmatrix}$$

$$BA=\begin{bmatrix}5 & -1\\-11 & 3\end{bmatrix}$$

(iii) $\boldsymbol{A^{-1}}$ **and** $\boldsymbol{B^{-1}}$

Solution:

$\boldsymbol{A^{-1}}$

$$A=\begin{bmatrix}2 & 0\\-3 & 1\end{bmatrix}$$

$A^{-1}=\frac{1}{|A|}\text{adj } A$ **……. Equ (i)**

$$|A|=\begin{vmatrix}2 & 0\\-3 & 1\end{vmatrix}$$

$$|A|=2-0$$

$$|A|=2\neq 0$$

$$\text{Adj } A=\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$$

Put the values in equ (i)

$$A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$$

$\boldsymbol{B^{-1}}$

$$B=\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$$

$B^{-1}=\frac{1}{|B|}\text{Adj B}$ **……. Equ (i)**

$$|B|=\begin{vmatrix}1 & -1\\-1 & 3\end{vmatrix}$$

$$|B|=3-1$$

$$|B|=2\neq 0$$

$$\text{Adj } B=\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}$$

Put the values in equ (i)

$$B^{-1}=\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}$$

Show that $(\boldsymbol{AB})^{-1}=\boldsymbol{B^{-1}A^{-1}}$

Solution:

$(\boldsymbol{AB})^{-1}=\boldsymbol{B^{-1}A^{-1}}$

LHS: $(\boldsymbol{AB})^{-1}$

As $AB=\begin{bmatrix}2 & -2\\-4 & 6\end{bmatrix}$

So

$(AB)^{-1}=\frac{1}{|AB|}\text{Adj (AB)}$ **……. Equ (i)**

Now

$$|AB|=\begin{vmatrix}2 & -2\\-4 & 6\end{vmatrix}$$

$$|AB|=12-8$$

$$|AB|=4\neq 0$$

$$\text{Adj (AB)}=\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}$$

Put the values in equ (i)

$$(AB)^{-1}=\frac{1}{4}\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}$$

RHS: $\boldsymbol{B^{-1}A^{-1}}$

As $A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$

And $B^{-1}=\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}$

Now

$$B^{-1}A^{-1}=\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}\times\frac{1}{2}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{2}\times\frac{1}{2}\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{4}\begin{bmatrix}3+3 & 0+2\\1+3 & 0+2\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{4}\begin{bmatrix}6 & 2\\4 & 2\end{bmatrix}$$

Hence $(\boldsymbol{AB})^{-1}=\boldsymbol{B^{-1}A^{-1}}$ **Proved:**

Ex # 1.5

Show that $(\boldsymbol{BA})^{-1} = \boldsymbol{A}^{-1}\boldsymbol{B}^{-1}$

Solution:

$(\boldsymbol{BA})^{-1} = \boldsymbol{A}^{-1}\boldsymbol{B}^{-1}$

LHS: $(\boldsymbol{BA})^{-1}$

As $BA = \begin{bmatrix} 5 & -1 \\ -11 & 3 \end{bmatrix}$

So

$$(BA)^{-1} = \frac{1}{|BA|}\text{Adj (BA)}$$ **……. Equ (i)**

$$|BA| = \begin{vmatrix} 5 & -1 \\ -11 & 3 \end{vmatrix}$$

$$|BA| = 15 - 11$$

$$|BA| = 4 \neq 0$$

$$\text{Adj (BA)} = \begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

Put the values in equ (i)

$$(BA)^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

RHS: $\boldsymbol{A}^{-1}\boldsymbol{B}^{-1}$

As $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

And $B^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$

Now

$$A^{-1}B^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{2} \times \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{4}\begin{bmatrix} 3+0 & 1+0 \\ 9+2 & 3+2 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

Hence $(\boldsymbol{BA})^{-1} = \boldsymbol{A}^{-1}\boldsymbol{B}^{-1}$ **<u>Proved:</u>**

Ex # 1.5

Q6: (i) **If** $A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ **then show that** $(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$

Solution:

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$

LHS: $(\boldsymbol{AB})^{-1}$

Now

$$AB = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (0)(2)+(-1)(1) & (0)(3)+(-1)(0) \\ (2)(2)+(1)(1) & (2)(3)+(1)(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 0-1 & 0+0 \\ 4+1 & 6+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 \\ 5 & 6 \end{bmatrix}$$

As we have:

$$(AB)^{-1} = \frac{1}{|AB|}\text{Adj (AB)}$$ **……. Equ (i)**

$$|AB| = \begin{vmatrix} -1 & 0 \\ 5 & 6 \end{vmatrix}$$

$$|AB| = -6 - 0$$

$$|AB| = -6 \neq 0$$

So solution exists

$$\text{Adj (AB)} = \begin{bmatrix} 6 & 0 \\ -5 & -1 \end{bmatrix}$$

Put the values in equ (i)

$$(AB)^{-1} = \frac{1}{-6}\begin{bmatrix} 6 & 0 \\ -5 & -1 \end{bmatrix}$$

RHS: $\boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$

First we find $\boldsymbol{A}^{-1}$

As $A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$

As we have

$$A^{-1}=\frac{1}{|A|}\text{Adj A}$$ ……. **Equ (i)**

$$|A|=\begin{vmatrix}0 & -1\\2 & 1\end{vmatrix}$$

$$|A|=0-(-2)$$

$$|A|=0+2$$

$$|A|=2\neq 0$$

$$\text{Adj A}=\begin{bmatrix}1 & 1\\-2 & 0\end{bmatrix}$$

Put the values in equ (i)

$$A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 1\\-2 & 0\end{bmatrix}$$

Now we find B^{-1}

As $B=\begin{bmatrix}2 & 3\\1 & 0\end{bmatrix}$

As we have:

$$B^{-1}=\frac{1}{|B|}\text{Adj B}$$ ……. **Equ (i)**

$$|B|=\begin{vmatrix}2 & 3\\1 & 0\end{vmatrix}$$

$$|B|=0-3$$

$$|B|=-3\neq 0$$

$$\text{Adj B}=\begin{bmatrix}0 & -3\\-1 & 2\end{bmatrix}$$

Put the values in equ (i)

$$B^{-1}=\frac{1}{-3}\begin{bmatrix}0 & -3\\-1 & 2\end{bmatrix}$$

Now

$$B^{-1}A^{-1}=\frac{1}{-3}\begin{bmatrix}0 & -3\\-1 & 2\end{bmatrix}\times\frac{1}{2}\begin{bmatrix}1 & 1\\-2 & 0\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-3}\times\frac{1}{2}\begin{bmatrix}0 & -3\\-1 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\-2 & 0\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}0+6 & 0+0\\-1-4 & -1+0\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{-6}\begin{bmatrix}6 & 0\\-5 & -1\end{bmatrix}$$

Hence $(AB)^{-1}=B^{-1}A^{-1}$ **Proved:**

Q6: (ii) If $A=\begin{bmatrix}0 & -1\\2 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\1 & 0\end{bmatrix}$ then show that $(BA)^{-1}=A^{-1}B^{-1}$

Solution:

$A=\begin{bmatrix}0 & -1\\2 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\1 & 0\end{bmatrix}$

$(BA)^{-1}=A^{-1}B^{-1}$

LHS: $(BA)^{-1}$

$$BA=\begin{bmatrix}2 & 3\\1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\2 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}(2)(0)+(3)(2) & (2)(-1)+(3)(1)\\(1)(0)+(0)(2) & (1)(-1)+(0)(1)\end{bmatrix}$$

$$BA=\begin{bmatrix}0+6 & -2+3\\0+0 & -1+0\end{bmatrix}$$

$$BA=\begin{bmatrix}6 & 1\\0 & -1\end{bmatrix}$$

As we have

$$(BA)^{-1}=\frac{1}{|BA|}\text{Adj (BA)}$$ ……. Equ (i)

$$|BA|=\begin{vmatrix}6 & 1\\0 & -1\end{vmatrix}$$

$$|BA|=-6-0$$

$$|BA|=-6\neq 0$$

So solution exists

$$\text{Adj (BA)}=\begin{bmatrix}-1 & -1\\0 & 6\end{bmatrix}$$

Put the values in equ (i)

$$(BA)^{-1}=\frac{1}{-6}\begin{bmatrix}-1 & -1\\0 & 6\end{bmatrix}$$

RHS: $A^{-1}B^{-1}$

First we find A^{-1}

As $A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}\text{Adj A}$ **....... Equ (i)**

$|A|=\begin{vmatrix}0 & -1\\ 2 & 1\end{vmatrix}$

$|A|=0-(-2)$

$|A|=0+2$

$|A|=2\neq 0$

$\text{Adj A}=\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$

Put the values in equ (i)

$A^{-1}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$

Now we find B^{-1}

As $B=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$

As we have:

$B^{-1}=\frac{1}{|B|}\text{Adj B}$ **....... Equ (i)**

$|B|=\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix}$

$|B|=0-3$

$|B|=-3\neq 0$

$\text{Adj B}=\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

Put the values in equ (i)

$B^{-1}=\frac{1}{-3}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

Now

$A^{-1}B^{-1}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}\times\frac{1}{-3}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

$A^{-1}B^{-1}=\frac{1}{2}\times\frac{1}{-3}\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$

$A^{-1}B^{-1}=\frac{1}{-6}\begin{bmatrix}0-1 & -3+2\\ 0+0 & 6+0\end{bmatrix}$

$A^{-1}B^{-1}=\frac{1}{-6}\begin{bmatrix}-1 & -1\\ 0 & 6\end{bmatrix}$

Hence $(BA)^{-1}=A^{-1}B^{-1}$ Proved:

Exercise # 1.6

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Q1: Solve the following system of linear equation using Inversion Method.

(i) $2x+3y=-1,\quad x-y=2$

Solution:

$2x+3y=-1$

$x-y=2$

In matrix form:

$\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$

Let $A=\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}-1\\ 2\end{bmatrix}$

As $AX=B$

$X=A^{-1}B$

$X=\frac{1}{|A|}\text{Adj }A\times B$ **....... Equ (i)**

$|A|=\begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix}$

$|A|=-2-3$

$|A|=-5\neq 0$

Thus Solution exists

$\text{Adj A}=\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}$

Put the values in equ (i)

$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-5}\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}\times\begin{bmatrix}-1\\ 2\end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} (-1)(-1)+(-3)(2) \\ (-1)(-1)+(2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} 1-6 \\ 1+4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5\times\frac{1}{-5} \\ 5\times\frac{1}{-5} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$x = 1$

$y = -1$

Thus Solution Set= $\{(\mathbf{1}\,,\ -\mathbf{1})\}$

(ii) $\boldsymbol{x + 2y = -13,}$ $\quad$ $\boldsymbol{3x + 6y = 11}$

Solution:

$x + 2y = -13$

$3x + 6y = 11$

In matrix form:

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$

As $AX = B$

$X = A^{-1}B$

$X = \frac{1}{|A|}\text{Adj A}\times B$ **……. Equ (i)**

First we find $|A|$

$$|A| = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$|A| = 6 - 6$

$|A| = 0$

As $|A| = 0$

So Solution is not possible.

(iii) $\boldsymbol{x + 2y = 1,}$ $\qquad 2x + 3y = \frac{5}{2}$

Solution:

First Method

$x + 2y = 1$

$2x + 3y = \frac{5}{2}$

In matrix form:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$

As $AX = B$

$X = A^{-1}B$

$X = \frac{1}{|A|}\text{Adj A}\times B$ **……. Equ (i)**

First we find $|A|$

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$|A| = 3 - 4$

$|A| = -1 \neq 0$

Thus solution exists

$$\text{Adj A} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}\times\begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} (3)(1)+(-2)\left(\frac{5}{2}\right) \\ (-2)(1)+(1)\left(\frac{5}{2}\right) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 3+(-1)(5) \\ -2+\frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}3-5\\ \dfrac{-4+5}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=-1\begin{bmatrix}-2\\ \dfrac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\ -\dfrac{1}{2}\end{bmatrix}$$

$$x=2$$

$$y=-\frac{1}{2}$$

Thus Solution Set= $\left\{\left(2\ ,\ \ -\dfrac{1}{2}\right)\right\}$

(iii) $\boldsymbol{x+2y=1,}$ $\quad 2x+3y=\dfrac{5}{2}$

Solution:

Second Method

$$x+2y=1$$

$$2x+3y=\frac{5}{2}$$

Multiply B.S by 2

$$2(2x+3y)=2\times\frac{5}{2}$$

$$4x+6y=5$$

So write in matrix form:

$$\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\5\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$, $B=\begin{bmatrix}1\\5\end{bmatrix}$

As $AX=B$

$$X=A^{-1}B$$

$X=\dfrac{1}{|A|}\text{Adj A}\times B$ ……. **Equ (i)**

First we find $|A|$

$$|A|=\begin{vmatrix}1 & 2\\4 & 6\end{vmatrix}$$

$$|A|=6-8$$

$$|A|=-2\neq 0$$

Thus solution exists

$$\text{Adj A}=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}\times\begin{bmatrix}1\\5\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}(6)(1)+(-2)(5)\\(-4)(1)+(1)(5)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}6-10\\-4+5\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}-4\\1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-4\times\dfrac{1}{-2}\\ 1\times\dfrac{1}{-2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\ -\dfrac{1}{2}\end{bmatrix}$$

$$x=2$$

$$y=-\frac{1}{2}$$

Thus Solution Set= $\left\{\left(\mathbf{2}\ ,\ \ -\dfrac{1}{2}\right)\right\}$

(iv) $\boldsymbol{x-2y-1=0,}$ $\quad \boldsymbol{2x+y+3=0}$

Solution:

$$x-2y-1=0$$

$$2x+y+3=0$$

Hence

$$x-2y=1$$

$$2x+y=-3$$

In matrix form:

$$\begin{bmatrix}1 & -2\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-3\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}1\\ -3\end{bmatrix}$

As $AX=B$

$$X=A^{-1}B$$

$$X=\frac{1}{|A|}\text{Adj A}\times B$$ **....... Equ (i)**

First we find $|A|$

$$|A|=\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix}$$

$$|A|=1-(-4)$$

$$|A|=1+5$$

$$|A|=5\neq 0$$

Thus solution exists

$$\text{Adj A}=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\times\begin{bmatrix}1\\ -3\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}(1)(1)+(2)(-3)\\ (-2)(1)+(1)(-3)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1+(-6)\\ -2+(-3)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1-6\\ -2-3\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}-5\\ -5\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-5\times\frac{1}{5}\\ -5\times\frac{1}{5}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-1\\ -1\end{bmatrix}$$

$$x=-1$$

$$y=-1$$

Thus Solution Set= $\{(-1\,,\ -1)\}$

Ex # 1.6

Q2: **Solve the following system of linear equations using Cramer's Rule**

(i) $x-2y=5,\quad 2x-y=6$

Solution:

$$x-2y=5$$

$$2x-y=6$$

In matrix form:

$$\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 6\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}5\\ 6\end{bmatrix}$

First we find $|A|$

$$|A|=\begin{vmatrix}1 & -2\\ 2 & -1\end{vmatrix}$$

$$|A|=-1-(-4)$$

$$|A|=-1+4$$

$$|A|=3\neq 0$$

Thus solution exists.

To find the value of x**, Replace the coefficient of** x **in A by Matrix B.**

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{\begin{vmatrix}5 & -2\\ 6 & -1\end{vmatrix}}{3}$$

$$x=\frac{-5-(-12)}{3}$$

$$x=\frac{-5+12}{3}$$

$$x=\frac{7}{3}$$

To find the value of y**, Replace the coefficient of** y **in A by Matrix B.**

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{\begin{vmatrix}1 & 5\\ 2 & 6\end{vmatrix}}{3}$$

$$y=\frac{6-10}{3}$$

$$y=\frac{-4}{3}$$

$$x=\frac{7}{3}$$

$$y=\frac{-4}{3}$$

Thus Solution Set$=\left\{\left(\frac{7}{3},\frac{-4}{3}\right)\right\}$

(ii) $\mathbf{4x+3y=-2,\quad x-2y=5}$

Solution:

$$4x+3y=-2$$

$$x-2y=5$$

In matrix form:

$$\begin{bmatrix}4 & 3\\1 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\5\end{bmatrix}$$

Let $A=\begin{bmatrix}4 & 3\\1 & -2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$, $B=\begin{bmatrix}-2\\5\end{bmatrix}$

First we find $|A|$

$$|A|=\begin{vmatrix}4 & 3\\1 & -2\end{vmatrix}$$

$$|A|=-8-3$$

$$|A|=-11\neq 0$$

Thus solution exists.

To find the value of x**, Replace the coefficient of** x **in A by Matrix B.**

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{\begin{vmatrix}-2 & 3\\5 & -2\end{vmatrix}}{-11}$$

$$x=\frac{4-15}{-11}$$

$$x=\frac{-11}{-11}$$

$$x=1$$

To find the value of$y$**, Replace the coefficient of** y **in A by Matrix B.**

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{\begin{vmatrix}4 & -2\\1 & 5\end{vmatrix}}{-11}$$

$$y=\frac{20-(-2)}{-11}$$

$$y=\frac{20+2}{-11}$$

$$y=\frac{22}{-11}$$

$$y=-2$$

$$x=1$$

$$y=-2$$

Thus Solution Set$=\{(\mathbf{1},\ -\mathbf{2})\}$

(iii) $\mathbf{5x+7y=3,\quad 3x+y=5}$

Solution:

$$5x+7y=3$$

$$3x+y=5$$

In matrix form

$$\begin{bmatrix}5 & 7\\3 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\5\end{bmatrix}$$

Let $A=\begin{bmatrix}5 & 7\\3 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$, $B=\begin{bmatrix}3\\5\end{bmatrix}$

First we find $|A|$

$$|A|=\begin{vmatrix}5 & 7\\3 & 1\end{vmatrix}$$

$$|A|=5-21$$

$$|A|=-16\neq 0$$

Thus solution exists.

To find the value of x**, Replace the coefficient of** x **in A by Matrix B.**

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{\begin{vmatrix}3 & 7\\ 5 & 1\end{vmatrix}}{-16}$$

$$x=\frac{3-35}{-16}$$

$$x=\frac{-32}{-16}$$

$$x=2$$

To find the value of y, Replace the coefficient of y in A by Matrix B.

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{\begin{vmatrix}5 & 3\\ 3 & 5\end{vmatrix}}{-16}$$

$$y=\frac{25-9}{-16}$$

$$y=\frac{16}{-16}$$

$$y=-1$$

$$x=2$$

$$y=-1$$

Thus Solution Set= $\{(\mathbf{2}\,,\ -\mathbf{1})\}$

Q3: Amjad thought of two numbers whose sum is 12 and whose difference is 4. Find the numbers.

Solution:

Let the one number= x

And second number= y

According to given condition:

$$x+y=12$$

$$x-y=4$$

In matrix form

$$\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}12\\ 4\end{bmatrix}$$

Let $A=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix},\ X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}12\\ 4\end{bmatrix}$

By Cramer's Method:

First we find $|A|$

$$|A|=\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}$$

$$|A|=-1-1$$

$$|A|=-2\neq 0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{\begin{vmatrix}12 & 1\\ 4 & -1\end{vmatrix}}{-2}$$

$$x=\frac{-12-4}{-2}$$

$$x=\frac{-16}{-2}$$

$$x=8$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{\begin{vmatrix}1 & 12\\ 1 & 4\end{vmatrix}}{-2}$$

$$y=\frac{4-12}{-2}$$

$$y=\frac{-8}{-2}$$

$$y=4$$

So one number= 8

And second number= 4

Q4: **The length of a rectangular playground is twice its width. The perimeter is 30. Find its dimensions.**

Solution:

Let the width $= x$

And length $= y$

According to first condition:

$2x = y$

$2x - y = 0$……. Equ (i)

As perimeter $= 30$

As we have

$2(x+y) = P$

$2(x+y) = 30$

$x + y = 15$……. Equ (ii)

Equ (i) and Equ (ii) in Matrix form

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$

First we find $|A|$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$|A| = 2 - (-1)$$

$$|A| = 2 + 1$$

$$|A| = 3 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 15 & 1 \end{vmatrix}}{3}$$

$$x = \frac{0 - (-15)}{3}$$

$$x = \frac{15}{3}$$

$$x = 5$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 2 & 0 \\ 1 & 15 \end{vmatrix}}{3}$$

$$y = \frac{30 - 0}{3}$$

$$y = \frac{30}{3}$$

$$y = 10$$

So the width $= 5$

And length $= 10$

Q5: **3 bags and 4 pens together cost 257 rupees whereas 4 bags and 3 pens together cost 324 rupees. Find the cost of a bag and 10 pens.**

Solution:

According to condition:

Let the cost of bag $= x$

And the cost of pen$= y$

$3x + 4y = 257$

$4x + 3y = 324$

$$\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 257 \\ 324 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 257 \\ 324 \end{bmatrix}$

By Cramer's Rule

First we find $|A|$

$$|A| = \begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix}$$

$$|A| = 9 - 16$$

$$|A| = -7 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 257 & 4 \\ 324 & 3 \end{vmatrix}}{-7}$$

$$x = \frac{771 - 1296}{-7}$$

Ex # 1.6

$$x = \frac{-525}{-7}$$

$$x = 75$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 3 & 257 \\ 4 & 324 \end{vmatrix}}{-7}$$

$$y = \frac{972 - 1028}{-7}$$

$$y = \frac{-56}{-7}$$

$$y = 8$$

So the cost of bag $= Rs.\ 75$

And the cost of 10 pens= $10 \times 8 = Rs. 80$

Q6: **If twice the son's age in years is added to the father's age, the sum is 70. But if the father's age is added to the son's age, the sum is 95. Find the ages of father and son.**

Solution:

Let the age of Son= x

And the age of father= y

According to condition:

$$2x + y = 70$$

$$x + 2y = 95$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 95 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 70 \\ 95 \end{bmatrix}$

By Cramer's Rule

First we find $|A|$

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$|A| = 4 - 1$$

$$|A| = 3 \neq 0$$

Ex # 1.6

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 70 & 1 \\ 95 & 2 \end{vmatrix}}{3}$$

$$x = \frac{140 - 95}{3}$$

$$x = \frac{45}{3}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 2 & 70 \\ 1 & 95 \end{vmatrix}}{3}$$

$$y = \frac{190 - 70}{3}$$

$$y = \frac{120}{3}$$

$$y = 40$$

The age of Son= 15

And the age of father= 40

REVIEW EXERSICE 1

Page # 47

Q2: Find x and y

$$\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$$

Compare the corresponding elements

$x - 1 = 0$

$x = 1$

$y + 3 = -2$

$y = -2 - 3$

$y = -5$

Q3: Find the product if possible

$$\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \begin{bmatrix} -6 & 5 & 8 \\ 0 & 4 & 1 \end{bmatrix}$$

As number of Columns in first matrix = 1

And number of Rows in second matrix = 2

Thus these are not conformable for multiplication.

Q4: Find the inverse of the matrix $A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|}\text{Adj A}$ .... Equ (i)

First we find $|A|$:

$$|A| = \begin{vmatrix} 6 & -3 \\ 5 & -2 \end{vmatrix}$$

$$|A| = -12 - (-15)$$

$$|A| = -12 + 15$$

$$|A| = 3 \neq 0$$

$$\text{Adj A} = \begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

Put the values in equ (i)

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

Q5: Solve the system: $2x + 5y = 9$, $5x - 2y = 8$

Solution:

$$2x + 5y = 9$$

$$5x - 2y = 8$$

In matrix form:

$$\begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$

$$AX = B$$

$$X = A^{-1}B$$

$X = \frac{1}{|A|}\text{Adj } A \times B$ .... Equ (i)

$$|A| = \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix}$$

$$|A| = -4 - 25$$

$$|A| = -29 \neq 0$$

$$\text{Adj A} = \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix} \times \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} (-2)(9) + (-5)(8) \\ (-5)(9) + (2)(8) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -18 - 40 \\ -45 + 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29}\begin{bmatrix} -58 \\ -29 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -58\times\dfrac{1}{-29} \\ -29\times\dfrac{1}{-29} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x = 2$

$y = 1$

Thus Solution Set= $\{(\mathbf{2}\ ,\ \mathbf{1})\}$

Q6: Qasim and Farzana are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Qasim sold 3 small boxes of oranges and 14 large boxes of oranges for a total of Rs. 203. Farzana sold 11 small boxes of oranges ad 11 large boxes of oranges for a total of Rs. 220. Find the cost of one small box of oranges and one large box of oranges.

Solution:

Let small box of oranges = x

And large box of oranges = y

According to given condition:

$3x + 14y = 203$

$11x + 11y = 220$

In matrix form:

$$\begin{bmatrix} 3 & 14 \\ 11 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 203 \\ 220 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & 14 \\ 11 & 11 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 203 \\ 220 \end{bmatrix}$

$AX = B$

$X = A^{-1}B$

$X = \dfrac{1}{|A|}\text{Adj } A \times B$ .... Equ (i)

First find $|A|$

$$|A| = \begin{vmatrix} 3 & 14 \\ 11 & 11 \end{vmatrix}$$

$|A| = 33 - 154$

$|A| = -121$

$$\text{Adj A} = \begin{bmatrix} 11 & -14 \\ -11 & 3 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121}\begin{bmatrix} 11 & -14 \\ -11 & 3 \end{bmatrix}\times\begin{bmatrix} 203 \\ 220 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121}\begin{bmatrix} (11)(203)+(-14)(220) \\ (-11)(203)+(3)(220) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121}\begin{bmatrix} 2233-3080 \\ -2233+660 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121}\begin{bmatrix} -847 \\ -1573 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -847\times\dfrac{1}{-121} \\ -1573\times\dfrac{1}{-121} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$x = 7$

$y = 13$

Thus small box of oranges = $Rs.7$

And large box of oranges = $Rs.13$

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MATHEMATICS

Class 9th (KPK)

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Ex # 2.1

Set of Natural Numbers

$$N = \{1, 2, 3, 4, \dots\}$$

Set of Whole Numbers

$$W = \{0, 1, 2, 3, 4, \dots\}$$

Set of Integers

$$Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$$

OR

$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3\}$$

Rational Numbers

The word Rational means "Ratio".

A rational number is a number that can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Rational numbers is denoted by Q.

Set of Rational Numbers

$$Q = \left\{\frac{p}{q} \mid p, q \in Z, q \neq 0\right\}$$

Irrational Numbers

The word Irrational means "Not Ratio".

Irrational number consists of all those numbers which are not rational. Irrational numbers is denoted by $Q^{/}$.

Real numbers

The set of rational and irrational numbers is called Real Numbers. Real numbers is denoted by R.

Thus $Q \cup Q^{/} = R$

Note:

All the numbers on the number line are real numbers.

Terminating Decimal Fraction:

A decimal number that contains a finite number of digits after the decimal point.

Non-Terminating Decimal Fraction:

A decimal number that has no end after the decimal point.

Non-Terminating Repeating Decimal Fraction

In non-terminating decimal fraction, some digits are repeated in same order after decimal point.

Non-Terminating Non-Repeating Decimal Fraction.

In non-terminating decimal fraction, the digits are not repeated in same order after decimal point.

Decimal Representation of Rational and Irrational Numbers.

(i) All terminating and repeating decimals are rational numbers.

(ii) Non-terminating recurring (repeating) decimals are rational numbers.

(iii) Never terminating or repeating decimals are irrational numbers.

Non-terminating and non-recurring (repeating) decimals are irrational numbers.

Note:

(i) Repeating decimals are called recurring decimals.

(ii) Non-repeating decimals are called non-recurring decimals.

Ex # 2.1
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In Questions 1 – 10, consider the numbers.

$$\mathbf{2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}, 0.333\ldots}$$

1. Which are whole numbers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad 1$

2. Which are integers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad -9, \quad 1$

3. Which are irrational numbers?

Ans: $\sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi$

4. Which are natural numbers?

Ans: $3, \quad \sqrt{36}, \quad 1$

5. Which are rational numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, -9, 1, \quad 4\frac{2}{3},$ $0.333\ldots$

11. Write the decimal representation of each of the following numbers.

$\frac{1}{6}, \frac{6}{7}, \frac{2}{9}, \frac{1}{8}$

$\frac{1}{6} = 0.1666\ldots.$

$\frac{6}{7} = 0.8571\ldots.$

$\frac{2}{9} = 0.222\ldots.$

$\frac{1}{8} = 0.125$

6. Which are real numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}$ $, 0.333.$

7. Which are rational numbers but not integers?

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad 4\frac{2}{3}, \quad 0.333\ldots$

8. Which are integers but not whole numbers?

Ans: -9

9. Which are integers but not natural numbers?

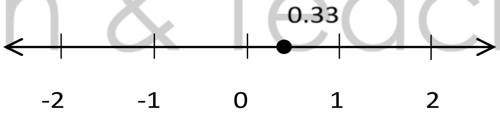
Ans: $0, -9$

10. Which are real numbers but not integers?

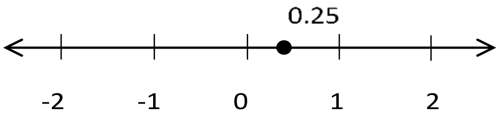
Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad \sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi,$ $4\frac{2}{3}, \quad 0.333\ldots$

12 Depict each number on a number line.

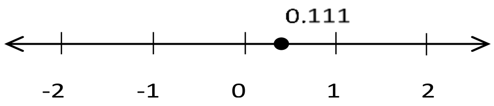
(i) $\mathbf{\frac{1}{3} = 0.333\ldots}$



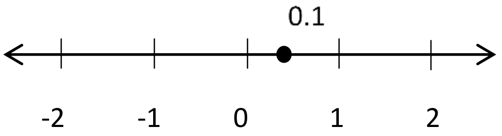
(ii) $\mathbf{\frac{1}{4} = 0.25}$



(ii) $\frac{1}{9} = 0.111\ldots$



(iv) $\frac{1}{10} = 0.1$



Ex # 2.2

Properties of Real Number

The set R of real number is the union of two disjoint sets. Thus $R = Q \cup Q^{/}$

Note:

$$Q \cap Q^{/} = \emptyset$$

Real Number System

Closure Property w.r.t Addition

The sum of real number is also a real number.

If $a,\ b\ \in R$ then $a + b\ \in R$

Example:

$$7 + 9 = 16$$

Where 16 is a real number.

Closure Property w.r.t Multiplication

The Product of real number is also a real number.

If $a,\ b\ \in R$ then $a\,.\,b\ \in R$

Example:

$$7 \times 9 = 63$$

Where 63 is a real number.

Commutative Property w.r.t Addition

If $a,\ b\ \in R$ then $a + b = b + a$

Example:

$$7 + 9 = 9 + 7$$
$$16 = 16$$

Commutative Property w.r.t Multiplication

If $a,\ b\ \in R$ then $a\,.\ b = b\,.\ a$

Example:

$$7 \times 9 = 9 \times 7$$
$$63 = 63$$

Associative Property w.r.t Addition

If $a,\ b,\ c\ \in R$ then

$$a + (b + c) = (a + b) + c$$

Example:

$$2 + (3 + 5) = (2 + 3) + 5$$
$$2 + 8 = 5 + 5$$
$$10 = 10$$

Associative Property w.r.t Multiplication

If $a,\ b,\ c\ \in R$ then

$$a(bc) = (ab)c$$

Example:

$$2(3 \times 5) = (2 \times 3)5$$
$$2(15) = (6)5$$
$$30 = 30$$

Additive Identity

Zero (0) is called Additive identity because adding "0" to a number does not change that number.

If $a \in R$ there exists $O\ \in R$ then

$$a + 0 = 0 + a = a$$

Example:

$$3 + 0 = 0 + 3 = 3$$

Multiplicative Identity

1 is called Multiplicative identity because multiplying "1" to a number does not change that number.

If $a \in R$ there exists $1\ \in R$ then

$$a\,.\,1 = 1\,.\,a = a$$

Example:

$$3 \times 1 = 1 \times 3 = 3$$

Additive Inverse

When the sum of two numbers is zero (0)

If $a \in R$ there exists an element $a^{/}$ then $a + a^{/} = a^{/} + a = 0$ then $a^{/}$ is called additive inverse of a

Or

$$a + (-a) = -a + a = 0$$

Example:

$$3 + (-3) = 3 - 3 = 0$$
$$-3 + 3 = 0$$

Ex # 2.2

Multiplicative Inverse

When the Product of two numbers is “1”.

If $a \in R$ and $a \neq 0$ there exists an element $a^{-1} \in R$ then $a \, . \, a^{-1} = a^{-1} \, . \, a = 1$ then a^{-1} is called multiplicative inverse of a

Or

$$a \, . \, \frac{1}{a} = \frac{1}{a} . a = 1$$

Example:

$$3 \times \frac{1}{3} = \frac{1}{3} \times 3 = 1$$

Distributive Property of Multiplication over Addition

If $a, \ b, \ c \ \in R$ then

$$a(b+c) = ab + ac$$
$$(b+c)a = ba + ca$$

Example:

$$2(3+5) = 2 \times 3 + 2 \times 5$$
$$2(8) = 6 + 10$$
$$16 = 16$$

Properties of Equality of Real Numbers

Reflexive Property of equality

Every number is equal to itself.

$$a = a$$

Example:

$$3 = 3$$

Symmetric Property of Equality

If $a = b$ then also $b = a$

Examples:

$$x = 5$$
$$or \ 5 = x$$
$$x^2 = y$$
$$or \ y = x^2$$

Transitive Property of Equality

If $a = b$ and $b = c$ then $a = c$

Example:

$if \ x + y = z$ and $z = a + b$

Then $x + y = a + b$

Ex # 2.2

Additive Property of Equality

If $a = b$ then also $a + c = b + c$

Examples:

$$x - 3 = 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

$$x + 3 = 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property of Equality

If $a = b$ then also $a. \ c = b \ . \ c$

Or

$a = b$ then $\frac{a}{c} = \frac{b}{c}$

Examples:

$$\frac{x}{3} = 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

$$2x = 24$$

Divide B.S by 2

$$\frac{2x}{2} = \frac{24}{2}$$
$$x = 12$$

Cancellation Property w.r.t Addition

If $a + c = b + c$ then $a = b$

Examples:

$$2x + 5 = y + 5$$
$$2x = y$$
$$2x - 5 = y - 5$$
$$2x = y$$

Ex # 2.2

Cancellation Property w.r.t Multiplication

If $a\,.\,c = b\,.\,c$ then $a = b$

OR

If $\frac{a}{c} = \frac{b}{c}$ then $a = b$

Examples:

$$2x \times 5 = y \times 5$$
$$2x = y$$
$$\frac{2x}{5} = \frac{y}{5}$$
$$2x = y$$

Properties of Inequality of Real Numbers

Trichotomy Property

Trichotomy property means when comparing two numbers, one of the following must be true:

$$a = b$$
$$a < b$$
$$a > b$$

Examples:

$$5 = 5$$
$$3 < 5$$
$$3 > 5$$

Transitive Property

(i) **If $a > b$ and $b > c$ then $a > c$**

Example:

If $7 > 5$ and $5 > 3$ then $\mathbf{7 > 3}$

(ii) **If $a < b$ and $b < c$ then $a < c$**

Example:

If $3 < 5$ and $5 < 7$ then $\mathbf{3 < 7}$

Additive Property

(i) **If $a < b$ then $a + c < b + c$**

Example:

$3 < 5$ then $3 + 2 < 5 + 2$

$$x - 3 > 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

Ex # 2.2

(ii) **If $a > b$ then $a + c > b + c$**

Example:

(a) $5 > 3$ then $5 - 2 > 3 - 2$

(b) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(c) $$x + 3 > 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property

When $c > 0$:

(i) If $a < b$ then $ac < bc$

(ii) If $a > b$ then $ac > bc$

Example:

(a) $5 > 3$ then $5 \times 2 > 3 \times 2$

(b) $$\frac{x}{3} > 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 > 5 \times 3$$
$$x > 15$$

$$2x > 24$$

Divide B.S by 2

$$\frac{2x}{2} > \frac{24}{2}$$
$$x > 12$$

When $c < 0$:

(i) If $a < b$ then $ac > bc$

(ii) If $a > b$ then $ac < bc$

Example:

(a) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

(b) $$\frac{x}{-3} < 5$$

Multiply B.S by -3

$$\frac{x}{-3} \times -3 > 5 \times -3$$
$$x > -15$$

Example: 4

Page # 58

Solve the following equation using properties of real numbers.

$$2x - 5 = 3x + 4$$

Solution:

$2x - 5 = 3x + 4$	
$2x - 5 + 5 = 3x + 4 + 5$	∴ $a = b$ then $a + c = b + c$
$2x - 5 + 5 = 3x + 9$	∴ Closure Property w.r.t Additon
$2x + 0 = 3x + 9$	∴ -5 & 5 are additive inverse
$2x = 3x + 9$	∴ 0 is the additive identity
$3x + 9 = 2x$	∴ Symmetric Property
$3x + 9 - 2x = 2x - 2x$	∴ $a = b$ then $a - c = b - c$
$3x - 2x + 9 = 0$	∴ $2x$ & $-2x$ are additive inverse
$(3 - 2)x + 9 = 0$	∴ Distributive Property
$1.x + 9 = 0$	
$x + 9 = 0$	∴ 1 is Multiplicative Identity
$x + 9 - 9 = 0 - 9$	∴ $a = b$ then $a - c = b - c$
$x + 9 - 9 = -9$	∴ 0 is the Additive Identity
$x + 0 = -9$	∴ 9 & -9 are additive inverse
$x = -9$	∴ 0 is the Additive Identity

Ex # 2.2

Page # 59

Q1: Name the properties used in following equations.

(i) $\mathbf{1 + (4 + 3) = (1 + 4) + 3}$

Ans: Associative law of addition

(ii) $\mathbf{5(a + b) = 5a + 5b}$

Ans: Distributive law of multiplication over addition

(iii) $\mathbf{a + 0 = 0 + a = a}$

Ans: Additive identity

(iv) $\mathbf{5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1}$

Ans: Multiplicative inverse

Q2: Write the missing number.

(i) $\mathbf{2 + (\_\_\_ + 4) = (2 + 6) + 4}$

Answer: 6

(ii) $\mathbf{7 + (4 + 2) = 13}$**, so** $\mathbf{(7 + 4) + 2 =}$____

Answer: 13

(iii) $\mathbf{9 \times (3 \times 4) = 108}$**, so** $\mathbf{(9 \times 3) \times 4 =}$_____

Answer: 108

(iv) $\mathbf{5 \times (8 \times 9) = (5 \times \_\_\_\_) \times 9}$

Answer: 8

Q3: Chose the correct option

(i) $\mathbf{8 \times (6 \times 7)}$ **is equal to:**

(a) $8 \times 6 - 7$ **(b)** $8 - (6 - 7)$
(c) 8×12 **(d)** $(8 \times 6) \times 7$

Answer: d. $(8 \times 6) \times 7$

(ii) Which one of the following illustrates the Associative Law of Addition?

(a) $3 + (2 + 4) = (4 + 4) + 1$
(b) $3 + (2 + 4) = (3 + 2) + 4$
(c) $3 + (2 + 4) = (5 + 2) + 2$
(d) $3 + (2 + 4) = (2 + 6) + 1$

Answer: b. $3 + (2 + 4) = (3 + 2) + 4$

Ex # 2.2

(iii) Which one of the following illustrates the Associative Law of Multiplication?

(a) $4 \times (3 \times 6) = (6 \times 6) \times 2$

(b) $4 \times (3 \times 6) = (3 \times 12) \times 2$

(c) $4 \times (3 \times 6) = (4 \times 3) \times 6$

(d) $4 \times (3 \times 6) = (3 \times 8) \times 3$

Answer: c. $4 \times (3 \times 6) = (4 \times 3) \times 6$

Q4: Do this without using distributive property.

(i) $\mathbf{39 \times 63 + 39 \times 37}$

Solution:

$39 \times 63 + 39 \times 37$
$= 2457 + 1443$
$= 3900$

(ii) $\mathbf{81 \times 450 + 81 \times 550}$

Solution:

$81 \times 450 + 81 \times 550$
$= 36450 + 44550$
$= 81000$

(iii) $\mathbf{50 \times 161 - 50 \times 81}$

Solution:

$50 \times 161 - 50 \times 81$
$= 8050 - 4050$
$= 4000$

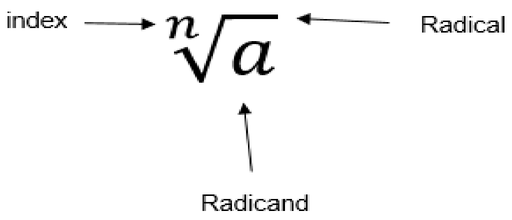
(iv) $\mathbf{827 \times 60 - 327 \times 60}$

Solution:

$827 \times 60 - 327 \times 60$
$= 49620 - 19620$
$= 30000$

Ex # 2.3

RADICALS AND RADICANDS



$\sqrt[n]{a}$ is the radical form of the nth root of a.

$a^{\frac{1}{n}}$ is the exponential form of the nth root of a.

If $n = 2$ then it becomes square root and write $\sqrt{a}$ instead of $\sqrt[2]{a}$

If $n = 3$ then it is called cube root like $\sqrt[3]{a}$

If $n = 5$ then it is called 5th root like $\sqrt[5]{625}$

Important Notes

(i) If a is positive, then the nth root of a is also positive.

Example:

$$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$$

(ii) If a is negative, then n must be odd for the nth root of a to be a real number.

Example:

$$\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$$

(iii) If a is zero, then $\sqrt[n]{0} = 0$

Properties of Radicals:

Product Rule of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} \,.\, \sqrt[n]{b}$

Example:

$\sqrt{6x}\sqrt{6y^2}$
$\sqrt{(6x)(6y^2)} = \sqrt{36y^2x} = \sqrt{36}\sqrt{y^2}\sqrt{x}$
$= 6y\sqrt{x}$

$\sqrt{6x}\sqrt{6x^2}$
$\sqrt{(6x)(6x^2)} = \sqrt{36x^2x} = \sqrt{36}\sqrt{x^2}\sqrt{x}$
$= 6y\sqrt{x}$

Ex # 2.3

Quotient Rule of Radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example:

Simplify: $2\sqrt{\frac{150xy}{3x}}$

Solution:

$$2\sqrt{\frac{150xy}{3x}} = 2\sqrt{50y} = 2\sqrt{5 \times 5 \times 2y}$$

$$= 2\sqrt{5^2}\sqrt{2y} = 2(5)\sqrt{2y} = 10\sqrt{2y}$$

Radical form of an Expression:

The number or quantity that is written under a radical sign ($\sqrt{\ }$ or $\sqrt[n]{\ }$) is called radical form of an expression.

Example:

$\sqrt{9}$ is the radical form of 3.

Exponential form of an Expression:

The number or quantity that is written in the form of exponent is called exponential form of an expression.

Example:

3^2 is the exponential form of 9.

Radical Form	**Exponential Form**
$\sqrt[n]{a}$	$a^{\frac{1}{n}}$
$\sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$	$a^{\frac{m}{n}}$
$\sqrt[n]{a^n}$	a

Some frequently used radicals are given in the following table

Square Root	**Cube Root**	**Fourth Root**
$\sqrt{1} = 1$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$
$\sqrt{4} = 2$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$
$\sqrt{9} = 3$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$
$\sqrt{16} = 4$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$
$\sqrt{25} = 5$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$
$\sqrt{36} = 6$	$\sqrt[3]{216} = 6$	$\sqrt[4]{1296} = 6$

Example 5 Page # 61

What is the difference between (i) $x^2 = 16$ (ii) $x = \sqrt{16}$?

(i) $x^2 = 16$

Solution:

$$x^2 = 16$$

This means what numbers squared becomes 16. Thus x can be 4 or -4 like $(4)^2 = 16$ and also $(-4)^2 = 16$.

Hence the value of $x = \pm 4$.

(ii) $x = \sqrt{16}$

Solution:

$$x = \sqrt{16}$$

Here x is the principal square root of 16, which has always a positive value such is $x = 4$.

Ex # 2.3

Page # 64

Q1: 64 **Write down the index and radicand for each of the following expressions.**

(i) $\sqrt{\frac{11}{y}}$

$index = 2, radicand = \frac{11}{y}$

(ii) $\sqrt[3]{\frac{13}{3x}}$

$index = 3, radicand = \frac{13}{3x}$

(iii) $\sqrt[5]{ab^2}$

$index = 5, radicand = ab^2$

Q2: 64 **Transform the following radical forms into exponential forms. Do not simplify.**

(i) $\sqrt{36}$

Exponential form= $(36)^{\frac{1}{2}}$

(ii) $\sqrt{1000}$

Exponential form= $(1000)^{\frac{1}{2}}$

(iii) $\sqrt[3]{8}$

Exponential form= $(8)^{\frac{1}{3}}$

(iv) $\sqrt[n]{q}$

Exponential form= $(q)^{\frac{1}{n}}$

(v) $\sqrt{(5-6a^2)^3}$

$((5-6a^2)^3)^{\frac{1}{2}}$

Exponential form= $(5-6a^2)^{\frac{3}{2}}$

(vi) $\sqrt[3]{-64}$

Exponential form= $(-64)^{\frac{1}{3}}$

Ex # 2.3

Q3: 64 **Transform the following exponential form of an expression into radical form.**

(i) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(ii) $x^{-\frac{3}{2}}$

$(x^{-3})^{\frac{1}{2}}$

$\sqrt{x^{-3}}$

(iii) $(-8)^{\frac{1}{5}}$

$\sqrt[5]{-8}$

(iv) $y^{\frac{3}{4}}$

$(y^3)^{\frac{1}{4}}$

$\sqrt[4]{y^3}$

(v) $b^{\frac{4}{5}}$

$(b^4)^{\frac{1}{5}}$

$\sqrt[5]{b^4}$

(vi) $(3x)^{\frac{1}{q}}$

$\sqrt[q]{3x}$

Q4: **Simplify:**

(i) $\sqrt[3]{125x}$

Solution:

$$\sqrt[3]{125x}$$

$$= (125x)^{\frac{1}{3}}$$

$$= (125)^{\frac{1}{3}}(x)^{\frac{1}{3}}$$

$$= (5 \times 5 \times 5)^{\frac{1}{3}}(x)^{\frac{1}{3}}$$

$$= (5^3)^{\frac{1}{3}}(x)^{\frac{1}{3}}$$

$$= 5(x)^{\frac{1}{3}}$$

$$= 5\sqrt[3]{x}$$

Ex # 2.3

(ii) $\sqrt[3]{\frac{8}{27}}$

$= \left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \left(\frac{2 \times 2 \times 2}{3 \times 3 \times 3}\right)^{\frac{1}{3}}$

$= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}}$

$$\frac{= (2^3)^{\frac{1}{3}}}{= (3^3)^{\frac{1}{3}}}$$

$= \frac{2}{3}$

(iii) $\sqrt{\frac{\mathbf{625x^3y^4}}{\mathbf{25xy^2}}}$

Solution:

$\sqrt{\frac{625x^3y^4}{25xy^2}}$

$= \sqrt{25x^2y^2}$

$= (25x^2y^2)^{\frac{1}{2}}$

$= (25)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}(y^2)^{\frac{1}{2}}$

$= 5xy$

(iv) $\sqrt{\mathbf{(3y-5)^2}}$

Solution:

$\sqrt{(3y-5)^2}$

$= [(3y-5)^2]^{\frac{1}{2}}$

$= 3y - 5$

Ex # 2.3

(v) $\mathbf{6\sqrt{18}}$

Solution:

$6\sqrt{18}$

$= 6(18)^{\frac{1}{2}}$

$= 6(3 \times 3 \times 2)^{\frac{1}{2}}$

$= 6(3^2 \times 2)^{\frac{1}{2}}$

$= 6(3^2)^{\frac{1}{2}}(2)^{\frac{1}{2}}$

$= 6(3)\sqrt{2}$

$= 18\sqrt{2}$

(vi) $\sqrt[3]{\mathbf{54x^3y^3z^2}}$

Solution:

$\sqrt[3]{54x^3y^3z^2}$

$= (54x^3y^3z^2)^{\frac{1}{3}}$

$= (54)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= (3 \times 3 \times 3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}(2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3)(x)(y)(2)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= 3xy(2z^2)^{\frac{1}{3}}$

$= 3xy\sqrt[3]{2z^2}$

Ex # 2.4

Base

جس کے اوپر power ہو اسے Base کہتے ہیں۔

Exponent /Power

Base کے اوپر جو چھوٹا سا نمبر ہوتا ہے اسے power کہتے ہیں۔ اس کو index بھی کہتے ہیں۔

Co-efficient

Base کے Left طرف جو نمبر ہوتا ہے اسے Co-efficient کہتے ہیں۔

Base اور Co-efficient آپس میں Multiply ہوتے ہیں

$4x^2$ Base: x Power: 2 Co-efficient: 4	**$5y^{-3}$** Base: y Power: −3 Co-efficient: 5	**$-2y^3$** Base: y Power: 3 Co-efficient: −2
x Base: x Power: 1 Co-efficient: 1	**x^3** Base: x Power: 3 Co-efficient: 1	**$5z$** Base: z Power: 1 Co-efficient: 5

Note:

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16} \qquad \frac{1}{3^{-3}} = 3^3 = 27$$

$$-4x^{-2} = \frac{-4}{x^2} \qquad (a+b)^{-1} = \frac{1}{(a+b)}$$

Laws of Exponents

Multiplication of Same Bases

To multiply powers of the same base, keep the same base and add the exponents.

اگر ایک جیسے bases آپس میں multiply ہوتے ہیں تو:

- ❖ **Co-efficient کو multiply کریں گے**
- ❖ **Base ایک لکھیں گے**
- ❖ **Powers کو Add کریں گے**

Example:

$$\boldsymbol{a^m . a^n = a^{m+n}}$$

Ex # 2.4

Multiplication of Different Bases

When different bases are multiplied just multiply the co-efficient or constant.

اگر مختلف bases آپس میں multiply ہوتے ہیں تو صرف Co-efficient کو multiply کریں گے

Law of Quotient

To divide two expressions with the same bases and different exponents, keep the same base and subtract the exponents.

جب fraction میں ایک جیسے bases ہو تو اس base کو اوپر لے جائیں گے لیکن اس کے power کا sign تبدیل ہو جائے گا۔

- ❖ اگر plus ہو گا تو minus ہو جائے گا
- ❖ اگر minus ہو گا تو plus ہو جائے گا

Law of Power of Power

To raise an exponential expression to a power, keep the same base multiply the exponents.

جب کسی بریکٹ کے اوپر Power آجائیں تو اس کو تمام Bases کے Powers ساتھ Multiply کریں گے۔

اگر Base یا Co-efficient کے ساتھ minus کا sign ہو تو:

1) جب power میں even نمبر ہو تو expression کے ساتھ plus کا sign لگائیں گے۔

$$(-x)^{22} = x^{22} \qquad (-4y)^2 = 16y^2$$

2) جب power میں Odd نمبر ہو تو expression کے ساتھ plus کا minus لگائیں گے۔

$$(-x)^{25} = -x^{25} \qquad (-2y)^3 = -8y^3$$

Zero Exponent Rule

Any non-zero number raised to the zero power equals one.

کسی بھی Base کا Power اگر Zero ہو تو 1 کے برابر ہو گا۔

$100^0 = 1$ and $(xy)^0 = 1$

Ex # 2.4

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Q1: **Write the base, exponent and value of the following.**

(i) $(\mathbf{2})^{-9} = \dfrac{\mathbf{1}}{\mathbf{1024}}$

$base = 2,\ Exponent = -9, value = \frac{1}{1024}$

(ii) $\left(\dfrac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{\boldsymbol{p}} = \dfrac{\boldsymbol{a}^{\boldsymbol{p}}}{\boldsymbol{b}^{\boldsymbol{p}}}$

$base = \frac{a}{b},\ Exponent = p,\ \ value = \frac{a^p}{b^p}$

(iii) $(-\mathbf{4})^{\mathbf{2}} = \mathbf{16}$

$base = -4,\ \ Exponent = 2,\ \ value = 16$

Q2: 67 **If a, b denote the real numbers then simplify the following.**

(i) $\boldsymbol{a}^{\mathbf{3}} \times \boldsymbol{a}^{\mathbf{5}}$

Solution:

$a^3 \times a^5$

$= a^{3+5}$

$= a^8$

(ii) $\left(\dfrac{\boldsymbol{b}}{\boldsymbol{a}}\right)^{\frac{\mathbf{3}}{\mathbf{2}}}\left(\dfrac{\boldsymbol{b}}{\boldsymbol{a}}\right)^{-\frac{\mathbf{2}}{\mathbf{3}}}$

Solution:

$\left(\dfrac{b}{a}\right)^{\frac{3}{2}}\left(\dfrac{b}{a}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}$

$= \left(\dfrac{b}{a}\right)^{\frac{9-4}{6}}$

$= \left(\dfrac{b}{a}\right)^{\frac{5}{6}}$

(iii) $(-\boldsymbol{a})^{\mathbf{4}} \times (-\boldsymbol{a})^{\mathbf{3}}$

Solution:

$(-a)^4 \times (-a)^3$

$= (-a)^{4+3}$

$= (-a)^7$

$= -a^7$

(iv) $\left(-\mathbf{2}\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b}^{\mathbf{3}}\right)^{\mathbf{3}}$

Solution:

$(-2a^2b^3)^3$

$= (-2)^3a^{2\times3}b^{3\times3}$

$= -8a^6b^9$

(v) $\boldsymbol{a}^{\mathbf{3}}(-\mathbf{2}\boldsymbol{b})^{\mathbf{2}}$

Solution:

$= a^3(-2b)^2$

$= a^3(-2)^2(b)^2$

$= a^3 \times 4b^2$

$= 4a^3b^2$

(vi) $\left(\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b}\right)\left(\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b}\right)$

Solution:

$(a^2b)(a^2b)$

$= a^{2+2}b^{1+1}$

$= a^4b^2$

(vii) $\dfrac{\mathbf{a}^{\mathbf{0}}.\,\boldsymbol{b}^{\mathbf{0}}}{\mathbf{2}}$

Solution:

$\dfrac{\mathrm{a}^0.\,b^0}{2}$

$= \dfrac{1 \times 1}{2}$

$= \dfrac{1}{2}$

(viii) $\left(-\mathbf{3}\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b}^{\mathbf{2}}\right)^{\mathbf{2}}$

Solution:

$(-3a^2b^2)^2$

$= (-3)^2a^{2\times2}b^{2\times2}$

$= 9a^4b^4$

Ex # 2.4

(ix) $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

Solution:

$$\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$$

$$=\frac{a^{2\times\frac{3}{2}}}{b^{4\times\frac{3}{2}}}$$

$$=\frac{a^{1\times3}}{b^{2\times3}}$$

$$=\frac{a^3}{b^6}$$

Q3: Simplify the following.

(i) $\frac{7^6}{7^4}$

Solution:

$$\frac{7^6}{7^4}$$

$$=7^6.7^{-4}$$

$$=7^{6-4}$$

$$=7^2$$

(ii) $\frac{2^4.5^3}{10^2}$

Solution:

$$\frac{2^4.5^3}{10^2}$$

$$=\frac{2^4.5^3}{(2\times5)^2}$$

$$=\frac{2^4.5^3}{2^2.5^2}$$

$$=2^4.5^3.2^{-2}.5^{-2}$$

$$=2^{4-2}.5^{3-2}$$

$$=2^2.5^1$$

$$=4\times5$$

$$=20$$

Ex # 2.4

(iii) $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

Solution:

$$\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$$

$$=\frac{(a+b)^{2\times3}.(c+d)^{3\times3}}{(a+b)^{1\times3}.(c+d)^{2\times3}}$$

$$=\frac{(a+b)^6.(c+d)^9}{(a+b)^3.(c+d)^6}$$

$$=(a+b)^6.(c+d)^9.(a+b)^{-3}.(c+d)^{-6}$$

$$=(a+b)^{6-3}.(c+d)^{9-6}$$

$$=(a+b)^3.(c+d)^3$$

(iv) $(\sqrt[3]{a})^{\frac{1}{2}}$

Solution:

$$(\sqrt[3]{a})^{\frac{1}{2}}$$

$$=\left(a^{\frac{1}{3}}\right)^{\frac{1}{2}}$$

$$=a^{\frac{1}{3}\times\frac{1}{2}}$$

$$=a^{\frac{1}{6}}$$

(v) $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

Solution:

$$\sqrt[5]{x^5}.\sqrt[4]{x^4}$$

$$=(x^5)^{\frac{1}{5}}(x^4)^{\frac{1}{4}}$$

$$=(x)^{5\times\frac{1}{5}}.(x)^{4\times\frac{1}{4}}$$

$$=x.x$$

$$=x^2$$

Ex # 2.4

Q4: 67 **Simplify the following in such a way that no answers should contain fractional or negative exponent.**

(i) $\left(\frac{25}{81}\right)^{\frac{1}{2}}$

Solution:

$$\left(\frac{25}{81}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5\times5}{9\times9}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5^2}{9^2}\right)^{\frac{1}{2}}$$

$$= \frac{5^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$

$$= \frac{5}{9}$$

(ii) $\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$

Solution:

$$\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$$

$$= \frac{(ab)^{\frac{1}{b}}}{((ab)^{-1})^{\frac{1}{a}}}$$

$$= \frac{(ab)^{\frac{1}{b}}}{(ab)^{-\frac{1}{a}}}$$

$$= (ab)^{\frac{1}{b}}.(ab)^{\frac{1}{a}}$$

$$= (ab)^{\frac{1}{b}+\frac{1}{a}}$$

$$= (ab)^{\frac{a+b}{ba}}$$

$$= (ab)^{\frac{a+b}{ab}}$$

$$= a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$$

Ex # 2.4

(iii) $\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$

Solution:

$$\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$$

$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.(2\times3)^q}{(2\times3)^p.(2\times5)^{q+2}.(3\times5)^p}$$

$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.2^q.3^q}{2^p.3^p.2^{q+2}.5^{q+2}.3^p.5^p}$$

$$= \frac{2^{p+1+q}.3^{2p-q+q}.5^{p+q}}{2^{p+q+2}.3^{p+p}.5^{q+2+p}}$$

$$= \frac{2^{p+1+q}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+2+p}}$$

$$= 2^{p+1+q}.3^{2p}.5^{p+q}.2^{-p-q-2}.3^{-2p}.5^{-q-2-p}$$

$$= 2^{p+1+q-p-q-2}.3^{2p-2p}.5^{p+q-q-2-p}$$

$$= 2^{1-2}.3^0.5^{-2}$$

$$= 2^{-1}.3^0.5^{-2}$$

$$= \frac{1}{2}\times1\times\frac{1}{5^2}$$

$$= \frac{1}{2}\times1\times\frac{1}{25}$$

$$= \frac{1}{50}$$

(iv) $\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$

Solution:

$$\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$$

$$= (x^p.x^{-q})^{p+q}(x^q.x^{-r})^{q+r}(x^r.x^{-p})^{r+p}$$

$$= (x^{p-q})^{p+q}(x^{q-r})^{q+r}(x^{r-p})^{r+p}$$

$$= (x)^{(p-q)(p+q)}.(x)^{(q-r)(q+r)}.(x)^{(r-p)(r+p}$$

$$= (x)^{p^2-q^2}.(x)^{q^2-r^2}.(x)^{r^2-p^2}$$

$$= x^{p^2-q^2+q^2-r^2+r^2-p^2}$$

$$= x^0$$

$$= 1$$

Ex # 2.4

Q5: 67 **Prove that** $\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$

Solution:

$$\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$$

L.H.S

$$= \left(\frac{(2^2)^5.(2^6)^3.2^3}{(2^3)^5.(2^7)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10}.2^{18}.2^3}{2^{15}.2^{14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10+18+3}}{2^{15+14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{31}}{2^{29}}\right)^{\frac{1}{2}}$$

$$= (2^{31-29})^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}}$$

$$= 2^{2\times\frac{1}{2}}$$

$$= 2$$

=R.H.S

Ex # 2.5

Complex Number

A number of the form $a + bi$ where a and b are real numbers is called complex number where "a" is called real part and "b" is called imaginary part.

Conjugate of a Complex Numbers

A conjugate of a complex number is obtained by changing the sign of imaginary part. The conjugate of $a + bi$ is $a - bi$ or the conjugate of $a + bi$ is denoted by $\overline{a + bi} = a - bi$.

Ex # 2.5

Equality of Two Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then $Z_1 = Z_2$ if real parts are equal i.e. $a = c$ and imaginary parts are equal i.e. $b = d$.

Operation on Complex Numbers

Addition of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$Z_1 + Z_2 = (a + bi) + (c + di)$$
$$Z_1 + Z_2 = a + bi + c + di$$
$$Z_1 + Z_2 = a + c + bi + di$$
$$Z_1 + Z_2 = (a + c) + (b + d)i$$

Subtraction of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$Z_1 - Z_2 = (a + bi) - (c + di)$$
$$Z_1 - Z_2 = a + bi - c - di$$
$$Z_1 - Z_2 = a - c + bi - di$$
$$Z_1 - Z_2 = (a - c) + (b - d)i$$

Multiplication of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$Z_1.Z_2 = (a + bi)(c + di)$$
$$Z_1.Z_2 = ac + adi + bci + bdi^2$$
$$Z_1.Z_2 = ac + (ad + bc)i + bd(-1) \text{ as } i^2 = -1$$
$$Z_1.Z_2 = ac + (ad + bc)i - bd$$
$$Z_1.Z_2 = (ac - bd) + (ad + bc)i$$

Division of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di}$$

Multiply and Divide by $c - di$

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$$

$$\frac{Z_1}{Z_2} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$\frac{Z_1}{Z_2} = \frac{ac - adi + bci - bdi^2}{c^2 - (di)^2}$$

$$\frac{Z_1}{Z_2}=\frac{ac+bci-adi-bd(-1)}{c^2-d^2i^2} \qquad As\ i^2=-1$$

$$\frac{Z_1}{Z_2}=\frac{ac+(bc-ad)i+bd}{c^2-d^2(-1)}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)}{c^2+d^2}+\frac{(bc-ad)i}{c^2+d^2}$$

Ex # 2.5

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Q1: Add the following complex number

(i) $8+9\mathrm{i}, 5+2\mathrm{i}$

Solution:

$8+9i, 5+2i$

Let $Z_1=8+9i$

And $Z_2=5+2i$

Now

$$Z_1+Z_2=(8+9i)+(5+2i)$$
$$Z_1+Z_2=8+9i+5+2i$$
$$Z_1+Z_2=8+5+9i+2i$$
$$Z_1+Z_2=13+11i$$

(ii) $6+3i, 3-5i$

Solution:

$6+3i, 3-5i$

Let $Z_1=6+3i$

And $Z_2=3-5i$

Now

$$Z_1+Z_2=(6+3i)+(3-5i)$$
$$Z_1+Z_2=6+3i+3-5i$$
$$Z_1+Z_2=6+3+3i-5i$$
$$Z_1+Z_2=9-2i$$

(iii) $2i+3, 8-5\sqrt{-1}$

Solution:

$2i+3, 8-5\sqrt{-1}$

Let $Z_1=2i+3$

And $Z_2=8-5\sqrt{-1}$

$8-5i \quad \therefore \sqrt{-1}=i$

Now

$$Z_1+Z_2=(2i+3)+(8-5i)$$
$$Z_1+Z_2=2i+3+8-5i$$
$$Z_1+Z_2=3+8+2i-5i$$
$$Z_1+Z_2=11-3i$$

(iv) $\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Solution:

$\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Let $Z_1=\sqrt{3}+\sqrt{2}i$

And $Z_2=3\sqrt{3}-2\sqrt{2}i$

Now

$$Z_1+Z_2=\left(\sqrt{3}+\sqrt{2}i\right)+\left(3\sqrt{3}-2\sqrt{2}i\right)$$
$$Z_1+Z_2=\sqrt{3}+\sqrt{2}i+3\sqrt{3}-2\sqrt{2}i$$
$$Z_1+Z_2=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$$
$$Z_1+Z_2=4\sqrt{3}-\sqrt{2}i$$

Q2: Subtract:

(i) $-2+3i$ from $6-3i$

Solution:

$-2+3i$ from $6-3i$

Let $Z_1=-2+3i$

And $Z_2=6-3i$

Now

$$Z_2-Z_1=(6-3i)-(-2+3i)$$
$$Z_2-Z_1=6-3i+2-3i$$
$$Z_2-Z_1=6+2-3i-3i$$
$$Z_2-Z_1=8-6i$$

(ii) $9+4i$ from $9-8i$

Solution:

$9+4i$ from $9-8i$

Let $Z_1=9+4i$

And $Z_2=9-8i$

Now

$$Z_2-Z_1=(9-8i)-(9+4i)$$
$$Z_2-Z_1=9-8i-9-4i$$
$$Z_2-Z_1=9-9-8i-4i$$
$$Z_2-Z_1=0-12i$$
$$Z_2-Z_1=-12i$$

Ex # 2.5

(iii) $\mathbf{1-3i}$ **from** $\mathbf{8-i}$

Solution:

$1-3i$ from $8-i$

Let $Z_1 = 1-3i$

And $Z_2 = 8-i$

Now

$Z_2 - Z_1 = (8-i)-(1-3i)$

$Z_2 - Z_1 = 8-i-1+3i$

$Z_2 - Z_1 = 8-1-i+3i$

$Z_2 - Z_1 = 7+2i$

(iv) $6-7i$ from $6+7i$

Solution:

$6-7i$ from $6+7i$

Let $Z_1 = 6-7i$

And $Z_2 = 6+7i$

Now

$Z_2 - Z_1 = (6+7i)-(6-7i)$

$Z_2 - Z_1 = 6+7i-6+7i$

$Z_2 - Z_1 = 6-6+7i+7i$

$Z_2 - Z_1 = 0+14i$

$Z_2 - Z_1 = 14i$

Q3: Multiply the following complex numbers

(i) $\mathbf{1+2i, 3-8i}$

Solution:

$1+2i, 3-8i$

Let $Z_1 = 1+2i$

And $Z_2 = 3-8i$

Now

$Z_1.Z_2 = (1+2i)(3-8i)$

$Z_1.Z_2 = 1(3-8i)+2i(3-8i)$

$Z_1.Z_2 = 3-8i+6i-16i^2$

$Z_1.Z_2 = 3-2i-16(-1)$

$Z_1.Z_2 = 3-2i+16$

$Z_1.Z_2 = 3+16-2i$

$Z_1.Z_2 = 19-2i$

(ii) $\mathbf{2i, 4-7i}$

Solution:

$2i, 4-7i$

Let $Z_1 = 2i$

And $Z_2 = 4-7i$

Ex # 2.5

Now

$Z_1.Z_2 = (2i)(4-7i)$

$Z_1.Z_2 = 2i(4-7i)$

$Z_1.Z_2 = 8i-14i^2$

$Z_1.Z_2 = 8i-14(-1)$

$Z_1.Z_2 = 8i+14$

$Z_1.Z_2 = 14+8i$

(iii) $\mathbf{5-3i, 2-4i}$

Solution:

$5-3i, 2-4i$

Let $Z_1 = 5-3i$

And $Z_2 = 2-4i$

Now

$Z_1.Z_2 = (5-3i)(2-4i)$

$Z_1.Z_2 = 5(2-4i)-3i(2-4i)$

$Z_1.Z_2 = 10-20i-6i+12i^2$

$Z_1.Z_2 = 10-26i+12(-1)$

$Z_1.Z_2 = 10-26i-12$

$Z_1.Z_2 = 10-12-26i$

$Z_1.Z_2 = -2-26i$

(iv) $\mathbf{\sqrt{2}+i, 1-\sqrt{2}i}$

Solution:

$\sqrt{2}+i, 1-\sqrt{2}i$

Let $Z_1 = \sqrt{2}+i$

And $Z_2 = 1-\sqrt{2}i$

Now

$Z_1.Z_2 = (\sqrt{2}+i)(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}(1-\sqrt{2}i)+i(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}-\sqrt{2\times 2}i+1i-\sqrt{2}i^2$

$Z_1.Z_2 = \sqrt{2}-2i+1i-\sqrt{2}(-1)$

$Z_1.Z_2 = \sqrt{2}-i+\sqrt{2}$

$Z_1.Z_2 = \sqrt{2}+\sqrt{2}-i$

$Z_1.Z_2 = 2\sqrt{2}-i$

Ex # 2.5

Q4: Divide the first complex number by the second.

(i) $\boldsymbol{Z_1 = 2 + i, Z_2 = 5 - i}$

Solution:

$$Z_1 = 2 + i, Z_2 = 5 - i$$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i}$$

Multiply and divide by $5 + i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i} \times \frac{5+i}{5+i}$$

$$\frac{Z_1}{Z_2} = \frac{(2+i)(5+i)}{(5-i)(5+i)}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 2i + 5i + i^2}{(5)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i + (-1)}{25 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i - 1}{25 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{10 - 1 + 7i}{25 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{9 + 7i}{26}$$

$$\frac{Z_1}{Z_2} = \frac{9}{26} + \frac{7}{26}i$$

(ii) $\boldsymbol{Z_1 = 3i + 4, Z_2 = 1 - i}$

Solution:

$$Z_1 = 3i + 4$$

$$4 + 3i$$

$$Z_2 = 1 - i$$

$$\frac{Z_1}{Z_2} = \frac{4 + 3i}{1 - i}$$

Multiply and divide by $1 + i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i} \times \frac{1+i}{1+i}$$

Ex # 2.5

$$\frac{Z_1}{Z_2} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 4i + 3i + 3i^2}{(1)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i + 3(-1)}{1 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i - 3}{1 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{4 - 3 + 7i}{1 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{1 + 7i}{2}$$

$$\frac{Z_1}{Z_2} = \frac{1}{2} + \frac{7}{2}i$$

Q5: Perform the indicated operations and reduce to the form $\boldsymbol{a + bi}$

(i) $\boldsymbol{(4 - 3i) + (2 - 3i)}$

Solution:

$(4 - 3i) + (2 - 3i)$
$= 4 - 3i + 2 - 3i$
$= 4 + 2 - 3i - 3i$
$= 6 - 6i$

(ii) $\boldsymbol{(5 - 2i) - (4 - 7i)}$

Solution:

$(5 - 2i) - (4 - 7i)$
$= 5 - 2i - 4 + 7i$
$= 5 - 4 - 2i + 7i$
$= 1 + 5i$

(iii) $\boldsymbol{2i(4 - 5i)}$

Solution:

$2i(4 - 5i)$
$= 2i - 10i^2$
$= 2i - 10(-1)$
$= 2i + 10$
$= 10 + 2i$

Ex # 2.5

(iv) $(\mathbf{2-3}\boldsymbol{i})\div(\mathbf{4-5}\boldsymbol{i})$

Solution:

$(2-3i)\div(4-5i)$

$=\dfrac{2-3i}{4-5i}$

Multiply and divide by $4+5i$

$=\dfrac{2-3i}{4-5i}\times\dfrac{4+5i}{4+5i}$

$=\dfrac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$

$=\dfrac{8+10i-12i-15i^2}{(4)^2-(5i)^2}$

$=\dfrac{8-2i-15(-1)}{16-25i^2}$

$=\dfrac{8-2i+15}{16-25(-1)}$

$=\dfrac{8+15-2i}{16+25}$

$=\dfrac{23-2i}{41}$

$=\dfrac{23}{41}-\dfrac{2}{41}i$

Q6: **Find the complex conjugate of the following complex numbers.**

(i) $-8-3\text{i}$

The complex conjugate of $-8-3i$ is $-8+3i$

(i) $\mathbf{-4+9}\boldsymbol{i}$

The complex conjugate of $-4+9i$ is $-4-9i$

(iii) $\mathbf{7+6}\boldsymbol{i}$

The complex conjugate of $7+6i$ is $7-6i$

(iv) $\sqrt{\mathbf{5}}-\boldsymbol{i}$

The complex conjugate of $\sqrt{5}-i$ is $\sqrt{5}+i$

Review Ex # 2

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Q3: **Simplify each of the following.**

(i) $\left(\dfrac{\mathbf{-2}}{\mathbf{3}}\right)^{\mathbf{3}}$

Solution:

$\left(\dfrac{-2}{3}\right)^3$

$=\dfrac{(-2)^3}{(3)^3}$

$=\dfrac{-8}{27}$

(ii) $(\mathbf{-2})^{\mathbf{3}}.(\mathbf{3})^{\mathbf{2}}$

Solution:

$(-2)^3.(3)^2$

$=-8\times9$

-72

(iii) $\mathbf{-3}\sqrt{\mathbf{48}}$

Solution:

$-3\sqrt{48}$

$-3\sqrt{4\times4\times3}$

$-3\sqrt{4\times4}\times\sqrt{3}$

$-3\times4\sqrt{3}$

$-12\sqrt{3}$

(iv) $\dfrac{\mathbf{5}}{\sqrt[3]{\mathbf{9}}}$

Solution:

$\dfrac{5}{\sqrt[3]{9}}$

$=\dfrac{5}{(9)^{\frac{1}{3}}}$

$=\dfrac{5}{(3^2)^{\frac{1}{3}}}$

$=\dfrac{5}{(3)^{\frac{2}{3}}}$

Multiply and Divide by $\sqrt[3]{3}$

$$\frac{5}{(3)^{\frac{2}{3}}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

$$\frac{5 \times \sqrt[3]{3}}{(3)^{\frac{2}{3}} \times (3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{2}{3}+\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{3}$$

Q4: **Multiply $8i, \quad -8i$**

Solution:

$8i, \quad -8i$

Now

$(8i)(-8i) = -64i^2$

$= -64(-1)$

$= 64$

Q5: **Divide $2 - 5i$ by $1 - 6i$**

Solution:

$$\frac{2 - 5i}{1 - 6i}$$

i

Multiply and divide by $1 + 6i$

$$= \frac{2 - 5i}{1 - 6} \times \frac{1 + 6i}{1 + 6i}$$

$$= \frac{(2 - 5i)(1 + 6i)}{(1 - 6i)(1 + 6i)}$$

$$= \frac{2 + 12i - 5i - 30i^2}{(1)^2 - (6i)^2}$$

$$= \frac{2 + 7i - 30(-1)}{1 - 36i^2}$$

$$= \frac{2 + 7i + 30}{1 - 36(-1)}$$

$$= \frac{2 + 30 + 7i}{1 + 36}$$

$$= \frac{32 + 7i}{37}$$

$$= \frac{32}{37} - \frac{7}{37}i$$

Q7: **Use laws of exponents to simplify:**

$$\frac{(\mathbf{81})^{n}.\mathbf{3}^{5} + (\mathbf{3})^{4n-1}(\mathbf{243})}{(\mathbf{9}^{2n})(\mathbf{3}^{3})}$$

Solution:

$$\frac{(81)^{n}.3^{5} + (3)^{4n-1}(243)}{(9^{2n})(3^{3})}$$

$$= \frac{(3^4)^{n}.3^{5} + 3^{4n-1}.(3^{5})}{(3^2)^{2n}(3^{3})}$$

$$= \frac{3^{4n}.3^{5} + 3^{4n}.3^{-1}.3^{5}}{3^{4n}.3^{3}}$$

$$= \frac{3^{4n}.3^{5}(1 + 3^{-1})}{3^{4n}.3^{3}}$$

$$= \frac{3^{4n}.3^{3}.3^{2}(1 + 3^{-1})}{3^{4n}.3^{3}}$$

$= 3^2(1 + 3^{-1})$

$$= 9\left(1 + \frac{1}{3}\right)$$

$$= 9\left(\frac{3 + 1}{3}\right)$$

$$= 9\left(\frac{4}{3}\right)$$

$= 3 \times 4$

$= 12$

Q6: **Name the property used**

$$\mathbf{7} \times \frac{\mathbf{1}}{\mathbf{7}} = \frac{\mathbf{1}}{\mathbf{7}} \times \mathbf{7} = \mathbf{1}$$

Answer:

Multiplicative Property

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MATHEMATICS

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UNIT # 3
LOGARITHM

Exercise # 3.1

SCIENTIFIC NOTATION:

Scientific notation is a way of writing numbers that are too big or too small to be easily written in decimal form.

Representation

The positive number "x" is represented in scientific notation as the product of two numbers where the first number "a" is a real number greater than 1 and less than 10 and the second is the integral power of "n" of 10.

$$x = a \times 10^n$$

Rules for Standard Notation to Scientific Notation

(i) In a given number, place the decimal after first non-zero digit.

(ii) If the decimal point is moved towards left, then the power of "10" will be positive.

(iii) If the decimal is moved towards right, then the power of "10" will be negative.
The numbers of digits through which the decimal point has been moved will be the exponent.

Rules for Standard Notation to Scientific Notation

(i) If the exponent of 10 is Positive, move the decimal towards Right.

(ii) If the exponent of 10 is Negative, move the decimal toward Left.

(iii) Move the decimal point to the same number of digits as the exponent of 10.

Example # 7 Page # 80

How many miles does light travel in 1 day? The speed of the light is 186,000 mi/ sec. write the answer in scientific notation.

Solution:

$$Time = t = 1\ day = 24\ hr$$
$$t = 24 \times 60 \times 60\ sec = 86400$$
$$t = 8.64 \times 10^4\ sec$$
$$Speed = v = 186000\ mi/sec$$
$$v = 1.86 \times 10^5\ \ mi/sec$$

As we know that

$$s = vt$$

Put the values

$$s = 1.86 \times 10^5 \times 8.64 \times 10^4$$
$$s = 1.86 \times 8.64 \times 10^5 \times 10^4$$
$$s = 16.0704 \times 10^{5+4}$$
$$s = 16.0704 \times 10^9$$
$$s = 1.60704 \times 10^1 \times 10^9$$
$$s = 1.60704 \times 10^{10}$$

Thus light travels $1.60704 \times 10^1 \times 10^9$ miles in a day

Exercise # 3.1
Page # 80

Q1: Write each number in scientific notation.

(i) 405,000

Solution:

405,000

In Scientific Form:

$$4.05 \times 10^4$$

(ii) 1,670,000

Solution:

1,670,000

In Scientific Form:

$$1.67 \times 10^6$$

(iii) 0.00000039

Solution:

0.00000039

In Scientific Form:

$$3.9 \times 10^{-7}$$

(iv) 0.00092

Solution:

0.00092

In Scientific Form:

$$9.2 \times 10^{-4}$$

Ex # 3.1

(v) **234,600,000,000**

Solution:

234,600,000,000

In Scientific Form:

$$2.346 \times 10^{11}$$

(vi) **8,904,000,000**

Solution:

8,904,000,000

In Scientific Form:

$$8.904 \times 10^{9}$$

(vii) **0.00104**

Solution:

0.00104

In Scientific Form:

1.04×10^{-3}

(viii) **0.00000000514**

Solution:

0.00000000514

In Scientific Form:

$$5.14 \times 10^{-9}$$

(ix) $\mathbf{0.05 \times 10^{-3}}$

Solution:

$$0.05 \times 10^{-3}$$

In Scientific Form:

$$5.0 \times 10^{-2} \times 10^{-3}$$
$$5.0 \times 10^{-2-3}$$
$$5.0 \times 10^{-5}$$

Q2: Write each number in standard notation.

(i) $\mathbf{8.3 \times 10^{-5}}$

Solution:

8.3×10^{-5}

In Standard Form:

0.000083

(ii) $\mathbf{4.1 \times 10^{6}}$

Solution:

4.1×10^{6}

In Standard Form:

410000

Ex # 3.1

(iii) $\mathbf{2.07 \times 10^{7}}$

Solution:

2.07×10^{7}

In Standard Form:

20700000

(iv) $\mathbf{3.15 \times 10^{-6}}$

Solution:

3.15×10^{-6}

In Standard Form:

0.00000315

(v) $\mathbf{6.27 \times 10^{-10}}$

Solution:

6.27×10^{-10}

In Standard Form:

0.000000000627

(vi) $\mathbf{5.41 \times 10^{-8}}$

Solution:

5.41×10^{-8}

In Standard Form:

0.0000000541

(vii) $\mathbf{7.632 \times 10^{-4}}$

Solution:

7.632×10^{-4}

In Standard Form:

0.0007632

(viii) $\mathbf{9.4 \times 10^{5}}$

Solution:

9.4×10^{5}

In Standard Form:

940000

(ix) $\mathbf{-2.6 \times 10^{9}}$

Solution:

-2.6×10^{9}

In Standard Form:

−2600000000

Ex # 3.1

Q3: How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7miles from Earth, and light travels 1.86×10^5 mi/s.

Solution:

Given:

Distance between earth and sun $= 9.3 \times 10^7$ miles

Speed of light $= 1.86 \times 10^5$ mi/s

As we have:

$$s = vt$$

$$\frac{s}{v} = t$$

Or

$$t = \frac{s}{v}$$

Put the values:

$$t = \frac{9.3 \times 10^7}{1.86 \times 10^5}$$

$$t = 5 \times 10^7 \times 10^{-5}$$

$$t = 5 \times 10^{7-5}$$

$$t = 5 \times 10^2$$

$$t = 500\ sec$$

$$t = 480\ \ \text{sec} + 20\ sec$$

$$t = 8\ \text{min}\ 20\ sec$$

Exercise # 3.2

Logarithm

If $a^x = y$ then the index x is called the logarithm of y to the base a and written as $\log_a y = x$.

We called $\log_a y = x$ like log of y to the base a equal to x.

Logarithm Form	Exponential Form
$\log_a y = x$	$a^x = y$
$\log_8 64 = 2$	$8^2 = 64$

Ex # 3.2

Page # 83

Q1: Write the following in logarithm form.

(i) $4^4 = 256$

Solution:

$4^4 = 256$

In logarithm form

$\log_4 256 = 4$

(ii) $2^{-6} = \frac{1}{64}$

Solution:

$2^{-6} = \frac{1}{64}$

In logarithm form

$\log_2 \frac{1}{64} = -6$

(iii) $10^0 = 1$

Solution:

$10^0 = 1$

In logarithm form

$\log_{10} 1 = 0$

(iv) $x^{\frac{3}{4}} = y$

Solution:

$x^{\frac{3}{4}} = y$

In logarithm form

$\log_x y = \frac{3}{4}$

(v) $3^{-4} = \frac{1}{81}$

Solution:

$3^{-4} = \frac{1}{81}$

In logarithm form

$\log_3 \frac{1}{81} = -4$

(vi) $64^{\frac{2}{3}} = 16$

Solution:

$64^{\frac{2}{3}} = 16$

In logarithm form

$\log_{64} 16 = \frac{2}{3}$

Ex # 3.2

Q2: Write the following in exponential form.

(i) $\log_a\left(\frac{1}{a^2}\right) = -1$

Solution:

$\log_a\left(\frac{1}{a^2}\right) = -1$

In exponential form

$a^{-1} = \frac{1}{a^2}$

(ii) $\log_2 \frac{1}{128} = -7$

Solution:

$\log_2 \frac{1}{128} = -7$

In exponential form

$2^{-7} = \frac{1}{128}$

(iii) $\log_b 3 = 64$

Solution:

$\log_b 3 = 64$

In exponential form

$b^{64} = 3$

(iv) $\log_a a = 1$

Solution:

$\log_a a = 1$

In exponential form

$a^1 = 1$

(v) $\log_a 1 = 0$

Solution:

$\log_a 1 = 0$

In exponential form

$a^0 = 1$

(vi) $\log_4 \frac{1}{8} = \frac{-3}{2}$

Solution:

$\log_4 \frac{1}{8} = \frac{-3}{2}$

In exponential form

$4^{\frac{-3}{2}} = \frac{1}{8}$

Ex # 3.2

Q3: Solve:

(i) $\log_{\sqrt{5}} 125 = x$

Solution:

$\log_{\sqrt{5}} 125 = x$

In exponential form

$(\sqrt{5})^x = 125$

$\left(5^{\frac{1}{2}}\right)^x = 5 \times 5 \times 5$

$5^{\frac{x}{2}} = 5^3$

Now

$\frac{x}{2} = 3$

Multiply B.S by 2

$2 \times \frac{x}{2} = 2 \times 3$

$x = 6$

(ii) $\log_4 x = -3$

Solution:

$\log_4 x = -3$

In exponential form

$4^{-3} = x$

Now

$\frac{1}{4^3} = x$

$\frac{1}{4 \times 4 \times 4} = x$

$\frac{1}{64} = x$

Or

$x = \frac{1}{64}$

(iii) $\log_{81} 9 = x$

Solution:

$\log_{81} 9 = x$

In exponential form

$81^x = 9$

$(9^2)^x = 9^1$

$9^{2x} = 9^1$

Now $2x = 1$

Divide B.S by 2

$\frac{2x}{2} = \frac{1}{2}$

$2x = \frac{1}{2}$

Ex # 3.2

(iv) $\mathbf{\log_3(5x+1)=2}$

Solution:

$\log_3(5x+1)=2$

In exponential form

$3^2=5x+1$

$9=5x+1$

Subtract 1 form B.S

$9-1=5x+1-1$

$8=5x$

Divide B.S by 5

$\frac{8}{5}=\frac{5x}{5}$

$\frac{8}{5}=x$

$x=\frac{8}{5}$

(v) $\mathbf{\log_2 x=7}$

Solution:

$\log_2 x=7$

In exponential form

$2^7=x$

Now

$2\times2\times2\times2\times2\times2\times2=x$

$128=x$

$x=128$

(vi) $\mathbf{\log_x 0.25=2}$

Solution:

$\log_x 0.25=2$

In exponential form

$x^2=0.25$

$\mathbf{x^2=\frac{25}{100}}$

Taking square root on B.S

$\sqrt{x^2}=\sqrt{\frac{25}{100}}$

$x=\frac{5}{10}$

$x=\frac{1}{2}$

Ex # 3.2

(vii) $\mathbf{\log_x(0.001)=-3}$

Solution:

$\log_x(0.001)=-3$

In exponential form

$x^{-3}=0.001$

$x^{-3}=\frac{1}{1000}$

$x^{-3}=\frac{1}{10^3}$

$x^{-3}=10^{-3}$

So

$x=10$

(viii) $\mathbf{\log_x\frac{1}{64}=-2}$

Solution:

$\log_x\frac{1}{64}=-2$

In exponential form

$x^{-2}=\frac{1}{64}$

$x^{-2}=\frac{1}{8\times8}$

$x^{-2}=\frac{1}{8^2}$

$x^{-2}=8^{-2}$

So

$x=8$

(ix) $\mathbf{\log_{\sqrt{3}}x=16}$

Solution:

$\log_{\sqrt{3}}x=16$

In exponential form

$(\sqrt{3})^{16}=x$

$\left(3^{\frac{1}{2}}\right)^{16}=x$

$3^{\frac{16}{2}}=x$

$3^8=x$

$3\times3\times3\times3\times3\times3\times3\times3=x$

$6561=x$

$x=6561$

Exercise # 3.3

COMMON LOGARITHM

Introduction

The common logarithm was invented by a British Mathematician Prof. Henry Briggs (1560-1631).

Definition

Logarithms having base 10 are called common logarithms or Briggs logarithms.

Note:

The base of logarithm is not written because it is considered to be 10.

Logarithm of the number consists of two parts. Characteristics and Mantissa

Example: 1.5377

Characteristics

The digit before the decimal point or Integral part is called characteristics

Mantissa

The decimal fraction part is mantissa.

In above example

1 is Characteristics and .5377 is Mantissa.

USE OF LOG TABLE TO FIND MANTISSA:

A logarithm table is divided into three parts.

(i) The first part of the table is the extreme left column contains number from10 to 99.

(ii) The second part of the table consists of 10 columns headed by 0, 1, 2, …. 9. The number under these columns are taken to find mantissa.

(iii) The third part consists of small columns known as mean difference headed by 1, 2, 3, … 9. These columns are added to the Mantissa found in second column.

To Find Mantissa

Let we have an example: 763.5

Solution:

(i) First ignore the decimal point.

(ii) Take first two digits e.g. 76 and proceed along this row until we come to column headed by third digit 3 of the number which is 8825.

(iii) Now take fourth digit i.e. 5 and proceed along this row in mean difference column which is 5.

(iv) Now add $8825 + 3 = 8828$

Ex # 3.3

Page # 86

Q1: Find the characteristics of the common logarithm of each of the following numbers.

(i) 57

In Scientific form:

5.7×10^1

Thus Characteristics $= 1$

(ii) 7.4

In Scientific form:

7.4×10^0

Thus Characteristics $= 0$

(iii) 5.63

In Scientific form:

5.63×10^0

Thus Characteristics $= 0$

(iv) 56.3

In Scientific form:

5.63×10^1

Thus Characteristics $= 1$

(v) 982.5

In Scientific form:

9.825×10^2

Thus Characteristics $= 2$

(vi) 7824

In Scientific form:

7.824×10^3

Thus Characteristics $= 3$

(vii) 186000

In Scientific form:

1.86×10^5

Thus Characteristics $= 5$

viii. 0.71

In Scientific form:

7.1×10^{-1}

Thus Characteristics $= -1$

Ex # 3.3

Q2: Find the following.

(i) $\log 87.2$

Solution:

$\log 87.2$

In Scientific form:

8.72×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9405$

Hence $\log 87.2 = 1.9405$

(ii) $\log 37300$

Solution:

$\log 37300$

In Scientific form:

3.73×10^4

Thus Characteristics $= 4$

To find Mantissa, using Log Table:

So Mantissa $= .5717$

Hence $\log 37300 = 4.5717$

(iii) $\log 753$

Solution:

$\log 753$

In Scientific form:

7.53×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8768$

Hence $\log 753 = 2.8768$

(iv) $\log 9.21$

Solution:

$\log 9.21$

In Scientific form:

9.21×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .9643$

Hence $\log 9.21 = 0.9643$

Ex # 3.3

(v) $\log 0.00159$

Solution:

$\log 0.00159$

In Scientific form:

1.59×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .2014$

Hence $\log 0.00159 = \bar{3}.2014$

(vi) $\log 0.0256$

Solution:

$\log 0.0256$

In Scientific form:

2.56×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .4082$

Hence $\log 0.0256 = \bar{2}.4082$

(vii) $\log 6.753$

Solution:

$\log 6.753$

In Scientific form:

6.753×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table

Mantissa $= .8295$

Hence $\log 6.753 = 0.8295$

$R.W$
$8293 + 2$ $= 8295$

Q3: Find logarithms of the following numbers.

(i) 2476

Solution:

2476

Let $x = 2476$

Taking log on B.S

$\log x = \log 2476$

In Scientific form:

2.476×10^3

Thus Characteristics $= 3$

To find Mantissa, using Log Table

So Mantissa $= .3927 + 11$

Mantissa $= .3938$

Hence $\log 2476 = 3.3938$

$R.W$
$3927 + 11$ $= 3938$

Ex # 3.3

(ii) 2.4

Solution:

2.4

$Let\ x = 2.4$

Taking log on B.S

$\log x = \log 2.4$

In Scientific form:

2.4×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .3802$

Hence $\log 2.4 = 0.3802$

(iii) 92.5

Solution:

92.5

$Let\ x = 92.5$

Taking log on B.S

$\log x = \log 92.5$

In Scientific form:

9.25×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9661$

Hence $\log 92.5 = 1.9661$

(iv) 482.7

Solution:

482.7

$Let\ x = 482.7$

Taking log on B.S

$\log x = \log 482.7$

In Scientific form:

4.827×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .6836$

Hence $\log 482.7 = 2.6836$

$R.W$
$6830 + 6$ $= 6836$

Ex # 3.3

(v) 0.783

Solution:

0.783

$Let\ x = 0.783$

Taking log on B.S

$\log x = \log 0.783$

In Scientific form:

7.83×10^{-1}

Thus Characteristics $= -1$

To find Mantissa, using Log Table:

So Mantissa $= .8938$

Hence $\log 0.783 = \bar{1}.8938$

(vi) 0.09566

Solution:

0.09566

$Let\ x = 0.09566$

Taking log on B.S

$\log x = \log 0.09566$

In Scientific form:

9.566×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .9808$

Hence $\log 0.09566 = \bar{2}.9808$

$R.W$
$9805 + 3$ $= 9808$

(vii) 0.006753

Solution:

0.006753

$Let\ x = 0.006753$

Taking log on B.S

$\log x = \log 0.006753$

In Scientific form:

6.753×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .8295$

Hence $\log 0.006735 = \bar{3}.8295$

$R.W$
$8293 + 2$ $= 8295$

Ex # 3.3

(viii) **700**

Solution:

700

$Let\ x = 700$

Taking log on B.S

$\log x = \log 700$

In Scientific form:

7.00×10^2

Thus Characteristics = 2

To find Mantissa, using Log Table:

So Mantissa $= .8451$

Hence $\log 700 = 2\,.8451$

Exercise # 3.4

ANTI-LOGARITHM

If $\log x = y$ then x is the anti-logarithm of y and written as $x = anti - \log y$

Explanation with Example:

$2\,.3456$

(i) Here the digit before decimal point is Characteristics i.e. 2

(ii) And Mantissa$= .3456$

To find anti-log, we see Mantissa in Anti-log Table

(i) Take first two digits i.e. .34 and proceed along this row until we come to column headed by third digit 5 of the number which is 2213.

(ii) Now take fourth digit i.e. 6 and proceed along this row which is 3.

(iii) Now add $2213 + 3 = 2216$

So to find anti-log, write it in Scientific form like

$anti - \log 2.3456 = 2\,.2216 \times 10^{char}$

$anti - \log 2.3456 = 2\,.216 \times 10^2$

$anti - \log 2.3456 = 221\,.6$

Ex # 3.4

Page # 88

Q1: **Find anti-logarithm of the following numbers.**

(i) **1.2508**

Solution:

1.2508

$Let\ \log x = 1.2508$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 1.2508$

$x = \text{Anti} - \log 1.2508$

Characteristics = 1

Mantissa = .2508

So

$x = 1.781 \times 10^1$

$x = 17.81$

$R.W$
1778+3 = 1781

(ii) **0.8401**

Solution:

0.8401

$Let\ \log x = 0.8401$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 0.8401$

$x = \text{Anti} - \log 0.8401$

Characteristics = 0

Mantissa = .8401

So

$x = 6.920 \times 10^0$

$x = 6.920$

$R.W$
6918+2 = 6920

(iii) **2.540**

Solution:

2.540

$Let\ \log x = 2.540$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 2.540$

$x = \text{Anti} - \log 2.540$

Characteristics = 2

Mantissa = .540

So

$x = 3.467 \times 10^2$

$x = 346.7$

Ex # 3.4

(iv) $\mathbf{\bar{2}.2508}$

Solution:

$\bar{2}.2508$

$Let\ \log x = \bar{2}.2508$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log \bar{2}.2508$

$x = \text{Anti} - \log \bar{2}.2508$

Characteristics $= -2$

Mantissa $= .2508$

So

$x = 1.781 \times 10^{-2}$

$x = 0.01781$

$R.W$
1778+3
= 1781

(v) $\mathbf{\bar{1}.5463}$

Solution:

$\bar{1}.5463$

$Let\ \log x = \bar{1}.5463$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log \bar{1}.5463$

$x = \text{Anti} - \log \bar{1}.5463$

Characteristics $= -1$

Mantissa $= .5463$

So

$x = 3.518 \times 10^{-1}$

$x = 0.3518$

$R.W$
3516+2
= 3518

(vi) **3.5526**

Solution:

3.5526

$Let\ \log x = 3.5526$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 3.5526$

$x = \text{Anti} - \log 3.5526$

Characteristics $= 3$

Mantissa $= .5526$

So

$x = 3.570 \times 10^{3}$

$x = 3570$

$R.W$
3565+5
= 3570

Ex # 3.4

Q2: **Find the values of x from the following equations:**

(i) $\mathbf{\log x = \bar{1}.8401}$

Solution:

$\log x = \bar{1}.8401$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{1}.8401$

$x = \text{Anti} - \log \bar{1}.8401$

Characteristics $= -1$

Mantissa $= .8401$

So

$x = 6.920 \times 10^{-1}$

$x = 0.6920$

$R.W$
6918 + 2
= 6920

(ii) $\mathbf{\log x = 2.1931}$

Solution:

$\log x = 2.1931$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 2.1931$

$x = \text{Anti} - \log 2.1931$

Characteristics $= 2$

Mantissa $= .1931$

So

$x = 1.560 \times 10^{2}$

$x = 156.0$

$R.W$
1560 + 0
= 1560

(iii) $\mathbf{\log x = 4.5911}$

Solution:

$\log x = 4.5911$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 4.5911$

$x = \text{Anti} - \log 4.5911$

Characteristics $= 4$

Mantissa $= .5911$

So

$x = 3.900 \times 10^{4}$

$x = 39000.0$

$R.W$
3899 + 1
= 3900

Ex # 3.4

(i) $\log x = \bar{3}.0253$

Solution:

$\log x = \bar{3}.0253$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{3}.0253$

$x = \text{Anti} - \log \bar{3}.0253$

Characteristics $= -3$

Mantissa $= .0253$

So

$x = 1.060 \times 10^{-3}$

$x = 0.001060$

$R.W$
$1059 + 1$ $= 1060$

(ii) $\log x = 1.8716$

Solution:

$\log x = 1.8716$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 1.8716$

$x = \text{Anti} - \log 1.8716$

Characteristics $= 1$

Mantissa $= .8716$

So

$x = 7.440 \times 10^{1}$

$x = 74.40$

$R.W$
$7430 + 10$ $= 7440$

(iii) $\log x = \bar{2}.8370$

Solution:

$\log x = \bar{2}.8370$

Taking anti – log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{2}.8370$

$x = \text{Anti} - \log \bar{2}.8370$

Characteristics $= -2$

Mantissa $= .8370$

So

$x = 6.871 \times 10^{-2}$

$x = 0.06781$

Ex # 3.5

LAWS OF LOGARITHM

(i) $\log_a mn = \log_a m + \log_a n$

or $\log mn = \log m + \log n$

Example:

$\log 2 \times 3 = \log 2 + \log 3$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

or $\log \frac{m}{n} = \log m - \log n$

Example:

$\log \frac{3}{5} = \log 3 - \log 5$

$\log 6 - \log 3 = \log \frac{6}{3} = \log 2$

(iii) $\log_a m^n = n \log_a m$

or $\log m^n = n \log m$

Example:

$\log 2^3 = 3 \log 2$

$\log_a m \log_m n = \log_a n$

$\log_2 3 \log_3 5 = \log_3 5$

$\log_m n = \frac{\log_a n}{\log_a m}$

Example:

(iv) $\frac{\log_7 r}{\log_7 t} = \log_t r$

Note:

(i) $\log_a a = 1$

(ii) $\log_{10} 10 = 1$

(iii) $\log 10 = 1$

(iv) $\log_{10} 1 = 0$

(v) $\log 1 = 0$

(vi) $\log_m n = \frac{\log_a n}{\log_a m}$

This is called Change of Base Law

Ex # 3.5

Proof of Laws of Logarithm one by one

(i) $\log_a mn = \log_a m + \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now multiply these:

$a^x \times a^y = mn$

Or $mn = a^x \times a^y$

$mn = a^{x+y}$

Taking $\log_a$ on B.S

$\log_a mn = \log_a a^{x+y}$

$\log_a mn = (x+y)\log_a a$

$\log_a mn = (x+y)(1) \qquad \therefore \log_a a = 1$

$\log_a mn = x + y$

$\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now Divide these:

$\frac{a^x}{a^y} = \frac{m}{n}$

Or

$\frac{m}{n} = \frac{a^x}{a^y}$

$\frac{m}{n} = a^{x-y}$

Taking $\log_a$ on B.S

$\log_a \frac{m}{n} = \log_a a^{x-y}$

$\log_a \frac{m}{n} = (x-y)\log_a a$

$\log_a \frac{m}{n} = (x-y)(1) \qquad \therefore \log_a a = 1$

$\log_a \frac{m}{n} = x - y$

$Hence\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

Ex # 3.5

(iii) $\log_a m^n = n\log_a m$

Proof:

$Let\ \ \log_a m = x$

In Exponential form:

$a^x = m$

Or

$m = a^x$

Taking power 'n' on B.S

$m^n = (a^x)^n$

$m^n = a^{nx}$

Taking $\log_a$ on B.S

$\log_a m^n = \log_a a^{nx}$

$\log_a m^n = nx\log_a a$

$\log_a m^n = nx(1) \qquad \therefore \log_a a = 1$

$\log_a m^n = nx$

$\log_a m^n = n\log_a m$

(iv) $\log_a m \log_m n = \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_m n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ m^y = n$

Now multiply these:

$As\ a^{xy} = (a^x)^y$

$But\ \ (a^x)^y = m$

$So\ a^{xy} = (m)^y = n$

$So\ a^{xy} = n$

Taking $\log_a$ on B.S

$\log_a a^{xy} = \log_a n$

$(xy)\log_a a = \log_a n$

$xy(1) = \log_a n \qquad \therefore \log_a a = 1$

Now

$\log_a m \log_m n = \log_a n$

Example # 14 page # 90

$-1 + \log y$

Solution:

$= -1 + \log y$

$= -\log 10 + \log y$

$= \log 10^{-1} + \log y$

$= \log \frac{1}{10} + \log y$

$= \log 0.1 + \log y$

$= \log 0.1\, y$

Ex # 3.5

Page # 91

Q1: Use logarithm properties to simplify the expression.

(i) $\log_7 \sqrt{7}$

Solution:

$\log_7 \sqrt{7}$

$Let\ x = \log_7 \sqrt{7}$

$x = \log_7 (7)^{\frac{1}{2}}$

$As\ \ \log_a m^n = n \log_a m$

$x = \frac{1}{2}\log_7 7$

$x = \frac{1}{2}(1) \qquad \therefore \log_a a = 1$

$x = \frac{1}{2}$

(ii) $\log_8 \frac{1}{2}$

Trick

Solution:

$\log_8 \frac{1}{2}$

$\log_8 \frac{1}{2}$

$Let\ \ \log_8 \frac{1}{2} = x$

In exponential form:

$8^x = \frac{1}{2}$

$(2^3)^x = 2^{-1}$

$2^{3x} = 2^{-1}$

Now

$3x = -1$

Divide B.S by 3, we get

$x = \frac{-1}{3}$

(iii) $\log_{10} \sqrt{1000}$

Solution:

$\log_{10} \sqrt{1000}$

$Let\ x = \log_{10} (10^3)^{\frac{1}{2}}$

$x = \log_{10} (10)^{\frac{3}{2}}$

Ex # 3.5

$As\ \ \log_a m^n = n \log_a m$

$x = \frac{3}{2}\log_{10} 10$

$x = \frac{3}{2}(1) \qquad \therefore \log_a a = 1$

$x = \frac{3}{2}$

(iv) $\log_9 3 + \log_9 27$

Solution:

$\log_9 3 + \log_9 27$

$Let\ x = \log_9 3 + \log_9 27$

$As\ \ \log_a mn = \log_a m + \log_a n$

$x = \log_9 3 \times 27$

$x = \log_9 81$

$x = \log_9 9^2$

$As\ \ \log_a m^n = n \log_a m$

$x = 2\log_9 9$

$x = 2(1) \qquad \therefore \log_a a = 1$

$x = 2$

(v) $\log \frac{1}{(0.0035)^{-4}}$

Solution:

$\log \frac{1}{(0.0035)^{-4}}$

$Let\ \ x = \log \frac{1}{(0.0035)^{-4}}$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$x = \log 1 - \log(0.0035)^{-4}$

$As\ \log 1 = 0$ and $\log_a m^n = n \log_a m$

Thus

$x = 0 - (-4)\log 0.0035$

Here $Ch = -3$

And $M = .5441$

So

$x = 4(-3 + .5441)$

$x = 4(-2.4559)$

$x = -9.8236$

$R.W$

3.5×10^{-3}

Ex # 3.5

(vi) $\mathbf{\log 45}$

Solution:

$\log 45$

$Let\ \ x = \log 45$

$x = \log 3 \times 3 \times 5$

$x = \log 3^2 \times 5$

$\boldsymbol{\log_a mn = \log_a m + \log_a n}$

and $\boldsymbol{\log_a m^n = n \log_a m}$

$x = 2\log 3 + \log 5$

$x = 2\log 3.00 + \log 5.00$

$x = 2(0 + .4771) + (0 + .6990)$

$x = 2(0.4771) + (0.6990)$

$x = 0.9542 + 0.6990$

$x = 1.6532$

Q2: Express each of the following as a single logarithm.

(i) $\mathbf{3\log 2 - 4\log 3}$

Solution:

$3\log 2 - 4\log 3$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$3\log 2 - 4\log 3 = \log 2^2 - \log 3^4$

$3\log 2 - 4\log 3 = \log 8 - \log 81$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$3\log 2 - 4\log 3 = \log\frac{8}{81}$

(ii) $\mathbf{2\log 3 + 4\log 2 - 3}$

Solution:

$2\log 3 + 4\log 2 - 3$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$2\log 3 + 4\log 2 - 3 = \log 3^2 + \log 2^4 - 3(1)$

$\boldsymbol{As\ \ \log 10 = 1}$

So

$2\log 3 + 4\log 2 - 3 = \log 9 + \log 16 - 3(\log 10)$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 10^3$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 1000$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$2\log 3 + 4\log 2 - 3 = \log\frac{144}{1000}$

$2\log 3 + 4\log 2 - 3 = \log 0.144$

(iii) $\mathbf{\log 5 - 1}$

Solution:

$\log 5 - 1$

$\boldsymbol{As\ \ \log 10 = 1}$

$\log 5 - 1 = \log 5 - \log 10$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\log 5 - 1 = \log\frac{5}{10}$

$\log 5 - 1 = \log 0.5$

(iv) $\boldsymbol{\frac{1}{2}\log x - 2\log 3y + 3\log z}$

Solution:

$\frac{1}{2}\log x - 2\log 3y + 3\log z$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$= \log x^{\frac{1}{2}} - \log(3y)^2 + \log z^3$

$= \log\sqrt{x} - \log 9y^2 + \log z^3$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\boldsymbol{And\ \ \log_a mn = \log_a m + \log_a n}$

$\frac{1}{2}\log x - 2\log 3y + 3\log z = \log\frac{\sqrt{x}z^3}{9y^2}$

Q3: Find the value of $'a'$ from the following equations.

(i) $\boldsymbol{\log_2 6 + \log_2 7 = \log_2 a}$

Solution:

$\log_2 6 + \log_2 7 = \log_2 a$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$\log_2 6 \times 7 = \log_2 a$

$\log_2 42 = \log_2 a$

Thus

$a = 42$

(ii) $\boldsymbol{\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2}$

Solution:

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{5 \times 8}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{40}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 20$

Thus

$a = 20$

(iii) $\boldsymbol{\frac{\log_7 r}{\log_7 t} = \log_a r}$

Solution:

$\frac{\log_7 r}{\log_7 t} = \log_a r$

$\boldsymbol{As\ \ \log_m n = \frac{\log_a n}{\log_a m}}$

$\log_t r = \log_a r$

Thus

$a = t$

(iv) $\boldsymbol{\log_6 25 - \log_6 5 = \log_6 a}$

Solution:

$\log_6 25 - \log_6 5 = \log_6 a$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\log_6 \frac{25}{5} = \log_6 a$

$\log_6 5 = \log_6 a$

Thus

$a = 5$

Q4: **Find** $\boldsymbol{\log_2 3 \,.\, \log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8}$

Solution:

$Let\ \ x = \log_2 3 \,.\, \log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

$\boldsymbol{As\ \ \log_a m^n = n \log_a m}$

So

$x = \log_2 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

$x = \log_2 5 \ \ .\ \ \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

$x = \log_2 6 \ \ .\, \log_6 7 \,.\, \log_7 8$

$x = \log_2 7 \ \ .\, \log_7 8$

$x = \log_2 8$

$x = \log_2 2^3$

$x = 3 \log_2 2$

$\boldsymbol{As\ \ \log_a a = 1}$

$x = 3(1)$

$x = 3$

Ex # 3.6

Page # 93

Q1: **Simplify** $\boldsymbol{3.81 \times 43.4}$ **with the help of logarithm.**

Solution:

(i) 3.81×43.4

Let $x = 3.81 \times 43.4$

Taking log on B.S

$\log x = \log 3.81 \times 43.4$

As $\log mn = \log m + \log n$

$\log x = \log 3.81 + \log 43.4$

$\log x = (0 + .5809) + (1 + .6375)$

$\log x = 0.5809 + 1.6375$

$\log x = 2.2184$

log 3.81
$Ch = 0$
$M = .5809$

log 43.4
$Ch = 1$
$M = .6375$

Taking anti – log on B.S

$\text{Anti} - \log\ (\log x) = \text{Anti} - \log 2.2184$

$x = \text{Anti} - \log 2.2184$

Here

Characteristics $= 2$

Mantissa $= .2184$

So

$x = 1.654 \times 10^2$

$x = 16.54$

$1652 + 2$
$= 1654$

Ex # 3.6

(ii) $\mathbf{73.42 \times 0.00462 \times 0.5143}$

Solution:

$73.42 \times 0.00462 \times 0.5143$

Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on B.S

$\log x = 73.42 \times 0.00462 \times 0.5143$

As $\log mn = \log m + \log n$

$\log x = \log 73.42 + \log 0.00462 + \log 0.5143$

$\log x = (1 + .8658) + (-3 + .6646) + (-1 + .7113)$

$\log x = 1.8658 + (-2.3354) + (-0.2887)$

$\log x = 1.8658 - 2.3354 - 0.2887$

$\log x = -0.7583$

Add and Subtract -1

$\log x = -1 + 1 - 0.7583$

$\log x = -1 + .2417$

$\log x = \bar{1}\,.2417$

Taking anti – log on B. S

anti – log $(\log x)$ = anti – log $\bar{1}\,.2417$

x = anti – log $\bar{1}\,.2417$

log 73.42
$Ch = 1$
$8657 + 1$
$M = .8658$

log 0.00462
$Ch = -3$
$M = .6646$

log 0.5143
$Ch = -1$
$7110 + 3$
$M = .7113$

Here

Characteristics = -1

Mantissa = .2417

So

$x = 1.745 \times 10^{-1}$

$x = 0.1745$

$1742 + 3$
$= 1745$

(iii) $\mathbf{\dfrac{784.6 \times 0.0431}{28.23}}$

Solution:

$$\frac{784.6 \times 0.0431}{28.23}$$

$$Let\ x = \frac{784.6 \times 0.0431}{28.23}$$

Taking log on B.S

$$\log x = \log \frac{784.6 \times 0.0431}{28.23}$$

$$\boldsymbol{As\ \ \log \frac{m}{n} = \log m - \log n}$$

$\log x = \log 784.6 \times 0.0431 - \log 28.23$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 784.6 + \log 0.0431 - \log 28.23$

Ex # 3.6

$\log x = (2 + .8946) + (-2 + .6345) + (1 + .4507)$

$\log x = 2.8946 + (-1.3655) + (1.4507)$

$\log x = 2.8946 - 1.3655 - 1.4507$

$\log x = 0\,.0784$

Taking anti − log on B.S

$\text{anti} - \log\ (\log x) = \text{anti} - \log 0\,.0784$

$x = \text{anti} - \log 0\,.0784$

Here

Characteristics = 0

Mantissa = .0784

So

$x = 1.198 \times 10^0$

$x = 1.198$

$\log 784.6$
$Ch = 2$
$8943 + 3$
$M = .8946$

$\log 0.0431$
$Ch = -2$
$M = .6345$

$\log 28.23$
$Ch = 1$
$4502 + 5$
$M = .4507$

$1197 + 1$
$= 1198$

(iv) $\dfrac{\mathbf{0.4932 \times 653.7}}{\mathbf{0.07213 \times 8456}}$

Solution:

$$\frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

$$Let\ x = \frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

Taking log on B.S

$$\log x = \log \frac{0.4932 \times 653.7}{0.07213 \times 8456}$$

$$\boldsymbol{As\ \ \log \frac{m}{n} = \log m - \log n}$$

$\log x = \log(0.4932 \times 653.7) - \log(0.07213 \times 8456)$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 0.4932 + \log 653.7 - (\log 0.07213 + \log 8456)$

$\log x = \log 0.4932 + \log 653.7 - \log 0.07213 - \log 8456$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-0.3070) + (2.8154) - (-1.1419) - (3.9271)$

$\log x = -0.3070 + 2.8154 + 1.1419 - 3.9271$

$\log x = -0.2768$

$\log 0.4932$
$Ch = -1$
$6928 + 2$
$M = .6930$

$\log 653.7$
$Ch = 2$
$8149 + 5$
$M = .8154$

$\log 0.07213$
$Ch = -2$
$8579 + 2$
$M = .8581$

$\log 8456$
$Ch = 3$
$9269 + 3$
$M = .9272$

Ex # 3.6

Add and Subtract -1

$\log x = -1 + 1 - 0.2768$

$\log x = -1 + .7232$

$\log x = \overline{1}\,.7232$

Taking anti – log on B.S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \overline{1}\,.7232$

$x = \text{anti} - \log \overline{1}\,..7232$

Here

Characteristics $= -1$

Mantissa $= .7232$

So

$x = 5.286 \times 10^{-1}$

$x = 0.5286$

$5284 + 2$
$= 5286$

(v) $\dfrac{\mathbf{(78.41)^3\sqrt{142.3}}}{\mathbf{\sqrt[4]{0.1562}}}$

Solution:

$\dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$Let\ x = \dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

Taking log on B.S

$\log x = \log\dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$\boldsymbol{As\ \ \log\dfrac{m}{n} = \log m - \log n}$

$\log x = \log(78.41)^3\sqrt{142.3} - \log\sqrt[4]{0.1562}$

$\mathbf{As\ \log \boldsymbol{mn} = \log \boldsymbol{m} + \log \boldsymbol{n}}$

$\log x = \log(78.41)^3 + \log\sqrt{142.3} - \log\sqrt[4]{0.1562}$

$\log x = \log(78.41)^3 + \log(142.3)^{\frac{1}{2}} - \log(0.1562)^{\frac{1}{4}}$

$\log x = 3\log 78.41 + \dfrac{1}{2}\log 142.3 - \dfrac{1}{4}\log 0.1562$

$\log x = 3\log(78.41) + \dfrac{1}{2}\log(142.3) - \dfrac{1}{4}\log(0.1562)$

$\log x = 3\,(1 + .8944) + \dfrac{1}{2}(\,2 + .1532) - \dfrac{1}{4}(-1 + .1937)$

$\log 78.41$ $Ch = 1$ $8943 + 1$ $M = .8944$
$\log 142.3$ $Ch = 2$ $1523 + 9$ $M = .1523$
$\log 0.1562$ $Ch = -1$ $1931 + 6$ $M = .1937$

$$\log x = 3\,(1.8944) + \frac{1}{2}(\,2.1532) - \frac{1}{4}(-0.8063)$$

$\log x = 5.6832 + 1.0766 + 0.2016$

$\log x = 6.9614$

Taking anti − log on B.S

$\text{anti} - \log\ (\log x) = \text{anti} - \log 6.9614$

$x = \text{anti} - \log 6.9614$

Here

Characteristics = 6

Mantissa = .9614

So

$x = 9.149 \times 10^6$

$x = 9149000$

$\boxed{\begin{array}{l}9141 + 8 \\ = 9149\end{array}}$

Q2: Find the following if $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, $\log 7 = 0.8451$

(i) $\log 105$

Solution:

$\log 105$

$\log 105 = \log 3 \times 5 \times 7$

As $\log mn = \log m + \log n$

$\log 105 = \log 3 + \log 5 + \log 7$

$\log 105 = 0.4771 + 0.6990 + 0.8451$

$\log 105 = 2.0211$

(ii) $\log 108$

$\log 108$

$\log 108 = \log 2 \times 2 \times 3 \times 3 \times 3$

$\log 108 = \log 2^2 \times 3^3$

As $\log mn = \log m + \log n$

$\log 108 = \log 2^2 + \log 3^3$

As $\boldsymbol{\log_a m^n = n \log_a m}$

$\log 108 = 2 \log 2 + 3 \log 3$

$\log 108 = 2(0.3010) + 3(0.4771)$

$\log 108 = 0.6020 + 1.4313$

$\log 108 = 2.0333$

(iii) $\log \sqrt[3]{72}$

Solution:

$\log \sqrt[3]{72}$

$$\log \sqrt[3]{72} = \log(72)^{\frac{1}{3}}$$

As $\boldsymbol{\log_a m^n = n \log_a m}$

$\log \sqrt[3]{72} = \frac{1}{3} \log 72$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2 \times 2 \times 2 \times 3 \times 3)$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3\ \times 3^2)$

As $\log mn = \log m + \log n$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3 + log\, 3^2)$

$log \sqrt[3]{72} = \frac{1}{3}(3\, log\, 2 + 2\, log\, 3)$

$log \sqrt[3]{72} = \frac{1}{3}[3(0.3010) + 2(0.4771)]$

$log \sqrt[3]{72} = \frac{1}{3}[0.9030 + 0.9542]$

$log \sqrt[3]{72} = \frac{1}{3}[1.8572]$

$log \sqrt[3]{72} = 0.6191$

(iv) $\log 2.4$

Solution:

$\log 2.4$

$\log 2.4 = \log \frac{24}{10}$

As $\boldsymbol{\log_a \frac{m}{n} = \log_a m - \log_a n}$

$\log 2.4 = \log 24 - \log 10$

$\log 2.4 = \log 2 \times 2 \times 2 \times 3 - \log 10$

$\log 2.4 = \log 2^3\ \times 3 - \log 10$

As $\log mn = \log m + \log n$

$\log 2.4 = \log 2^3 + \log 3 - \log 10$

As $\boldsymbol{\log_a m^n = n \log_a m}$

$\log 2.4 = 3 \log 2 + \log 3 - \log 10$

$\log 2.4 = 3(0.3010) + 0.4771 - \log 10$

$\log 2.4 = 0.9030 + 0.4771 - 1 \therefore \boldsymbol{\log 10 = 1}$

$\log 2.4 = 1.3801 - 1$

$\log 2.4 = 0.3801$

Ex # 3.6

(v) $\log 0.0081$

Solution:

$\log 0.0081$

$\log 0.0081 = \log \frac{81}{10000}$

$\log 0.0081 = \log \frac{3^4}{10^4}$

$\log 0.0081 = \log \left(\frac{3}{10}\right)^4$

As $\log_a m^n = n \log_a m$

$\log 0.0081 = 4 \log \frac{3}{10}$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 0.0081 = 4(\log 3 - \log 10)$

$\log 0.0081 = 4(0.4771 - 1) \quad \therefore \log 10 = 1$

$\log 0.0081 = 4(-0.5229)$

$$\log 0.0081 = -2.0916$$

REVIEW EXERCISE # 3

Page # 95

Q2: **Write 9473.2 in scientific notation.**

9473.2

In scientific notation:

$$9.4732 \times 10^3$$

Q3: **Write 5.4×10^6 in standard notation.**

$$5.4 \times 10^6$$

In standard form:

5400000

Q4: **Write in logarithm form: $3^{-3} = \frac{1}{27}$**

$$3^{-3} = \frac{1}{27}$$

In logarithm form:

$$\log_3 \frac{1}{27} = -3$$

Review Ex # 3

Q5: **Write in exponential form: $\log_5 1 = 0$**

$$\log_5 1 = 0$$

In exponential form:

$$5^0 = 1$$

Q6: **Solve for x: $\log_4 16 = x$**

$$\log_4 16 = x$$

In exponential form:

$$4^x = 16$$

$$4^x = 4^2$$

So

$$x = 2$$

Q7: **Find the characteristic of the common logarithm 0.0083.**

0.0083

In scientific notation:

$$8.3 \times 10^{-3}$$

So Characteristics -3

Q8: **Find $\log 12.4$**

$$\log 12.4$$

In Scientific form:

$$1.24 \times 10^1$$

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

Mantissa $= .0934$

Hence $\log 12.4 = 0.0934$

Q9: **Find the value of 'a',**

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

Solution:

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

As $\log_a mn = \log_a m + \log_a n$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} \frac{9 \times 2}{3}$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 3 \times 2$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 6$

Thus $3a = 6$

$a = \frac{6}{2}$

$a = 3$

Q10 $\dfrac{\mathbf{(63.28)^3(0.00843)^2(0.4623)}}{\mathbf{(412.3)(2.184)^5}}$

Solution:

$$\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$Let\ x=\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

Taking log on B.S

$$\log x=\log\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$As\quad \log\frac{m}{n}=\log m-\log n$$

$$\log x=\log((63.28)^3(0.00843)^2(0.4623))-\log((412.3)(2.184)^5)$$

As $\log mn=\log m+\log n$

$$\log x=\log(63.28)^3+\log(0.00843)^2+\log 0.4623-(\log 412.3+\log(2.184)^5)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-(\log 412.3+5\log 2.184)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-\log 412.3-5\log 2.184$$

$$\log x=3(1+.8012)+2(-3+.9258)+(-1+.6649)-(2+.6152)-5(0+.3393)$$

$$\log x=3(1.8012)+2(-2.0742)+(-0.3351)-(2.6152)-5(0.3393)$$

$$\log x=5.4036-4.1484-0.3351-2.6152-1.6965$$

$$\log x=-3.3916$$

Add and Subtract -4

$\log x=-4+4-3.3916$

$\log x=-4+.6084$

$\log x=\bar{4}.6084$

Taking anti $-$ log on B.S

anti $-$ log $(\log x)=$ anti $-\log \bar{4}.6084$

$x=$ anti $-\log \bar{4}.6084$

Here

Characteristics $=-4$

Mantissa $=.6084$

So

$x=4.059\times 10^{-4}$

$x=0.000405$

$\log 63.28$
$Ch=1$
$8007+5$
$M=.8012$

$\log 0.00843$
$Ch=-3$
$M=.9258$

$\log 0.4623$
$Ch=-1$
$6646+3$
$M=.6649$

$\log 412.3$
$Ch=2$
$6149+3$
$M=.6152$

$\log 2.184$
$Ch=0$
$3385+8$
$M=.3393$

$4055+4$
$=4059$

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Chapter # 4

UNIT # 4

ALGEBRAIC EXPRESSIONS & ALGEBRAIC FORMULAS

Ex # 4.1

Algebraic Expressions

When variables and constants are connected by algebraic operations like addition, subtraction, multiplication, division, root extraction & rising integral or fractional powers is called algebraic expressions.

Variable:

A quantity that value may change within the context of problem. It is unknown value.

Normally, we use English letters for variables

Example:

a, d, e, x, y, z

Constant:

A quantity that value doesn't change. It is a fixed value.

Example:

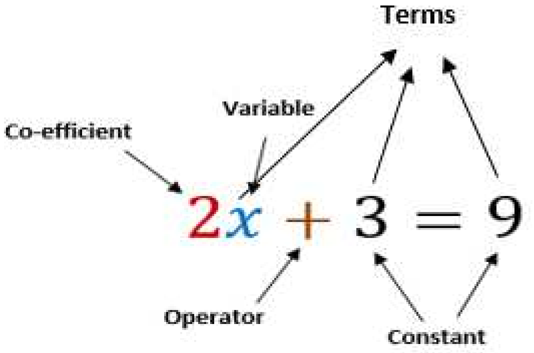
4, 6, 267, 983384

Constant

جس کی value تبدیل نہیں ہوتی یعنی 1,2,3,9,22

Variable

جس کی value تبدیل ہوتی یعنی a,b,c,x,y,z



For Addition and Subtraction and other important terminologies

Visit this video:

https://youtu.be/4jFH9OMmjXI

Polynomial

The algebraic expression in which powers of variables are whole numbers is called polynomial.

Rational Expression:

An expression of form of $\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials and $q(x) \neq 0$.

Example:

$$\frac{x^2 - 6x + 1}{x + 9}$$

$$\frac{4x^2 + 10x + 11}{5}$$

Note:

Every polynomial $p(x)$ is a rational expression but every rational expression need not to be a polynomial.

Irrational Expression:

An expression which cannot be written in the form of $\frac{p(x)}{q(x)}$

Term

Different parts of an algebraic expression joined by the operations of addition and subtraction are called term.

Example

$3x^3 + 5\sqrt{x} - 7$. The terms are $\mathbf{3x^3},\ \mathbf{5\sqrt{x}},\ \mathbf{-7}$

Rules to express a rational expression in its lowest term

$$Let\ \frac{p(x)}{q(x)}$$

Step 1: Factorize both the polynomial in the numerator and denominator.

Step 2: cancel the common factors between them.

Example # 9

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Chapter # 4

Ex # 4.1

Page # 106

Q1: Which of the following expressions are polynomials?

(i) $\mathbf{1 - 5y + 8y^2 + 6y^3}$

Ans: Polynomial and also Rational

(ii) $\mathbf{\frac{5}{x^2} + \frac{3}{4x+1}}$

Ans: Non-Polynomial but Rational

(iii) $\mathbf{\frac{\sqrt{x}}{6x-1}}$

Ans: Non-Polynomial but Irrational

Q2: Which of the following rational expressions are in their lowest terms?

(i) $\mathbf{\frac{5y^2-5}{y-1}}$

Solution:

$$\frac{5y^2 - -5}{y-1}$$

$$\frac{5y^2-5}{y-1} = \frac{5(y^2-1)}{y-1}$$

$$\frac{5y^2-5}{y-1} = \frac{5(y+1)(y-1)}{y-1}$$

$$\frac{5y^2-5}{y-1} = 5(y+1)$$

So it is **Not** in Lowest Term:

(ii) $\mathbf{\frac{x^2-9}{x-2}}$

Solution:

$$\frac{x^2-9}{x-2}$$

$$\frac{x^2-9}{x-2} = \frac{(x+3)(x-3)}{x-2}$$

We can't solve it more

So it is in Lowest Term

Ex # 4.1

(iii) $\mathbf{\frac{x+y}{x^2-y^2}}$

Solution:

$$\frac{x+y}{x^2-y^2}$$

$$\frac{x+y}{x^2-y^2} = \frac{x+y}{(x+y)(x-y)}$$

$$\frac{x+y}{x^2-y^2} = \frac{1}{x-y}$$

So it is **Not** in Lowest Term:

Q3: Reduce the following rational expression to their lowest term:

(i) $\mathbf{\frac{x-5}{x^2-5x}}$

Solution:

$$\frac{x-5}{x^2-5x}$$

$$\frac{x-5}{x^2-5x} = \frac{x-5}{x(x-5)}$$

$$\frac{x-5}{x^2-5x} = \frac{1}{x}$$

(ii) $\mathbf{\frac{t^3(t-3)}{(t-3)(t+5)}}$

Solution:

$$\frac{t^3(t-3)}{(t-3)(t+5)}$$

$$\frac{t^3(t-3)}{(t-3)(t+5)} = \frac{t^3}{(t+5)}$$

(iii) $\mathbf{\frac{x^4 + \frac{1}{x^4}}{x^2 - \frac{1}{x^2}}}$

Solution:

$$\frac{x^4 + \frac{1}{x^4}}{x^2 - \frac{1}{x^2}}$$

Ans: It cannot be reduced further

Chapter # 4

Ex # 4.1

(iv) $\mathbf{\frac{2a+6}{a^2-9}}$

Solution:

$\frac{2a+6}{a^2-9}$

$\frac{2a+6}{a^2-9} = \frac{2(a+3)}{(a+3)(a-3)}$

$\frac{2a+6}{a^2-9} = \frac{2}{(a-3)}$

Q4: Add the following rational expressions:

(i) $\mathbf{4x^2-5x-10, \quad 2x^2+5x+10}$

Solution:

$4x^2-5x-10, \quad 2x^2+5x+10$

Now

$(4x^2-5x-10)+(2x^2+5x+10)$

$= 4x^2-5x-10+2x^2+5x+10$

Write the like term

$= 4x^2+2x^2-5x+5x-10+10$

$= 6x^2$

(ii) $\mathbf{\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}}$

Solution:

$\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}$

$= \frac{y+9}{y^2+3} + \frac{-7y+7}{y^2+3}$

$= \frac{(y+9)+(-7y+7)}{y^2+3}$

$= \frac{y+9-7y+7}{y^2+3}$

$= \frac{y-7y+9+7}{y^2+3}$

$= \frac{-6y+16}{y^2+3}$

Ex # 4.1

(iii) $\mathbf{\frac{y}{y+4}, \quad \frac{2y}{y-4}}$

Solution:

$\frac{y}{y+4}, \quad \frac{2y}{y-4}$

$= \frac{y}{y+4} + \frac{2y}{y-4}$

$= \frac{y(y-4)+2y(y+4)}{(y+4)(y-4)}$

$= \frac{y^2-4y+2y^2+8y}{(y+4)(y-4)}$

$= \frac{y^2+2y^2-4y++8y}{x^2-4^2}$

$= \frac{3y^2+4y}{x^2-16}$

(iv) $\mathbf{\frac{t}{t^2-25}, \quad \frac{3t}{t+5}}$

Solution:

$\frac{t}{t^2-25}, \quad \frac{3t}{t+5}$

$\frac{t}{t^2-25} + \frac{3t}{t+5}$

$\frac{t}{(t+5)(t-5)} + \frac{3t}{t+5}$

$\frac{t+3t(t-5)}{(t+5)(t-5)}$

$\frac{t+3t^2-15t}{t^2-5^2}$

$\frac{3t^2+t-15t}{t^2-25}$

$\frac{3t^2-14t}{t^2-25}$

Chapter # 4

Ex # 4.1

Q5: Subtract the first expression from the second in the following.

(i) $\mathbf{y^2+4y-15,\quad 8y^2+2}$

Solution:

$y^2+4y-15,\quad 8y^2+2$

$=(8y^2+2)-(y^2+4y-15)$

$=8y^2+2-y^2-4y+15$

$=8y^2-y^2-4y+2+15$

$=7y^2-4y+17$

(ii) $\mathbf{\dfrac{8x^2-7}{x^2+1},\quad \dfrac{8x^2+7}{x^2+1}}$

Solution:

$$\frac{8x^2-7}{x^2+1},\quad \frac{8x^2+7}{x^2+1}$$

$$=\frac{8x^2+7}{x^2+1}-\frac{8x^2-7}{x^2+1}$$

$$=\frac{(8x^2+7)-(8x^2-7)}{x^2+1}$$

$$=\frac{8x^2+7-8x^2+7}{x^2+1}$$

$$=\frac{8x^2-8x^2+7+7}{x^2+1}$$

$$=\frac{14}{x^2+1}$$

(iii) $\mathbf{\dfrac{1}{a-3},\quad \dfrac{2a}{a^2-9}}$

Solution:

$$\frac{1}{a-3},\quad \frac{2a}{a^2-9}$$

$$=\frac{2a}{a^2-9}-\frac{1}{a-3}$$

$$=\frac{2a}{(a+3)(a-3)}-\frac{1}{a-3}$$

$$=\frac{2a-1(a+3)}{(a+3)(a-3)}$$

$$=\frac{2a-a-3}{(a+3)(a-3)}$$

Ex # 4.1

$$=\frac{a-3}{(a+3)(a-3)}$$

$$=\frac{1}{(a+3)}$$

(iv) $\mathbf{\dfrac{x}{3x-6},\quad \dfrac{x+2}{x-2}}$

Solution:

$$\frac{x}{3x-6},\quad \frac{x+2}{x-2}$$

$$=\frac{x+2}{x-2}-\frac{x}{3x-6}$$

$$=\frac{x+2}{x-2}-\frac{x}{3(x-2)}$$

$$=\frac{3(x+2)-x}{3(x-2)}$$

$$=\frac{3x+6-x}{3(x-2)}$$

$$=\frac{3x-x+6}{3(x-2)}$$

$$=\frac{2x+6}{3(x-2)}$$

$$=\frac{2(x+3)}{3(x-2)}$$

Q6: Simplify the following.

(i) $\mathbf{\dfrac{2x}{6x-9}\cdot\dfrac{4x-6}{x^2+x}}$

Solution:

$$\frac{2x}{6x-9}\cdot\frac{4x-6}{x^2+x}$$

$$=\frac{2x}{3(2x-3)}\cdot\frac{2(2x-3)}{x(x+1)}$$

$$=\frac{2}{3}\cdot\frac{2}{(x+1)}$$

$$=\frac{4}{3(x+1)}$$

Ex # 4.1

(ii) $\mathbf{\dfrac{x+4}{3-x}\cdot\dfrac{x^2-9}{x^2-16}}$

Solution:

$$\frac{x+4}{3-x}\cdot\frac{x^2-9}{x^2-16}$$
$$=\frac{x+4}{-x+3}\cdot\frac{x^2-3^2}{x^2-4^2}$$
$$=\frac{x+4}{-(x-3)}\cdot\frac{(x+3)(x-3)}{(x+4)(x-4)}$$
$$=\frac{1}{-1}\cdot\frac{(x+3)}{(x-4)}$$
$$=\frac{1(x+3)}{-1(x-4)}$$
$$=\frac{x+3}{-x+4}$$
$$=\frac{x+3}{4-x}$$

(iii) $\mathbf{\dfrac{3x-15}{2x+6}\cdot\dfrac{x^2-9}{x^2-25}}$

Solution:

$$\frac{3x-15}{2x+6}\cdot\frac{x^2-9}{x^2-25}$$
$$=\frac{3(x-5)}{2(x+3)}\cdot\frac{(x+3)(x-3)}{(x+5)(x-5)}$$
$$=\frac{3}{2}\cdot\frac{(x-3)}{(x-5)}$$
$$=\frac{3(x-3)}{2(x-5)}$$

Q7: Simplify the following.

(i) $\mathbf{\dfrac{2y-10}{3y}\div(y-5)}$

Solution:

$$\frac{2y-10}{3y}\div(y-5)$$
$$=\frac{2(y-5)}{3y}\times\frac{1}{y-5}$$
$$=\frac{2}{3y}$$

Ex # 4.1

(ii) $\mathbf{\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}}$

Solution:

$$\frac{p}{q}\div\frac{r}{q}\cdot\frac{p}{q}$$
$$=\frac{p}{q}\cdot\frac{q}{r}\cdot\frac{p}{q}$$
$$=\frac{p}{q}\cdot\frac{1}{r}\cdot\frac{p}{1}$$
$$=\frac{p^2}{qr}$$

(iii) $\mathbf{\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}}$

Solution:

$$\frac{a^2-9}{(a-6)(a+4)}\div\frac{a-3}{a-6}$$
$$=\frac{(a+3)(a-3)}{(a-6)(a+4)}\times\frac{a-6}{a-3}$$
$$=\frac{(a+3)}{(a+4)}$$
$$=\frac{a+3}{a+4}$$

Ex # 4.2

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Q1: Evaluate the following when $a=3$, $b=-1$, $c=2$.

(i) $\mathbf{5a-10}$

Solution:

$5a-10$

$5a-10=5(3)-10$

$5a-10=15-10$

$5a-10=5$

Ex # 4.2

(ii) $\boldsymbol{3b + 5c}$

Solution:

$3b + 5c$

$3b + 5c = 3(-1) + 5(2)$

$3b + 5c = -3 + 10$

$3b + 5c = 7$

(iii) $\boldsymbol{2a - 3b + 2c}$

Solution:

$2a - 3b + 2c$

$2a - 3b + 2c = 2(3) - 3(-1) + 2(2)$

$2a - 3b + 2c = 6 + 3 + 4$

$2a - 3b + 2c = 13$

Q2: **Evaluate the following for $\boldsymbol{x = -5}$ and $\boldsymbol{y = 2}$.**

(i) $\boldsymbol{7 - 3xy}$

Solution:

$7 - 3xy$

$7 - 3xy = 7 - 3(-5)(2)$

$7 - 3xy = 7 - 3(-10)$

$7 - 3xy = 7 + 30$

$7 - 3xy = 37$

(ii) $\boldsymbol{x^2 + xy + y^2}$

Solution:

$x^2 + xy + y^2$

$x^2 + xy + y^2 = (-5)^2 + (-5)(2) + (2)^2$

$x^2 + xy + y^2 = 25 + (-10) + 4$

$x^2 + xy + y^2 = 25 - 10 + 4$

$x^2 + xy + y^2 = 15 + 4$

$x^2 + xy + y^2 = 19$

(iii) $\boldsymbol{(3x)^2 - (4y)^2}$

Solution:

$(3x)^2 - (4y)^2$

$(3x)^2 - (4y)^2 = [3(-5)]^2 - [4(2)]^2$

$(3x)^2 - (4y)^2 = [-15]^2 - [8]^2$

$(3x)^2 - (4y)^2 = 225 - 64$

$(3x)^2 - (4y)^2 = 161$

Ex # 4.2

Q3: **Evaluate the following when $\boldsymbol{k = -2}$, $\boldsymbol{l = 3}$, $\boldsymbol{m = 4}$.**

(i) $\boldsymbol{k^2(2l - 3m)}$

Solution:

$k^2(2l - 3m)$

$k^2(2l - 3m) = (-2)^2[2(3) - 3(4)]$

$k^2(2l - 3m) = 4(6 - 12)$

$k^2(2l - 3m) = 4(-6)$

$k^2(2l - 3m) = -24$

(ii) $\boldsymbol{5m\sqrt{k^2 + l^2}}$

Solution:

$5m\sqrt{k^2 + l^2}$

$5m\sqrt{k^2 + l^2} = 5(4)\sqrt{(-2)^2 + (3)^2}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{4 + 9}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{13}$

(iii) $\boldsymbol{\dfrac{k + l + m}{k^2 + l^2 + m^2}}$

Solution:

$\dfrac{k + l + m}{k^2 + l^2 + m^2}$

Put the values

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{(-2) + (3) + (4)}{(-2)^2 + (3)^2 + (4)^2}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{-2 + 3 + 4}{4 + 9 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{1 + 4}{13 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{5}{29}$

Ex # 4.2

Q4: ***Evaluate*** $\boldsymbol{\frac{a+1}{4a^2+1}}$ ***when*** $\boldsymbol{a = \frac{1}{2}}$ **and** $\boldsymbol{a = -\frac{1}{2}}$.

Solution:

For $\boldsymbol{a = -\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{1}{2}+1}{4\left(\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{3}{2}}{1+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{3}{2}}{2}$$

$$\frac{a+1}{4a^2+1} = \frac{3}{2} \div 2$$

$$\frac{a+1}{4a^2+1} = \frac{3}{2} \times \frac{1}{2}$$

$$\frac{a+1}{4a^2+1} = \frac{3}{4}$$

For $\boldsymbol{a = -\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1} = \frac{-\frac{1}{2}+1}{4\left(-\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{-1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1} = \frac{\frac{1}{2}}{1+1}$$

Ex # 4.2

$$\frac{a+1}{4a^2+1} = \frac{\frac{1}{2}}{2}$$

$$\frac{a+1}{4a^2+1} = \frac{1}{2} \div 2$$

$$\frac{a+1}{4a^2+1} = \frac{1}{2} \times \frac{1}{2}$$

$$\frac{a+1}{4a^2+1} = \frac{1}{4}$$

Q5: ***If*** $\boldsymbol{a = 9\,,\quad b = 12, c = 15}$ ***and*** $\boldsymbol{s = \frac{a+b+c}{2}}$.

Find the value of $\boldsymbol{\sqrt{s(s-a)(s-b)(s-c)}}$

Solution:

Given:

$$a = 9\,, b = 12, c = 15 \text{ and } s = \frac{a+b+c}{2}$$

To Find:

$\sqrt{s(s-a)(s-b)(s-c)} = ?$

First we find:

$$s = \frac{a+b+c}{2}$$

Put the values:

$$s = \frac{a+b+c}{2}$$

$$s = \frac{9+12+15}{2}$$

$$s = \frac{36}{2}$$

$$s = 18$$

Now

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18(18-9)(18-12)(18-15)}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18(9)(6)(3)}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{2 \times 9 \times 9 \times 2 \times 3 \times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9 \times 9 \times 2 \times 2 \times 3 \times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9^2 \times 2^2 \times 3^2}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = 9 \times 2 \times 3$$

$$\sqrt{s(s-a)(s-b)(s-c)} = 9 \times 6$$

$$\sqrt{s(s-a)(s-b)(s-c)} = 54$$

Chapter # 4

Ex # 4.3

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $a^2-b^2 = (a+b)(a-b)$
4. $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$ **Q2, Q3(i)**
5. $(a+b)^2-(a-b)^2 = 4ab$ **Q2, Q3(ii)**
6. $(x+y)^2+(x-y)^2 = 2(x^2+y^2)$ **Q1, Q5**
7. $(x+y)^2-(x-y)^2 = 4xy$ **Q1, Q4, Q5**
8. $(u+v)^2-(u-v)^2 = 4uv$ **Q6**

Ex # 4.3

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Q1: Find the value of x^2+y^2 and xy, when:

(i) $\boldsymbol{x+y=8,\quad x-y=3}$

Solution:

$x+y=8,\quad x-y=3$

To Find:

x^2+y^2 =? and xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(8)^2+(3)^2=2(x^2+y^2)$

$64+9=2(x^2+y^2)$

$73=2(x^2+y^2)$

Divide B.S by 2

$$\frac{73}{2}=\frac{2(x^2+y^2)}{2}$$

$$\frac{73}{2}=x^2+y^2$$

$$x^2+y^2=\frac{73}{2}$$

$\underline{\boldsymbol{xy}}$

Also we have

$\boldsymbol{(x+y)^2-(x-y)^2=4xy}$

Put the values

$(8)^2-(3)^2=4xy$

$64-9=4xy$

$55=4xy$

Divide B.S by 4

$$\frac{55}{4}=\frac{4xy}{4}$$

$$\frac{55}{4}=xy$$

$$xy=\frac{55}{4}$$

Ex # 4.3

(ii) $\boldsymbol{x+y=10,\quad x-y=7}$

Solution:

$x+y=10,\quad x-y=7$

To Find:

x^2+y^2 =? And xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(10)^2+(7)^2=2(x^2+y^2)$

$100+49=2(x^2+y^2)$

$149=2(x^2+y^2)$

Divide B.S by 2

$$\frac{149}{2}=\frac{2(x^2+y^2)}{2}$$

$$\frac{149}{2}=x^2+y^2$$

$$x^2+y^2=\frac{149}{2}$$

$\underline{\boldsymbol{xy}}$

Also we have

$\boldsymbol{(x+y)^2-(x-y)^2=4xy}$

Put the values

$(10)^2-(7)^2=4xy$

$100-49=4xy$

$51=4xy$

Divide B.S by 4

$$\frac{51}{4}=\frac{4xy}{4}$$

$$\frac{51}{4}=xy$$

$$xy=\frac{51}{4}$$

(iii) $\boldsymbol{x+y=11,\quad x-y=5}$

Solution:

$x+y=11,\quad x-y=5$

To Find:

x^2+y^2 =? and xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(11)^2+(5)^2=2(x^2+y^2)$

Ex # 4.3

$121 + 25 = 2(x^2 + y^2)$

$146 = 2(x^2 + y^2)$

Divide B.S by 2

$\frac{146}{2} = \frac{2(x^2 + y^2)}{2}$

$73 = x^2 + y^2$

$x^2 + y^2 = 73$

xy

Also we have

$$(\boldsymbol{x} + \boldsymbol{y})^2 - (\boldsymbol{x} - \boldsymbol{y})^2 = \boldsymbol{4xy}$$

Put the values

$(11)^2 - (5)^2 = 4xy$

$121 - 25 = 4xy$

$96 = 4xy$

Divide B.S by 4

$\frac{96}{4} = \frac{4xy}{4}$

$24 = xy$

$xy = 24$

(iv) $\boldsymbol{x + y = 7, \quad x - y = 4}$

Solution:

$x + y = 7, \quad x - y = 4$

To Find:

$x^2 + y^2 =$? and $xy =$?

$\boldsymbol{x^2 + y^2}$

As we have

$$(\boldsymbol{x} + \boldsymbol{y})^2 + (\boldsymbol{x} - \boldsymbol{y})^2 = \boldsymbol{2}(\boldsymbol{x}^2 + \boldsymbol{y}^2)$$

Put the values

$(7)^2 + (4)^2 = 2(x^2 + y^2)$

$49 + 16 = 2(x^2 + y^2)$

$65 = 2(x^2 + y^2)$

Divide B.S by 2

$\frac{65}{2} = \frac{2(x^2 + y^2)}{2}$

$\frac{65}{2} = x^2 + y^2$

$x^2 + y^2 = \frac{65}{2}$

xy

Also we have

$$(\boldsymbol{x} + \boldsymbol{y})^2 - (\boldsymbol{x} - \boldsymbol{y})^2 = \boldsymbol{4xy}$$

Put the values

$(7)^2 - (4)^2 = 4xy$

Ex # 4.3

$49 - 16 = 4xy$

$33 = 4xy$

Divide B.S by 4

$\frac{33}{4} = \frac{4xy}{4}$

$\frac{33}{4} = xy$

$xy = \frac{33}{4}$

Q2: **Find the value of $\boldsymbol{a^2 + b^2}$ and $\boldsymbol{ab}$, when**

(i) $\boldsymbol{a + b = 7, \quad a - b = 3}$

Solution:

$a + b = 7$ and $a - b = 3$

To Find:

$a^2 + b^2 =$? and $ab =$?

$\boldsymbol{a^2 + b^2}$

As we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 + (\boldsymbol{a} - \boldsymbol{b})^2 = \boldsymbol{2}(\boldsymbol{a}^2 + \boldsymbol{b}^2)$$

Put the values

$(7)^2 + (3)^2 = 2(a^2 + b^2)$

$49 + 9 = 2(a^2 + b^2)$

$58 = 2(a^2 + b^2)$

Divide B.S by 2

$\frac{58}{2} = \frac{2(a^2 + b^2)}{2}$

$29 = a^2 + b^2$

$a^2 + b^2 = 29$

ab

Also we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 - (\boldsymbol{a} - \boldsymbol{b})^2 = \boldsymbol{4ab}$$

Put the values

$(7)^2 - (3)^2 = 4ab$

$49 - 9 = 4ab$

$40 = 4ab$

Divide B.S by 4

$\frac{40}{4} = \frac{4ab}{4}$

$10 = ab$

$ab = 10$

Chapter # 4

Ex # 4.3

Q2: (ii) **Find the value of $a^2 + b^2$ and ab, when $a + b = 9$, $a - b = 1$.**

Solution:

$a + b = 9$ and $a - b = 1$

To Find:

$a^2 + b^2 =?$ and $ab =?$

$\underline{\boldsymbol{a^2 + b^2}}$

As we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 + (\boldsymbol{a} - \boldsymbol{b})^2 = 2(\boldsymbol{a}^2 + \boldsymbol{b}^2)$$

Put the values

$(9)^2 + (1)^2 = 2(a^2 + b^2)$

$81 + 1 = 2(a^2 + b^2)$

$82 = 2(a^2 + b^2)$

Divide B.S by 2

$\frac{82}{2} = \frac{2(a^2 + b^2)}{2}$

$41 = a^2 + b^2$

$a^2 + b^2 = 41$

$\underline{\boldsymbol{ab}}$

Also we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 - (\boldsymbol{a} - \boldsymbol{b})^2 = 4\boldsymbol{ab}$$

Put the values

$(9)^2 - (1)^2 = 4ab$

$81 - 1 = 4ab$

$80 = 4ab$

Divide B.S by 4

$\frac{80}{4} = \frac{4ab}{4}$

$20 = ab$

$ab = 20$

Q3: (i) **If $a + b = 10$, $a - b = 6$, then find the value of $a^2 + b^2$.**

Solution:

$a + b = 10$ and $a - b = 6$

To Find:

$a^2 + b^2 =?$

As we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 + (\boldsymbol{a} - \boldsymbol{b})^2 = 2(\boldsymbol{a}^2 + \boldsymbol{b}^2)$$

Put the values

$(10)^2 + (6)^2 = 2(a^2 + b^2)$

$100 + 36 = 2(a^2 + b^2)$

$136 = 2(a^2 + b^2)$

Ex # 4.3

Divide B.S by 2

$\frac{136}{2} = \frac{2(a^2 + b^2)}{2}$

$68 = a^2 + b^2$

$a^2 + b^2 = 68$

Q3: (ii) **If $a + b = 5$, $a - b = \sqrt{17}$, then find the value of ab.**

Solution:

$a + b = 5$ and $a - b = \sqrt{17}$

To Find:

$ab =?$

Also we have

$$(\boldsymbol{a} + \boldsymbol{b})^2 - (\boldsymbol{a} - \boldsymbol{b})^2 = 4\boldsymbol{ab}$$

Put the values

$(5)^2 - (\sqrt{17})^2 = 4ab$

$25 - 17 = 4ab$

$8 = 4ab$

Divide B.S by 4

$\frac{8}{4} = \frac{4ab}{4}$

$2 = ab$

$ab = 2$

Q4: **Find the value of $4xy$ when $x + y = 17$, $x - y = 5$.**

Solution:

$x + y = 17, \quad x - y = 5$

To find:

$4xy =?$

Also we have

$$(\boldsymbol{x} + \boldsymbol{y})^2 - (\boldsymbol{x} - \boldsymbol{y})^2 = 4\boldsymbol{xy}$$

Put the values

$(17)^2 - (5)^2 = 4xy$

$289 - 25 = 4xy$

$264 = 4xy$

OR

$4xy = 264$

Chapter # 4

Ex # 4.3

Q5: **If $+y = 11$ *and* $x - y = 3$,**
find $8xy(x^2 + y^2)$.

Solution:

$x + y = 11, \qquad x - y = 3$

To Find:

$8xy(x^2 + y^2) = ?$

As we have

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

Put the values

$(11)^2 + (3)^2 = 2(x^2 + y^2)$

$121 + 9 = 2(x^2 + y^2)$

$130 = 2(x^2 + y^2)$

$2(x^2 + y^2) = 130 \quad --equ(i)$

Also we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(11)^2 - (3)^2 = 4xy$

$121 - 9 = 4xy$

$112 = 4xy$

$4xy = 112 \quad --equ(ii)$

Multiply equ (i) and (ii)

$2(x^2 + y^2) \times 4xy = 130 \times 112$

$8xy(x^2 + y^2) = 14560$

Q6: **If $u + v = 7$ and $uv = 12$, find $u - v$.**

Solution:

$u + v = 7, \; uv = 12$

To Find:

$u - v = ?$

As we know that

$$(u + v)^2 - (u - v)^2 = 4uv$$

Put the values

$(7)^2 - (u - v)^2 = 4(12)$

$49 - (u - v)^2 = 48$

$-(u - v)^2 = 48 - 49$

$-(u - v)^2 = -1$

$(u - v)^2 = 1$

Taking square root on B.S

$\sqrt{(u - v)^2} = \sqrt{1}$

$u - v = \pm 1$

Ex # 4.4

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
Q1, Q2, Q3
2. $2(x^2 + y^2 + z^2 - xy - yz - zx) =$
$(x - y)^2 + (y - z)^2 + (z - x)^2$ **Q4, Q5**
3. $2(a^2 + b^2 + c^2 - ab - bc - ca) =$
$(a - b)^2 + (b - c)^2 + (c - a)^2$ **Q6**

Ex # 4.4

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Q1: **Find the values of $a^2 + b^2 + c^2$, when**

(i) **$a + b + c = 5$ and $ab + bc + ca = -4$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -4$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-4)$

$25 = a^2 + b^2 + c^2 - 8$

$25 + 8 = a^2 + b^2 + c^2$

$33 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 33$

(ii) **$a + b + c = 5$ and $ab + bc + ca = -2$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -2$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-2)$

$25 = a^2 + b^2 + c^2 - 4$

$25 + 4 = a^2 + b^2 + c^2$

$29 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 29$

Ex # 4.4

Q2: Find the values of $a+b+c$, when

(i) $a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

Solution:

$a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

To Find:

$a+b+c=?$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(a+b+c)^2=38+2(-1)$

$(a+b+c)^2=38-2$

$(a+b+c)^2=36$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{36}$

$a+b+c=6$

(ii) $a^2+b^2+c^2=10$ and $ab+bc+ca=11$

Solution:

$a^2+b^2+c^2=10$ and $ab+bc+ca=11$

To Find:

$a+b+c=?$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(a+b+c)^2=10+2(11)$

$(a+b+c)^2=10+22$

$(a+b+c)^2=32$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{32}$

$a+b+c=\sqrt{16\times 2}$

$a+b+c=\sqrt{16}\times\sqrt{2}$

$a+b+c=4\sqrt{2}$

Q3: Find the values of $ab+bc+ca$, when

(i) $a^2+b^2+c^2=56$ and $a+b+c=12$

Solution:

$a^2+b^2+c^2=56$ and $a+b+c=12$

To Find:

$ab+bc+ca=?$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Ex # 4.4

Put the values

$(12)^2=56+2(ab+bc+ca)$

$144=56+2(ab+bc+ca)$

Subtract 56 from B.S

$144-56=56-56+2(ab+bc+ca)$

$88=2(ab+bc+ca)$

Divide B.S by 2

$\frac{88}{2}=\frac{2(ab+bc+ca)}{2}$

$44=ab+bc+ca$

$ab+bc+ca=44$

(ii) $a^2+b^2+c^2=12$ and $a+b+c=5$

Solution:

$a^2+b^2+c^2=12$ and $a+b+c=5$

To Find:

$ab+bc+ca=?$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(5)^2=12+2(ab+bc+ca)$

$25=12+2(ab+bc+ca)$

Subtract 12 from B.S

$25-12=12-12+2(ab+bc+ca)$

$13=2(ab+bc+ca)$

Divide B.S by 2

$\frac{13}{2}=\frac{2(ab+bc+ca)}{2}$

$\frac{13}{2}=ab+bc+ca$

$ab+bc+ca=\frac{13}{2}$

Chapter # 4

Ex # 4.4

Q #4 **Prove that $x^2+y^2+y^2-xy-yz-zx = \left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$**

Solution:

$x^2+y^2+y^2-xy-yz-zx =$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

R.H.S

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

$$=\frac{(x-y)^2}{\left(\sqrt{2}\right)^2}+\frac{(y-z)^2}{\left(\sqrt{2}\right)^2}+\frac{(z-x)^2}{\left(\sqrt{2}\right)^2}$$

$$=\frac{x^2+y^2-2xy}{2}+\frac{y^2+z^2-2yz}{2}+\frac{z^2+x^2-2zx}{2}$$

$$=\frac{x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx}{2}$$

$$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{2}$$

$$=x^2+y^2+z^2-xy-yz-zx$$

$$=\text{L.H.S}$$

Q #5 **Write $2\left[x^2+y^2+y^2-xy-yz-zx\right]$ as the sum of three squares.**

Solution:

$2[x^2+y^2+y^2-xy-yz-zx]$

$2x^2+2y^2+2z^2-2xy-2yz-2zx$

$x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$

Re-arranging the terms

$x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$

As we have

$$\boldsymbol{a^2+b^2-2ab=(a-b)^2}$$

$(x-y)^2+(y-z)^2+(z-x)^2$

Ex # 4.4

Q #6 **Find the value of**
$a^2+b^2+c^2-ab-bc-ca$
when $a-b=2,\ b-c=3, c-a=4.$

Solution:

Given that:

$$a-b=2, \qquad b-c=3, c-a=4$$

To find

$$a^2+b^2+c^2-ab-bc-ca=?$$

As we have

$\boldsymbol{2(a^2+b^2+c^2-ab-bc-ca)=(a-b)^2+(b-c)^2+(c-a)^2}$

Put the values

$2(a^2+b^2+c^2-ab-bc-ca)=(2)^2+(3)^2+(4)^2$

$2(a^2+b^2+c^2-ab-bc-ca)=4+9+16$

$2(a^2+b^2+c^2-ab-bc-ca)=29$

Divide B.S by 2

$\frac{2(a^2+b^2+c^2-ab-bc-ca)}{2}=\frac{29}{2}$

$a^2+b^2+c^2-ab-bc-ca=\frac{29}{2}$

Ex # 4.5

1. $(a+b)^3=a^3+b^3+3ab(a+b)$ **Q#1, 7**
2. $(a-b)^3=a^3-b^3-3ab(a-b)$ **Q#2**
3. $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$ **Q#3**
4. $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ **Q#4**
5. $\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\left(\frac{1}{a}\right)\left(3a+\frac{1}{a}\right)$ **Q#5**
6. $\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$ **Q#6**
7. $(u-v)^3=u^3-v^3-3uv(u-v)$ **Q#8**
8. $\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$ **Q#9**
9. $\left(a^2+\frac{1}{a^2}\right)^2=a^4+\frac{1}{a^4}+2(a^2)\left(\frac{1}{a^2}\right)$ **Q#9**
10. $\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

Ex # 4.5

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Q1: (i) **Find the value of $a^3 + b^3$, when $a + b = 4$ and $ab = 5$.**

Solution:

$$a + b = 4, \qquad ab = 5$$

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(5)(4)$

$64 = a^3 + b^3 + 60$

Subtract 60 from B.S

$64 - 60 = a^3 + b^3 + 60 - 60$

$4 = a^3 + b^3$

$a^3 + b^3 = 4$

(ii) **$a + b = 3$ and $ab = 20$.**

Solution:

$a + b = 3$ and $ab = 20$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(3)^3 = a^3 + b^3 + 3(3)(20)$

$27 = a^3 + b^3 + 180$

Subtract 180 from B.S

$27 - 180 = a^3 + b^3 + 180 - 180$

$-153 = a^3 + b^3$

$a^3 + b^3 = -153$

(iii) **$a + b = 4$ and $ab = 2$.**

Solution:

$a + b = 4$ and $ab = 2$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(2)(4)$

$64 = a^3 + b^3 + 24$

Ex # 4.5

Subtract 24 from B.S

$64 - 24 = a^3 + b^3 + 24 - 24$

$40 = a^3 + b^3$

$a^3 + b^3 = 40$

Q2: (i) **Find the value of $a^3 - b^3$, when $a - b = 5$ and $ab = 7$.**

Solution:

$a - b = 5, \ ab = 7$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Put the values

$(5)^3 = a^3 - b^3 - 3(7)(5)$

$125 = a^3 - b^3 - 105$

Add 105 on B.S

$125 + 105 = a^3 - b^3 - 105 + 105$

$230 = a^3 - b^3$

$a^3 - b^3 = 230$

(ii) **$a - b = 2$ and $ab = 15$.**

Solution:

$a - b = 2, \ ab = 15$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Put the values

$(2)^3 = a^3 - b^3 - 3(15)(2)$

$8 = a^3 - b^3 - 90$

Add 90 on B.S

$8 + 90 = a^3 - b^3 - 90 + 90$

$98 = a^3 - b^3$

$a^3 - b^3 = 98$

Ex # 4.5

(iii) $\boldsymbol{a - b = 7}$ **and** $\boldsymbol{ab = 6}$**.**

Solution:

$a - b = 7, \quad ab = 6$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$\boldsymbol{(a-b)^3 = a^3 - b^3 - 3ab(a-b)}$$

Put the values

$(7)^3 = a^3 - b^3 - 3(6)(7)$

$343 = a^3 - b^3 - 126$

Add 126 on B.S

$343 + 126 = a^3 - b^3 - 126 + 126$

$469 = a^3 - b^3$

$a^3 + b^3 = 469$

Q3: **Find the value of** $\boldsymbol{x^3 + \frac{1}{x^3}}$ **, when**

(i) $\boldsymbol{x + \frac{1}{x} = \frac{5}{2}}$

Solution:

$x + \frac{1}{x} = \frac{5}{2}$

To Find:

$x^3 + \frac{1}{x^3} = ?$

As we have

$$\boldsymbol{\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)}$$

Put the values

$\left(\frac{5}{2}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(\frac{5}{2}\right)$

$\frac{125}{8} = x^3 + \frac{1}{x^3} + \frac{15}{2}$

Subtract $\frac{15}{2}$ from B.S

$\frac{125}{8} - \frac{15}{2} = x^3 + \frac{1}{x^3} + \frac{15}{2} - \frac{15}{2}$

$\frac{125 - 60}{8} = x^3 + \frac{1}{x^3}$

$\frac{65}{8} = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = \frac{65}{8}$

Ex # 4.5

(ii) $\boldsymbol{x + \frac{1}{x} = 2}$

Solution:

$$x + \frac{1}{x} = 2$$

To Find:

$$x^3 + \frac{1}{x^3} = ?$$

As we have

$$\boldsymbol{\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)}$$

Put the values

$(2)^3 = x^3 + \frac{1}{x^3} + 3(2)$

$8 = x^3 + \frac{1}{x^3} + 6$

Subtract 6 from B.S

$8 - 6 = x^3 + \frac{1}{x^3} + 6 - 6$

$2 = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = 2$

Q3: **Find the value of** $\boldsymbol{x^3 - \frac{1}{x^3}}$ **, when**

(i) $\boldsymbol{x - \frac{1}{x} = \frac{3}{2}}$

Solution:

$$x - \frac{1}{x} = \frac{3}{2}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\boldsymbol{\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)}$$

Put the values

$\left(\frac{3}{2}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{3}{2}\right)$

$\frac{27}{8} = x^3 - \frac{1}{x^3} - \frac{9}{2}$

Add $\frac{9}{2}$ on B.S

$\frac{27}{8} + \frac{9}{2} = x^3 - \frac{1}{x^3} - \frac{9}{2} + \frac{9}{2}$

$$\frac{27+36}{8} = x^3 - \frac{1}{x^3}$$

$$\frac{63}{8} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{63}{8}$$

(ii) $\boldsymbol{x - \frac{1}{x} = \frac{7}{3}}$

Solution:

$$x - \frac{1}{x} = \frac{7}{3}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\boldsymbol{\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)}$$

Put the values

$$\left(\frac{7}{3}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{7}{3}\right)$$

$$\frac{343}{27} = x^3 - \frac{1}{x^3} - \frac{21}{3}$$

Add $\frac{21}{3}$ on B.S

$$\frac{343}{27} + \frac{21}{3} = x^3 - \frac{1}{x^3} - \frac{21}{3} + \frac{21}{3}$$

$$\frac{343+189}{27} = x^3 - \frac{1}{x^3}$$

$$\frac{532}{27} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{532}{27}$$

(iii) $\boldsymbol{x - \frac{1}{x} = \frac{15}{4}}$

Solution:

$$x - \frac{1}{x} = \frac{15}{4}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\boldsymbol{\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)}$$

Put the values

$$\left(\frac{15}{4}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{15}{4}\right)$$

$$\frac{3375}{64} = x^3 - \frac{1}{x^3} - \frac{45}{4}$$

Add $\frac{45}{4}$ on B.S

$$\frac{3375}{64} + \frac{45}{4} = x^3 - \frac{1}{x^3} - \frac{45}{4} + \frac{45}{4}$$

$$\frac{3375+720}{64} = x^3 - \frac{1}{x^3}$$

$$\frac{4095}{64} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{4095}{64}$$

Q5: $\boldsymbol{\text{If } 3a + \frac{1}{a} = 4, \text{find } 27a^3 + \frac{1}{a^3}}$

Solution:

$$3a + \frac{1}{a} = 4$$

To Find:

$$27a^3 + \frac{1}{a^3} = ?$$

As we have

$$\boldsymbol{\left(3a + \frac{1}{a}\right)^3 = 27a^3 + \frac{1}{a^3} + 3(3a)\left(\frac{1}{a}\right)\left(3a + \frac{1}{a}\right)}$$

Put the values

$$(4)^3 = 27a^3 + \frac{1}{a^3} + 9(4)$$

$$64 = 27a^3 + \frac{1}{a^3} + 36$$

Subtract 36 from B.S

$$64 - 36 = 27a^3 + \frac{1}{a^3} + 36 - 36$$

$$28 = 27a^3 + \frac{1}{a^3}$$

$$27a^3 + \frac{1}{a^3} = 28$$

Ex # 4.5

Q6: **If $x-\frac{1}{2x}=6$, find $x^3-\frac{1}{8x^3}$**

Solution:

$$x-\frac{1}{2x}=6$$

To Find:

$$x^3-\frac{1}{8x^3}=?$$

As we have

$$\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$$

Put the values

$$(6)^3=x^3-\frac{1}{8x^3}-\frac{3}{2}(6)$$

$$216=x^3-\frac{1}{8x^3}-3(3)$$

$$216=x^3-\frac{1}{8x^3}-9$$

Add 9 on B.S

$$216+9=x^3-\frac{1}{8x^3}-9+9$$

$$225=x^3-\frac{1}{8x^3}$$

$$x^3-\frac{1}{8x^3}=225$$

Q7: **If $a+b=6$, show that $a^3+b^3+18ab=216$.**

Solution:

$$a+b=6$$

To Prove:

$$a^3+b^3+18ab=216$$

As we have

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

Put the values

$$(6)^3=a^3+b^3+3ab(6)$$

$$216=a^3+b^3+18ab$$

$$a^3+b^3+18ab=216$$

Q8: **If $u-\text{v}=3$ then prove that $u^3-\text{v}^3-9uv=27$.**

Solution:

$$u-\text{v}=3$$

To Prove:

$$u^3-\text{v}^3-9uv=27$$

As we have

$$(u-v)^3=u^3-v^3-3uv(u-v)$$

Ex # 4.5

Put the values

$$(2)^3=a^3-b^3-3(15)(2)$$

$$8=a^3-b^3-90$$

Add 90 on B.S

$$8+90=a^3-b^3-90+90$$

$$98=a^3-b^3$$

$$a^3-b^3=98$$

Q9: **If $a+\frac{1}{a}=2$, find the values of $a^2+\frac{1}{a^2}$, $a^4+\frac{1}{a^4}$, $a^3+\frac{1}{a^3}$**

Solution:

Given

$$a+\frac{1}{a}=2$$

To prove

$$a^2+\frac{1}{a^2}=?$$

$$a^4+\frac{1}{a^4}=?$$

$$a^3+\frac{1}{a^3}=?$$

As we have

$$\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$$

Put the values

$$(2)^2=a^2+\frac{1}{a^2}+2$$

$$4=a^2+\frac{1}{a^2}+2$$

Subtract 2 from B.S

$$4-2=a^2+\frac{1}{a^2}+2-2$$

$$2=a^2+\frac{1}{a^2}$$

$$a^2+\frac{1}{a^2}=2$$

Now take square on B.S

$$\left(a^2+\frac{1}{a^2}\right)^2=(2)^2$$

$$(a^2)^2+\left(\frac{1}{a^2}\right)^2+2(a^2)\left(\frac{1}{a^2}\right)=4$$

$$a^4+\frac{1}{a^4}+2=4$$

Ex # 4.5

Subtract 2 from B.S

$$a^4+\frac{1}{a^4}+2-2=4-2$$

$$a^4+\frac{1}{a^4}=2$$

Now $\boldsymbol{a^3+\frac{1}{a^3}}$

Also we have

$$\boldsymbol{\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)}$$

Put the values

$$(2)^3=a^3+\frac{1}{a^3}+3(2)$$

$$8=a^3+\frac{1}{a^3}+6$$

Subtract 6 from B.S

$$8-6=a^3+\frac{1}{a^3}+6-6$$

$$2=a^3+\frac{1}{a^3}$$

$$a^3+\frac{1}{a^3}=2$$

Hence

$$a^2+\frac{1}{a^2}=a^4+\frac{1}{a^4}=a^3+\frac{1}{a^3}=2$$

Ex # 4.6

1. $a^3+b^3=(a+b)(a^2-ab+b^2)$
2. $a^3-b^3=(a-b)(a^2+ab+b^2)$
3. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)$
4. $x^3-\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)$

OR

1. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$
2. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

3. $(x+y)(x^2-xy+y^2)=x^3+y^3$
4. $(x-y)(x^2+xy+y^2)=x^3-y^3$
5. $(x+y)(x-y)=x^2-y^2$

Ex # 4.6

Page # 118

Q1: Find the following product.

(i) $\boldsymbol{(a-1)(a^2+a+1)}$

Solution:

$$(a-1)(a^2+a+1)$$
$$=(a-1)[(a)^2+(a)(1)+(1)^2]$$

As we know that

$\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}$

Here $a=a$ and $b=1$

So

$=(a)^3-(1)^3$

$=a^3-1$

(ii) $\boldsymbol{(3-b)(9+3b+b^2)}$

Solution:

$$(3-b)(9+3b+b^2)$$
$$=(3-b)[(3)^2+(3)(b)+(b)^2]$$

As we know that

$\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}$

Here $a=3$ and $b=b$

So

$=(3)^3-(b)^3$

$=27-b^3$

(iii) $\boldsymbol{(8+b)(64-8b+b^2)}$

Solution:

$$(8+b)(64-8b+b^2)$$
$$=(8+b)[(8)^2-(8)(b)+(b)^2]$$

As we know that

$\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}$

Here $a=8$ and $b=b$

So

$=(8)^3+(b)^3$

$=512+b^3$

(iv) $(a+2)(a^2-2a+4)$

Solution:

$$(a+2)(a^2-2a+4)$$
$$=(a+2)[(a)^2-(a)(2)+(2)^2]$$

As we know that

$\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}$

Here $a=a$ and $b=2$

So

$=(a)^3+(2)^3$

$=a^3+8$

Ex # 4.6

Q2: Find the following product.

(i) $$\left(\boldsymbol{2p+\frac{1}{2p}}\right)\left(\boldsymbol{4p^2+\frac{1}{4p^2}-1}\right)$$

Solution:

$$\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$$

$$\left(2p+\frac{1}{2p}\right)\left[(2p)^2+\frac{1}{(2p)^2}-(2p)\left(\frac{1}{2p}\right)\right]$$

As we know that

$$\left(\boldsymbol{x+\frac{1}{x}}\right)\left(\boldsymbol{x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)}\right)=\boldsymbol{x^3+\frac{1}{x^3}}$$

So

$$=(2p)^3+\left(\frac{1}{2p}\right)^3$$

$$=8p^3+\frac{1}{8p^3}$$

(ii) $$\left(\boldsymbol{\frac{3}{2}p-\frac{2}{3p}}\right)\left(\boldsymbol{\frac{9}{4}p^2+\frac{4}{9p^2}+1}\right)$$

Solution:

$$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$$

$$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left[\left(\frac{3}{2}p\right)^2+\left(\frac{2}{3p}\right)^2+\left(\frac{3}{2}p\right)\left(\frac{2}{3p}\right)\right]$$

As we know that

$$\left(\boldsymbol{x-\frac{1}{x}}\right)\left(\boldsymbol{x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)}\right)=\boldsymbol{x^3-\frac{1}{x^3}}$$

So

$$=\left(\frac{3}{2}p\right)^3-\left(\frac{2}{3p}\right)^3$$

$$=\frac{27}{8}p^3-\frac{8}{27p^3}$$

(iii) $$\left(\boldsymbol{3p-\frac{1}{3p}}\right)\left(\boldsymbol{9p^2+\frac{1}{9p^2}+1}\right)$$

Solution:

$$\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$$

$$\left(3p-\frac{1}{3p}\right)\left[(3p)^2+\frac{1}{(3p)^2}+(3p)\left(\frac{1}{3p}\right)\right]$$

As we know that

$$\left(\boldsymbol{x-\frac{1}{x}}\right)\left(\boldsymbol{x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)}\right)=\boldsymbol{x^3-\frac{1}{x^3}}$$

Ex # 4.6

So

$$=(3p)^3-\left(\frac{1}{3p}\right)^3$$

$$=27p^3+\frac{1}{27p^3}$$

(iv) $$\left(\boldsymbol{5p+\frac{1}{5p}}\right)\left(\boldsymbol{25p^2+\frac{1}{25p^2}-1}\right)$$

Solution:

$$\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$$

$$\left(5p+\frac{1}{5p}\right)\left[(5p)^2+\frac{1}{(5p)^2}-(5p)\left(\frac{1}{5p}\right)\right]$$

As we know that

$$\left(\boldsymbol{x+\frac{1}{x}}\right)\left(\boldsymbol{x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)}\right)=\boldsymbol{x^3+\frac{1}{x^3}}$$

So

$$=(5p)^3+\left(\frac{1}{5p}\right)^3$$

$$=125p^3+\frac{1}{125p^3}$$

Q3: Find the following continued product.

(i) $$\boldsymbol{(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)}$$

Solution:

$$(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$$

Using $\boldsymbol{a^2-b^2=(a+b)(a-b)}$

$$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$$

Arrange it

$$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$

By Using Formulas

$$=(x^3+y^3)(x^3-y^3)$$

Again by Formula

$$=(x^3)^2-(y^3)^2$$

$$=x^6-y^6$$

Ex # 4.6

(ii) $\boldsymbol{(x+y)(x-y)(x^2+y^2)(x^4+y^4)}$

Solution:

$(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^2-y^2)(x^2+y^2)(x^4+y^4)$

Again by Formula

$[(x^2)^2-(y^2)^2](x^4+y^4)$

$(x^4-y^4)(x^4+y^4)$

Now again by Formula

$(x^4)^2-(y^4)^2$

x^8-y^8

iii. $\boldsymbol{(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)}$

Solution:

$(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Arrange it

$(2x-y)(4x^2+2xy+y^2)(2x+y)(4x^2-2xy+y^2)$

$(2x-y)[(2x)^2+(2x)(y)+(y)^2](2x+y)[(2x)^2-(2x)(y)+(y)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(2x)^3-(y)^3][(2x)^3+(y)^3]$

$(8x^3-y^3)(8x^3+y^3)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(8x^3)^2-(y^3)^2$

$64x^6-y^6$

iv. $\boldsymbol{(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)}$

Solution:

$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Arrange it

$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

$(x-2)[(x)^2+(x)(2)+(2)^2](x+2)[(x)^2-(x)(2)+(2)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(x)^3-(2)^3][(x)^3+(2)^3]$

$(x^3-8)(x^3+8)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^3)^2-(8)^2$

x^6-64

Ex # 4.6

Q4: **Find the product with the help of formula. $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$**

Solution:

$(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

$=(\sqrt{x}-\sqrt{y})\left[(\sqrt{x})^2+(\sqrt{x})(\sqrt{y})+(\sqrt{y})^2\right]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

$=(\sqrt{x})^3-(\sqrt{y})^3$

$=\left(x^{\frac{1}{2}}\right)^3-\left(y^{\frac{1}{2}}\right)^3$

$=x^{\frac{3}{2}}-y^{\frac{3}{2}}$

Q5: **Simplify with the help of formula. $(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$**

Solution:

$(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

$=(x^p+y^q)[(x^p)^2-(x^p)(y^q)+(y^q)^2]$

As $(x+y)(x^2-xy+y^2)=x^3+y^3$

$=(x^p)^3+(y^p)^3$

$=x^{3p}+y^{3p}$

Examples Page # 116 and 117

Chapter # 4

Ex # 4.7

SURDS

A number of the form of $\sqrt[n]{a}$ is called Surd, where a is a positive rational number.

A number will be a surd, if

i. It is irrational
ii. It is a root
iii. A root of a rational number.

Examples:

$\sqrt{3}$ and $\sqrt{5+\sqrt{3}}$

In the above examples, both are irrational numbers. First number is a root of rational number 3, whereas the second number is a root of irrational number $5+\sqrt{3}$.

Thus $\sqrt{3}$ is a surd and $\sqrt{5+\sqrt{3}}$ is not a surd.

$\sqrt[3]{8}$ is not a surd because its value is 2 which is rational.

$\sqrt{-2}$, $\sqrt{-3}$ are not surds because -2 $and -3$ are negative.

Conjugate of Surds

The conjugate of $a\sqrt{x}+b\sqrt{y}$ is $a\sqrt{x}-b\sqrt{y}$.

Similarly the conjugate of $5+\sqrt{3}$ is $5-\sqrt{3}$

Ex # 4.7

Page # 122

Q1: State which of the following are surd quantities

(i) $\sqrt[3]{81}$

As 81 is a rational number and the result is irrational.

So it is surd.

(ii) $\sqrt{1+\sqrt{5}}$

As $1+\sqrt{5}$ is irrational.

So it is not surd.

(iii) $\sqrt{\sqrt{5}}$

As $\sqrt{5}$ is irrational.

So it is not surd.

(iv) $\sqrt[4]{32}$

As 32 is a rational number and the result is irrational.

So it is surd.

Ex # 4.7

(v) π

As π is irrational.

So it is not surd.

(vi) $\sqrt{1+\pi^2}$

As $1+\pi^2$ is irrational.

So it is not surd.

Q2: Express the following as the simplest possible surds.

(i) $\sqrt{12}$

Solution:

$\sqrt{12}$

$\sqrt{2\times2\times3}$

$\sqrt{2^2\times3}$

$\sqrt{2^2}\sqrt{3}$

$2\sqrt{3}$

2	12
2	6
3	3
	1

(ii) $\sqrt{48}$

Solution:

$\sqrt{48}$

$\sqrt{2\times2\times2\times2\times3}$

$\sqrt{2^2\times2^2\times3}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3}$

$2\times2\sqrt{3}$

$4\sqrt{3}$

2	48
2	24
2	12
2	6
3	3
	1

(iii) $\sqrt{240}$

Solution:

$\sqrt{240}$

$\sqrt{2\times2\times2\times2\times3\times5}$

$\sqrt{2^2\times2^2\times3\times5}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3\times5}$

$2\times2\sqrt{15}$

$4\sqrt{15}$

2	240
2	120
2	60
2	30
3	15
5	5
	1

Ex # 4.7

Q3: Simplify the following surds.

(i) $\mathbf{(2-\sqrt{3})(3+\sqrt{5})}$

Solution:

$(2-\sqrt{3})(3+\sqrt{5})$

$2(3+\sqrt{5})-\sqrt{3}(3+\sqrt{5})$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{3\times5}$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$

(ii) $\mathbf{(\sqrt{3}-4)(\sqrt{2}+1)}$

Solution:

$(\sqrt{3}-4)(\sqrt{2}+1)$

$\sqrt{3}(\sqrt{2}+1)-4(\sqrt{2}+1)$

$\sqrt{3\times2}+1\sqrt{3}-4\sqrt{2}-4$

$\sqrt{6}+\sqrt{3}-4\sqrt{2}-4$

(iii) $\mathbf{(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})}$

Solution:

$(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})$

$\sqrt{2}(\sqrt{5}+\sqrt{2})+\sqrt{3}(\sqrt{5}+\sqrt{2})$

$\sqrt{2\times5}+\sqrt{2\times2}+\sqrt{3\times5}+\sqrt{3\times2}$

$\sqrt{10}+2+\sqrt{15}+\sqrt{6}$

(iv) $\mathbf{(3-2\sqrt{3})(3+2\sqrt{3})}$

Solution:

$(3-2\sqrt{3})(3+2\sqrt{3})$

Using Formula: $(a+b)(a+b)=a^2-b^2$

So

$(3)^2-(2\sqrt{3})^2$

$9-(2)^2(\sqrt{3})^2$

$9-4(3)$

$9-12$

-3

Q4: Rationalize the denominator and simplify.

(i) $\mathbf{\frac{1}{\sqrt{7}}}$

Solution:

$\frac{1}{\sqrt{7}}$

Ex # 4.7

Multiply and divide by $\sqrt{7}$

$\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$

$\frac{1\sqrt{7}}{(\sqrt{7})^2}$

$\frac{\sqrt{7}}{7}$

(ii) $\mathbf{\frac{3}{\sqrt{45}}}$

Solution:

$\frac{3}{\sqrt{45}}$

$\frac{3}{\sqrt{3\times3\times5}}$

$\frac{3}{3\sqrt{5}}$

$\frac{1}{\sqrt{5}}$

Multiply and divide by $\sqrt{5}$

$\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$

$\frac{1\sqrt{5}}{(\sqrt{5})^2}$

$\frac{\sqrt{5}}{5}$

(iii) $\mathbf{\frac{1}{\sqrt{2}-1}}$

Solution:

$\frac{1}{\sqrt{2}-1}$

Multiply and divide by $\sqrt{2}+1$

$\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\frac{1(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2}$

$\frac{\sqrt{2}+1}{2-1}$

$\sqrt{2}+1$

Ex # 4.7

(iv) $\mathbf{\frac{5}{2+\sqrt{5}}}$

Solution:

$$\frac{5}{2+\sqrt{5}}$$

Multiply and divide by $2-\sqrt{5}$

$$\frac{5}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$\frac{5(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2}$$

$$\frac{5(2-\sqrt{5})}{4-5}$$

$$\frac{5(2-\sqrt{5})}{-1}$$

$$-5(2-\sqrt{5})$$

(v) $\mathbf{\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}}$

Solution:

$$\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$$

$$\frac{1(\sqrt{5}+2)+1(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$\frac{\sqrt{5}+2+\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$\frac{\sqrt{5}+\sqrt{5}}{5-4}$$

$$\frac{2\sqrt{5}}{1}$$

$$2\sqrt{5}$$

Q5: **If $x=\sqrt{5}+2$, find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x=\sqrt{5}+2$

To find:

$$x+\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}$$

Ex # 4.7

Multiply and divide by $\sqrt{5}-2$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{x}=\frac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{1}$$

$$\frac{1}{x}=\sqrt{5}-2$$

Now

$$x+\frac{1}{x}=(\sqrt{5}+2)+(\sqrt{5}-2)$$

$$x+\frac{1}{x}=\sqrt{5}+2+\sqrt{5}-2$$

$$x+\frac{1}{x}=2\sqrt{5}$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(2\sqrt{5})^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=(2)^2(\sqrt{5})^2$$

$$x^2+\frac{1}{x^2}+2=4(5)$$

$$x^2+\frac{1}{x^2}+2=20$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=20-2$$

$$x^2+\frac{1}{x^2}=18$$

Answers:

$$x+\frac{1}{x}=2\sqrt{5}$$

$$x^2+\frac{1}{x^2}=18$$

Chapter # 4

Ex # 4.7

Q6: **If $x = \sqrt{2}+\sqrt{3}$, find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x = \sqrt{2}+\sqrt{3}$

To find:

$$x-\frac{1}{x} = ?\text{ and } x^2+\frac{1}{x^2} = ?$$

$\frac{1}{x} = \frac{1}{\sqrt{2}+\sqrt{3}}$

Multiply and divide by $\sqrt{2}-\sqrt{3}$

$\frac{1}{x} = \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$\frac{1}{x} = \frac{1(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$\frac{1}{x} = \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$

$\frac{1}{x} = \frac{\sqrt{2}-\sqrt{3}}{2-3}$

$\frac{1}{x} = \frac{\sqrt{2}-\sqrt{3}}{-1}$

$\frac{1}{x} = -(\sqrt{2}-\sqrt{3})$

$\frac{1}{x} = -\sqrt{2}+\sqrt{3}$

Now

$x-\frac{1}{x} = (\sqrt{2}+\sqrt{3})-(-\sqrt{2}+\sqrt{3})$

$x-\frac{1}{x} = \sqrt{2}+\sqrt{3}+\sqrt{2}-\sqrt{3}$

$x-\frac{1}{x} = 2\sqrt{2}$

Taking Square on B.S

$\left(x-\frac{1}{x}\right)^2 = (2\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right) = (2)^2(\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2 = 4(2)$

$x^2+\frac{1}{x^2}-2 = 8$

Ex # 4.7

Add 2 on B.S

$x^2+\frac{1}{x^2}-2+2 = 8+2$

$x^2+\frac{1}{x^2} = 10$

Answers:

$$x-\frac{1}{x} = 2\sqrt{2}$$

$$x^2+\frac{1}{x^2} = 10$$

Q7: **If $x = 5-2\sqrt{6}$, find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x = 5-2\sqrt{6}$

To find:

$$x+\frac{1}{x} = ?\text{ and } x^2+\frac{1}{x^2} = ?$$

$\frac{1}{x} = \frac{1}{5-2\sqrt{6}}$

Multiply and divide by $5+2\sqrt{6}$

$\frac{1}{x} = \frac{1}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$

$\frac{1}{x} = \frac{1(5+2\sqrt{6})}{(5-2\sqrt{6})(5+2\sqrt{6})}$

$\frac{1}{x} = \frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}$

$\frac{1}{x} = \frac{5+2\sqrt{6}}{25-(2)^2(\sqrt{6})^2}$

$\frac{1}{x} = \frac{5+2\sqrt{6}}{25-(4)(6)}$

$\frac{1}{x} = \frac{5+2\sqrt{6}}{25-24}$

$\frac{1}{x} = \frac{5+2\sqrt{6}}{1}$

$\frac{1}{x} = 5+2\sqrt{6}$

Ex # 4.7

Now

$$x+\frac{1}{x}=\left(5-2\sqrt{6}\right)+\left(5+2\sqrt{6}\right)$$

$$x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}$$

$$x+\frac{1}{x}=10$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=100$$

$$x^2+\frac{1}{x^2}+2=100$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=100-2$$

$$x^2+\frac{1}{x^2}=98$$

Answers:

$$x+\frac{1}{x}=10$$

$$x^2+\frac{1}{x^2}=98$$

Q8: **If $x=\frac{1}{\sqrt{2}-1}$ find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$$x=\frac{1}{\sqrt{2}-1}$$

To find

$$x-\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$$\frac{1}{x}=\sqrt{2}-1$$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}$$

Multiply and divide by $\sqrt{5}-2$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

Ex # 4.7

$$\frac{1}{x}=\frac{1\left(\sqrt{5}-2\right)}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{1}$$

$$\frac{1}{x}=\sqrt{5}-2$$

Now

$$x-\frac{1}{x}=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)$$

$$x-\frac{1}{x}=\sqrt{2}+1-\sqrt{2}+1$$

$$x-\frac{1}{x}=2$$

Taking Square on B.S

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2$$

$$x^2+\frac{1}{x^2}-2=4$$

Add 2 on B.S

$$x^2+\frac{1}{x^2}-2+2=4+2$$

$$x^2+\frac{1}{x^2}=6$$

Answers:

$$x-\frac{1}{x}=2$$

$$x^2+\frac{1}{x^2}=6$$

Ex # 4.7

Q9: **If $x = \sqrt{10} + 3$, find the value of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$**

Solution:

$x = \sqrt{10} + 3$

To find

$$x - \frac{1}{x} = ?\text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$\frac{1}{x} = \frac{1}{\sqrt{10} + 3}$

Multiply and divide by $\sqrt{10} - 3$

$\frac{1}{x} = \frac{1}{\sqrt{10} + 3} \times \frac{\sqrt{10} - 3}{\sqrt{10} - 3}$

$\frac{1}{x} = \frac{1(\sqrt{10} - 3)}{(\sqrt{10} + 3)(\sqrt{10} - 3)}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{(\sqrt{10})^2 - (3)^2}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{10 - 9}$

$\frac{1}{x} = \frac{\sqrt{10} - 3}{1}$

$\frac{1}{x} = \sqrt{10} - 3$

Now

$x - \frac{1}{x} = (\sqrt{10} + 3) - (\sqrt{10} - 3)$

$x - \frac{1}{x} = \sqrt{10} + 3 - \sqrt{10} + 3$

$x - \frac{1}{x} = \sqrt{10} - \sqrt{10} + 3 + 3$

$x - \frac{1}{x} = 6$

Taking Square on B.S

$\left(x - \frac{1}{x}\right)^2 = (6)^2$

$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 36$

Ex # 4.7

$x^2 + \frac{1}{x^2} - 2 = 36$

Add 2 on B.S

$x^2 + \frac{1}{x^2} - 2 + 2 = 36 + 2$

$x^2 + \frac{1}{x^2} = 38$

Answers:

$$x - \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = 38$$

Q10: **If $x = 2 - \sqrt{3}$, find the value of $x^4 + \frac{1}{x^4}$**

Solution:

$x = 2 - \sqrt{3}$

To find

$$x + \frac{1}{x} = ?\text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$

Multiply and divide by $2 + \sqrt{3}$

$\frac{1}{x} = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$\frac{1}{x} = \frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{4 - 3}$

$\frac{1}{x} = \frac{2 + \sqrt{3}}{1}$

$\frac{1}{x} = 2 + \sqrt{3}$

Now

$x + \frac{1}{x} = (2 - \sqrt{3}) + (2 + \sqrt{3})$

$x + \frac{1}{x} = 2 - \sqrt{3} + 2 + \sqrt{3}$

Ex # 4.7

$$x+\frac{1}{x}=2+2-\sqrt{3}+\sqrt{3}$$

$$x+\frac{1}{x}=4$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(4)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=16$$

$$x^2+\frac{1}{x^2}+2=16$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=16-2$$

$$x^2+\frac{1}{x^2}=14$$

Again take the square on B.S

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$x^4+\frac{1}{x^4}+2(x^2)\left(\frac{1}{x^2}\right)=196$$

$$x^4+\frac{1}{x^4}+2=196$$

Subtract 2 from B.S

$$x^4+\frac{1}{x^4}+2-2=196-2$$

$$x^4+\frac{1}{x^4}=194$$

Answer:

$$x^4+\frac{1}{x^4}=194$$

Review Exercise # 4

Page # 124

Q2: **Simplify** $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

Solution:

$$\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$$

$$\frac{3y^3}{5}\cdot\frac{3a^2}{4x^3}$$

$$\frac{9y^3a^2}{20x^3}$$

$$\frac{9a^2y^3}{20x^3}$$

Q3: **Evaluate** $\frac{2x-3}{x^2-x+1}$ **for** $x=2$

Solution:

$$\frac{2x-3}{x^2-x+1}$$

Put the value

$$\frac{2x-3}{x^2-x+1}=\frac{2(2)-3}{(2)^2-(2)+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{4-3}{4-2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{3}$$

Q4: **Find the value of x^2+y^2 and xy when $x+y=7$, $x-y=3$.**

Solution:

$$x+y=7\ ,\ \ x-y=3$$

To Find:

$x^2+y^2=?$ and $xy=?$

$$\underline{\boldsymbol{x^2+y^2}}$$

As we have

$$(\boldsymbol{x}+\boldsymbol{y})^2+(\boldsymbol{x}-\boldsymbol{y})^2=\mathbf{2}(\boldsymbol{x}^2+\boldsymbol{y}^2)$$

Put the values

$$(7)^2+(3)^2=2(x^2+y^2)$$

$$49+9=2(x^2+y^2)$$

$$58=2(x^2+y^2)$$

Divide B.S by 2

$$\frac{58}{2}=\frac{2(x^2+y^2)}{2}$$

$29=x^2+y^2$

$29=x^2+y^2$

$\underline{xy}$

As we have

$$(\boldsymbol{x}+\boldsymbol{y})^2-(\boldsymbol{x}-\boldsymbol{y})^2=\boldsymbol{4xy}$$

Put the values

$(7)^2-(3)^2=4xy$

$49-9=4xy$

$40=4xy$

Divide B.S by 4

$$\frac{40}{4}=\frac{4xy}{4}$$

$10=xy$

$xy=10$

Q5: **Find the value of $a+b+c$ when $a^2+b^2+c^2=43$ and $ab+bc+ca=3$.**

Solution:

$a^2+b^2+c^2=43$ and $ab+bc+ca=3$

To Find:

$$a+b+c=?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(a+b+c)^2=43+2(3)$

$(a+b+c)^2=43+6$

$(a+b+c)^2=49$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{49}$

$a+b+c=7$

Q6: **If $a+b+c=6$ and $a^2+b^2+c^2=24$, then find the value of $ab+bc+ca$**

Solution:

$a+b+c=6\ and\ a^2+b^2+c^2=24$

To Find:

$$ab+bc+ca=?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2=\boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(6)^2=24+2(ab+bc+ca)$

$36=25+2(ab+bc+ca)$

Subtract 24 from B.S

$36-24=24-24+2(ab+bc+ca)$

$12=2(ab+bc+ca)$

Divide B.S by 2

$$\frac{12}{2}=\frac{2(ab+bc+ca)}{2}$$

$6=ab+bc+ca$

$ab+bc+ca=6$

Q7: **If $2x-3y=8$ and $xy=2$, then find the values of $8x^3-27y^3$.**

Solution:

$2x-3y=8$ and $xy=2$

To Find:

$$8x^3-27y^3=?$$

As we have

$$(\boldsymbol{2x}-\boldsymbol{3y})^3=(\boldsymbol{2x})^3-(\boldsymbol{3y})^3-3(\boldsymbol{2x})(\boldsymbol{3y})(\boldsymbol{2x}-\boldsymbol{3y})$$

Put the values

$(8)^3=8x^3-27y^3-18xy(8)$

$512=8x^3-27y^3-18(2)(8)$

$512=8x^3-27y^3-288$

Add 288 on B.S

$512+288=8x^3-27y^3-288+288$

$800=8x^3-27y^3$

$8x^3-27y^3=800$

Review Ex # 4

Q8: **Find the product** $\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$

Solution:

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$$

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2+\left(\frac{5}{4x}\right)^2+\left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)\right]$$

As we know that

$$\left(\boldsymbol{x}-\frac{\boldsymbol{1}}{\boldsymbol{x}}\right)\left(\boldsymbol{x}^2+\frac{\boldsymbol{1}}{\boldsymbol{x}^2}+(\boldsymbol{x})\left(\frac{\boldsymbol{1}}{\boldsymbol{x}}\right)\right)=\boldsymbol{x}^3-\frac{\boldsymbol{1}}{\boldsymbol{x}^3}$$

$$=\left(\frac{4}{5}x\right)^3-\left(\frac{5}{4x}\right)^3$$

$$=\frac{64}{125}x^3-\frac{125}{64x^3}$$

Q9: **Find the value of** $x^3+\frac{1}{x^3}$ **, when** $x+\frac{1}{x}=8$

Solution:

$$x+\frac{1}{x}=8$$

To Find:

$$x^3+\frac{1}{x^3}=?$$

As we have

$$\left(\boldsymbol{x}+\frac{\boldsymbol{1}}{\boldsymbol{x}}\right)^3=\boldsymbol{x}^3+\frac{\boldsymbol{1}}{\boldsymbol{x}^3}+\boldsymbol{3}(\boldsymbol{x})\left(\frac{\boldsymbol{1}}{\boldsymbol{x}}\right)\left(\boldsymbol{x}+\frac{\boldsymbol{1}}{\boldsymbol{x}}\right)$$

Put the values

$$(8)^3=x^3+\frac{1}{x^3}+3(8)$$

$$512=x^3+\frac{1}{x^3}+24$$

Subtract 24 from B.S

$$512-24=x^3+\frac{1}{x^3}+24-24$$

$$488=x^3+\frac{1}{x^3}$$

$$x^3+\frac{1}{x^3}=488$$

Review Ex # 4

Think Trick

Q10: **Simplify** $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

Solution:

$$\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$\frac{2x^2}{x^4-16}+\frac{1}{x+2}-\frac{x}{x^2-4}$$

$$\frac{2x^2}{(x^2)^2-(4)^2}+\frac{1}{x+2}-\frac{x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{1(x-2)-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-2-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-x-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{2}{x^2-4}$$

$$\frac{2x^2-2(x^2+4)}{(x^2+4)(x^2-4)}$$

$$\frac{2x^2-2x^2-8}{(x^2)^2-(4)^2}$$

$$\frac{-8}{x^4-16}$$

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MATHEMATICS

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Chapter # 5 FACTORIZATION

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UNIT # 5

FACTORIZATION

Ex # 5.1

Factorization

Writing an algebraic expression as the product of two or more algebraic expressions is called factorization of the algebraic expression.

Example

$$5x + 10x^2 = 5x(1 + 2x)$$

Here $5x$ and $1 + 2x$ are called factors of $5x + 10x^2$.

Type 1: $ka + kb + kc$

Common Techniques

اگر کسی سوال سے پہلے minus آجائیں تو اس کو common لیں گے۔ minus کو common لینے سے تمام terms کے sign تبدیل ہو جائیں گے۔

$-2x^3 + 12y - 7z = -(2x^3 - 12y + 7z)$

اگر تمام terms کے constants ایک ہی table میں آتے ہو تو اس میں بھی common لیں گے۔

$4x^3 - 24y + 64 = 4(x^3 - 6y + 16)$

اگر تمام terms میں ایک جیسے variable ہو تو سب سے کم power والے variable کو common لیں گے۔

$4x^3 - 5x^2 + 3xy = x(4x^2 - 5x + 3y)$

Example:

$-4x^3 + 24x^2 - 64x = -4x(x^2 - 6x + 16)$

Example 1:

(i) $15 + 10x - 5x^2 = 5(1 + 2x - x^2)$

(ii) $12x^2y^2 - 20x^3y = 4x^2y(3y - 5x)$

Type 2: $ac + ad + bc + bd.$

Example 2:

Factorize $a^2 - ab - 3a + 3b$

Solution:

$a^2 - ab - 3a + 3b$ **Making two pairs/groups**

Taking common from each group

$= a(a - b) - 3(a - b)$

$= (a - b)(a - 3)$ As $(a - b)$ is a common

Ex # 5.1

Type 3: $a^2 \pm 2ab + b^2$

Example 3:

$x^2 + 8x + 16 = (x)^2 + 2(x)(4) + (4)^2$

$x^2 + 8x + 16 = (x + 4)^2 = (x + 4)(x + 4)$

$25y^2 - 30y + 9 = (5y)^2 - 2(5y)(3) + (3)^2$

$25y^2 - 30y + 9 = (5y - 3)^2 = (5y - 3)(5y - 3)$

Type 4: $a^2 - b^2$

Example 4:

(i) $x^2 - 16 = (x)^2 - (4)^2 = (x + 4)(x - 4)$

(ii) $9a^2 - 25 = (3a)^2 - (5)^2 = (3a + 5)(3a - 5)$

(iii) $6x^4 - 6y^4 = 6(x^4 - y^4)$

$6x^4 - 6y^4 = 6[(x^2)^2 - (y^2)^2]$

$6x^4 - 6y^4 = 6(x^2 + y^2)(x^2 - y^2)$

$6x^4 - 6y^4 = 6(x^2 + y^2)(x + y)(x - y)$

Type 5: $a^2 \pm 2ab + b^2 - c^2$

Example 5:

Factorize $a^2 + 4ab + 4b^2 - c^2$

Solution:

$a^2 + 4ab + 4b^2 - c^2$

$= (a)^2 + 2(a)(2b) + (2b)^2 - c^2$

$= (a + 2b)^2 - (c)^2$

$= (a + 2b + c)(a + 2b - c)$

Example 6:

Factorize $a^2 - b^2 + 2b - 1$

Solution:

$a^2 - b^2 + 2b - 1$

$= a^2 - (b^2 - 2b + 1)$

$= a^2 - \{(b)^2 - 2(b)(1) + (1)^2\}$

$= a^2 - (b - 1)^2$

$= \{a + (b - 1)\}\{a - (b - 1)\}$

$(a + b - 1)(a - b + 1)$

Chapter # 5

Exercise# 5.1

Q1 $\boldsymbol{9s^2t + 15s^2t^3 - 3s^2t^2}$

Solution

$9s^2t + 15s^2t^3 - 3s^2t^2$

Take common $3s^2t$

$= 3s^2t(3 + 5t^2 - t)$

Q2 $\boldsymbol{10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2}$

Solution

$10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2$

Take common $5a^2b^2c^2$

$= 5a^2b^2c^2(2bc^2 - 3a + 6a^2b)$

Q3 $\boldsymbol{ax - a - x + 1}$

Solution

$ax - a - x + 1$

Taking common

$= a(x - 1) - 1(x - 1)$

Taking common

$= (x - 1)(a - 1)$

Q4 $\boldsymbol{x^2 - 2y^3 - 2xy^2 + xy}$

Solution

$x^2 - 2y^3 - 2xy^2 + xy$

Arrange it

$= x^2 + xy - 2xy^2 - 2y^3$

Taking common

$= x(x + y) - 2y^2(x + y)$

Taking common

$= (x + y)(x - 2y^2)$

Q5 $\boldsymbol{4x^2 + 4 + \frac{1}{x^2}}$

Solution

$$4x^2 + 4 + \frac{1}{x^2}$$

$$= (2x)^2 + 2(2x)\frac{1}{x} + \left(\frac{1}{x}\right)^2$$

As we know that

$= a^2 + 2ab + b^2 = (a + b)^2$

$$= \left(2x + \frac{1}{x}\right)^2$$

Q6 $\boldsymbol{4(x + y)^2 - 20(x + y)z + 25z^2}$

Solution

$4(x + y)^2 - 20(x + y)z + 25z^2$

$= [2(x + y)]^2 - 2[2(x + y)](5z) + (5z)^2$

As we know that $a^2 - 2ab + b^2 = (a - b)^2$

$= [2(x + y) - 5z]^2$

Q7 $\boldsymbol{\frac{x^4}{y^4} - \frac{y^4}{x^4}}$

Solution

$$\frac{x^4}{y^4} - \frac{y^4}{x^4}$$

$$= \frac{(x^2)^2}{(y^2)^2} - \frac{(y^2)^2}{(x^2)^2}$$

$$= \left(\frac{x^2}{y^2}\right)^2 - \left(\frac{y^2}{x^2}\right)^2$$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right)\left(\frac{x^2}{y^2} - \frac{y^2}{x^2}\right)$$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right)\left[\left(\frac{x}{y}\right)^2 - \left(\frac{y}{x}\right)^2\right]$$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right)\left(\frac{x}{y} + \frac{y}{x}\right)\left(\frac{x}{y} - \frac{y}{x}\right)$$

Q8 $\boldsymbol{2x^2 - 288}$

Solution

$2x^2 - 288$

Taking common

$= 2(x^2 - 144)$

$= 2[(x)^2 - (12)^2]$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$= 2(x + 12)(x - 12)$

Q9 $\mathbf{1 - u^2 + 2uv - v^2}$

Solution

$1 - (u^2 - 2uv + v^2)$

Using Formula $a^2 - 2ab + b^2 = (a - b)^2$

$= (1)^2 - (u - v)^2$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$= [1 + (u - v)][1 - (u - v)]$

$= (1 + u - v)(1 - u + v)$

Q10 $\mathbf{25a^2b^2 - 20abc + 4c^2 - 16d^2}$

Solution

$25a^2b^2 - 20abc + 4c^2 - 16d^2$

Using Formula $a^2 - 2ab + b^2 = (a - b)^2$

$= (5ab)^2 - 2(5ab)(2c) + (2c)^2 - (4d)^2$

$= (5ab - 2c)^2 - (4d)^2$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$= (5ab - 2c + 4d)(5ab - 2c - 4d)$

Exercise# 5.2

Q1 $\mathbf{x^4 + 64}$

Solution

$x^4 + 64$

$= (x^2)^2 + (8)^2$

Add and Subtract $2(x^2)(8)$

$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (x^2 + 8)^2 - 16x^2$

$= (x^2 + 8)^2 - (4x)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$

$= (x^2 + 4x + 8)\ (x^2 - 4x + 8)$

Q2 $\mathbf{4x^4 + 81}$

Solution

$4x^4 + 81$

$= (2x^2)^2 + (9)^2$

Add and Subtract $2(2x^2)(9)$

$= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9)$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (2x^2 + 9)^2 - 36x^2$

$= (2x^2 + 9)^2 - (6x)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$

$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$

Q3 $a^4 + a^2b^2 + b^4$

Solution

$a^4 + a^2b^2 + b^4$

$= a^4 + b^4 + a^2b^2$

$= (a^2)^2 + (b^2)^2 + a^2b^2$

Add and Subtract $2(a^2)(b^2)$

$= (a^2)^2 + (b^2)^2 + 2(a^2)(b^2) - 2(a^2)(b^2) + a^2b^2$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$

$= (a^2 + b^2)^2 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

Q4 $\mathbf{x^4 + x^2 + 1}$

Solution

$x^4 + x^2 + 1$

$= x^4 + 1 + x^2$

$= (x^2)^2 + (1)^2 + x^2$

Add and Subtract $2(x^2)(1)$

$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (x^2 + 1)^2 - 2x^2 + x^2$

$= (x^2 + 1)^2 - x^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (x^2 + 1 + x)(x^2 + 1 - x)$

$= (x^2 + x + 1)(x^2 - x + 1)$

Q5 $\mathbf{x^8 + x^4 + 1}$

Solution

$x^8 + x^4 + 1$

$= x^8 + 1 + x^4$

$= (x^4)^2 + (1)^2 + x^4$

Add and Subtract $2(x^4)(1)$

$(x^4)^2 + (1)^2 + 2(x^4)(1) - 2(x^4)(1) + x^4$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$(x^4 + 1)^2 - 2x^4 + x^4$

$(x^4 + 1)^2 - x^4$

$(x^4 + 1)^2 - (x^2)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$(x^4 + 1 + x^2)(x^4 + 1 - x^2)$

$[(x^2)^2 + (1)^2 + x^2](x^4 + 1 - x^2)$

$[(x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2](x^4 + 1 - x^2)$

Using Formula $a^2+b^2+2ab=(a+b)^2$

$= [(x^2+1)^2-2x^2+x^2](x^4+1-x^2)$

$= [(x^2+1)^2-x^2](x^4+1-x^2)$

As $a^2-b^2=(a+b)(a-b)$

$= [(x^2+1+x)(x^2+1-x)](x^4+1-x^2)$

$= [(x^2+x+1)(x^2-x+1)](x^4-x^2+1)$

Q6 $\boldsymbol{x^4+\frac{1}{x^4}-7}$

Solution

$x^4+\frac{1}{x^4}-7$

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2-7$

Add and Subtract $2(x^2)\left(\frac{1}{x^2}\right)$

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(x^2)\left(\frac{1}{x^2}\right)-2(x^2)\left(\frac{1}{x^2}\right)-7$

Using Formula $a^2+b^2+2ab=(a+b)^2$

$=\left(x^2+\frac{1}{x^2}\right)^2-2-7$

$=\left(x^2+\frac{1}{x^2}\right)^2-9$

$=\left(x^2+\frac{1}{x^2}\right)^2-(3)^2$

As $a^2-b^2=(a+b)(a-b)$

$=\left(x^2+\frac{1}{x^2}+3\right)\left(x^2+\frac{1}{x^2}-3\right)$

Q7 $\boldsymbol{81x^4+\frac{1}{81x^4}-14}$

Solution

$81x^4+\frac{1}{81x^4}-14$

$=(9x^2)^2+\left(\frac{1}{9x^2}\right)^2-14$

Add and Subtract $2(9x^2)\left(\frac{1}{9x^2}\right)$

$=(9x^2)^2+\left(\frac{1}{9x^2}\right)^2+2(9x^2)\left(\frac{1}{9x^2}\right)-2(9x^2)\left(\frac{1}{9x^2}\right)-14$

Using Formula $a^2+b^2+2ab=(a+b)^2$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-2-14$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-16$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-(4)^2$

As $a^2-b^2=(a+b)(a-b)$

$=\left(9x^2+\frac{1}{9x^2}+4\right)\left(9x^2+\frac{1}{9x^2}-4\right)$

Q8 $\boldsymbol{4x^4-4x^2y^2+64y^4}$

Solution

$4x^4-4x^2y^2+64y^4$

$=4(x^4-x^2y^2+16y^2)$

$=4(x^4+16y^4-x^2y^2)$

$=4[(x^2)^2+(4y^2)^2-x^2y^2]$

Add and Subtract $2(x^2)(4y^2)$

$=4[(x^2)^2+(4y^2)^2+2(x^2)(4y^2)-2(x^2)(4y^2)-x^2y^2]$

Using Formula $a^2+b^2+2ab=(a+b)^2$

$=4[(x^2+4y^2)^2-8x^2y^2-x^2y^2]$

$=4[(x^2+4y^2)^2-9x^2y^2]$

$=4[(x^2+4y^2)^2-(3xy)^2]$

As $a^2-b^2=(a+b)(a-b)$

$=4(x^2+4y^2+3xy)(x^2+4y^2-3xy)$

$=4(x^2+3xy+4y^2)(x^2-3xy+4y^2)$

Q9 $\boldsymbol{16m^4+4m^2n^2+n^4}$

Solution

$16m^4+4m^2n^2+n^4$

$=16m^4+n^4+4m^2n^2$

$=(4m^2)^2+(n^2)^2+4m^2n^2$

Add and Subtract $2(4m^2)(n^2)$

$=(4m^2)^2+(n^2)^2+2(4m^2)(n^2)-2(4m^2)(n^2)+4m^2n^2$

Using Formula $a^2+b^2+2ab=(a+b)^2$

$=(4m^2+n^2)^2-8m^2n^2+4m^2n^2$

$=(4m^2+n^2)^2-4m^2n^2$

$=(4m^2+n^2)^2-(2mn)^2$

As $a^2-b^2=(a+b)(a-b)$

$=(4m^2+n^2+2mn)(4m^2+n^2-2mn)$

$=(4m^2+2mn+n^2)(4m^2-2mn+n^2)$

Q10 $\mathbf{4x^5y + 11x^3y^3 + 9xy^5}$

Solution

$4x^5y + 11x^3y^3 + 9xy^5$

$= xy(4x^4 + 11x^2y^2 + 9y^4)$

$= xy(4x^4 + 9y^4 + 11x^2y^2)$

$= xy[(2x^2)^2 + (3y^2)^2 + 11x^2y^2]$

Add and Subtract $2(2x^2)(3y^2)$

$= xy[(2x^2)^2 + (3y^2)^2 + 2(2x^2)(3y^2) - 2(2x^2)(3y^2) + 11x^2y^2]$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= xy[(2x^2 + 3y^2)^2 - 12x^2y^2 + 11x^2y^2]$

$= xy[(2x^2 + 3y^2)^2 - x^2y^2]$

$= xy[(2x^2 + 3y^2)^2 - (xy)^2]$

As $a^2 - b^2 = (a+b)(a-b)$

$= xy(2x^2 + 3y^2 + xy)(2x^2 + 3y^2 - xy)$

$= xy(2x^2 + xy + 3y^2)(2x^2 - xy + 3y^2)$

Exercise# 5.3

Q1 $\mathbf{x^2 - 7x + 12}$

Solution

$x^2 - 7x + 12$

$= x^2 - 3x - 4x + 12$

$= x(x-3) - 4(x-3)$

$= (x-3)(x-4)$

$(x^2)(12) = 12x^2$	
Add	**Multiply**
$-3x$	$-3x$
$-4x$	$-4x$
$-7x$	$12x^2$

Q2 $\mathbf{x^2 + x - 12}$

Solution

$x^2 + x - 12$

$= x^2 - 3x + 4x - 12$

$= x(x-3) + 4(x-3)$

$= (x-3)(x+4)$

$(x^2)(-12) = -12x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+4x$	$+4x$
x	$-12x^2$

Q3 $\mathbf{20 - x - x^2}$

Solution

$20 - x - x^2$

$= 20 + 4x - 5x - x^2$

$= 4(5+x) - x(5+x)$

$= (5+x)(4-x)$

$(20)(-x^2) = -20x^2$	
Add	**Multiply**
$+4x$	$+4x$
$-5x$	$-5x$
$-x$	$-20x^2$

Q4 $\mathbf{2y^2 - 7y + 3}$

Solution

$= 2y^2 - 1y - 6y + 3$

$= y(2y-1) - 3(2y-1)$

$= (2y-1)(y-3)$

$(2y^2)(3) = 6y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-6y$	$-6y$
$-7y$	$6y^2$

Q5 $\mathbf{4x^2 + 8x + 3}$

Solution

$4x^2 + 8x + 3$

$= 4x^2 + 2x + 6x + 3$

$= 2x(2x+1) + 3(2x+1)$

$= (2x+1)(2x+3)$

$(4x^2)(3) = 12x^2$	
Add	**Multiply**
$+2x$	$+2x$
$+6x$	$+6x$
$8x$	$12x^2$

Q6 $\mathbf{10y^2 - 3y - 1}$

Solution

$10y^2 - 3y - 1$

$= 10y^2 + 2y - 5y - 1$

$= 2y(5y+1) - 1(5y+1)$

$= (5y+1)(2y-1)$

$(10y^2)(-1) = -10y^2$	
Add	**Multiply**
$+2y$	$+2y$
$-5y$	$-5y$
$-3y$	$-10y^2$

Q7 $6x^3 - 15x^2 - 9x$

Solution

$= 3x(2x^2 - 5x - 3)$

$= 3x(2x^2 + 1x - 6x - 3)$

$= 3x[x(2x+1) - 3(2x+1)]$

$= 3x(2x+1)(x-3)$

$(2x^2)(-3) = -6x^2$	
Add	**Multiply**
$+1x$ $-6x$	$+1x$ $-6x$
$-5x$	$-6x^2$

Q8 $2xy^2 + 8xy - 24x$

Solution

$= 2x(y^2 + 4y - 12)$

$= 2x(y^2 - 2y + 6y - 12)$

$= 2x[y(y-2) + 6(y-2)]$

$= 2x(y-2)(y+6)$

$(y^2)(-12) = -12y^2$	
Add	**Multiply**
$-2y$	$-2y$
$+6y$	$+6y$
$+4y$	$-12y^2$

Q10 $-16x^3y - 20x^2y^2 - 6xy^3$

Solution

$-16x^3y - 20x^2y^2 - 6xy^3$

$= -2xy(8x^2 + 10xy + 3y^2)$

$-2xy(8x^2 + 4xy + 6xy + 3y^2)$

$-2xy[4x(2x+y) + 3y(2x+y)]$

$-2xy(2x+y)(4x+3y)$

$(8x^2)(3y^2) = 24x^2y^2$	
Add	**Multiply**
$+4xy$	$+4xy$
$+6xy$	$+6xy$
$+10xy$	$24x^2y^2$

Q11 $\mathbf{(x+1)^2 + 3(x+1) + 2}$

Solution

$(x+1)^2 + 3(x+1) + 2$

$= x^2 + (1)^2 + 2(x)(1) + 3x + 3 + 2$

$= x^2 + 1 + 2x + 3x + 5$

$= x^2 + 5x + 6$

$= x^2 + 2x + 3x + 6$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)$

$(x^2)(6) = 6x^2$	
Add	**Multiply**
$+2x$	$+2x$
$+3x$	$+3x$
$5x$	$6x^2$

Q12 $\mathbf{4x^8y^{10} - 40x^5y^7 + 84x^2y^4}$

Solution

$4x^8y^{10} - 40x^5y^7 + 84x^2y^4$

$= 4x^2y^4(x^6y^6 - 10x^3y^3 + 21)$

$= 4x^2y^4(x^6y^6 - 3x^3y^3 - 7x^3y^3 + 21)$

$= 4x^2y^4[x^3y^3(x^3y^3 - 3) - 7(x^3y^3 - 3)]$

$= 4x^2y^4(x^3y^3 - 3)(x^3y^3 - 7)$

$(x^6y^6)(21) = 21x^6y^6$	
Add	Multiply
$-3x^3y^3$	$-3x^3y^3$
$-7x^3y^3$	$-7x^3y^3$
$-10x^3y^3$	$21x^6y^6$

Q13 **Find an expression for the perimeter of a rectangle with area given by $\mathbf{x^2 + 24x - 81}$**

Given

$Area\ of\ rectangle = x^2 + 24x - 81$

To find

Perimeter of rectangle = ?

As $Area = l \times w$

And $Perimeter = 2l + 2w$

Now

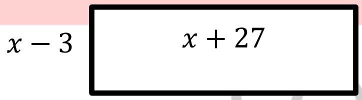
$x^2 + 24x - 81$

$= x^2 - 3x + 27x - 81$

$= x(x - 3) + 27(x - 3)$

$= (x - 3)(x + 27)$

$(x^2)(-81) = -81x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+27x$	$+27x$
$24x$	$-81x^2$



Now $l = (x + 27) and\ w = (x - 3)$

As

$Perimeter = 2l + 2w$

$Perimeter = 2(x + 27) + 2(x - 3)$

$Perimeter = 2x + 54 + 2x - 6$

$Perimeter = 4x + 48$

Q9 $\mathbf{2 + 5t - 12t^2}$

Solution

$2 + 5t - 12t^2$

$-12t^2 + 5t + 2$

$-(12t^2 - 5t - 2)$

$-(12t^2 + 3t - 8t - 2)$

$-[3t(4t + 1) - 2(4t + 1)]$

$-(4t + 1)(3t - 2)$

$(12t^2)(-2) = -24t^2$	
Add	**Multiply**
$+3t$	$+3t$
$-8t$	$-8t$
$-5t$	$-24t^2$

Exercise# 5.4

Q1 $\mathbf{(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16}$

Solution

$(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16$

Let $4x^2 - 16x = y$

$= (y + 7)(y + 15) + 16$

$= y^2 + 15y + 7y + 105 + 16$

$= y^2 + 22y + 121$

$= y^2 + 11y + 11y + 121$

$= y(y + 11) + 11(y + 11)$

$= (y + 11)(y + 11)$

But $y = 4x^2 - 16x$

$= (4x^2 - 16x + 11)\ (4x^2 - 16x + 11)$

$= (4x^2 - 16x + 11)^2$

Q2 $\mathbf{(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72}$

Solution

$(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72$

Let $9x^2 + 9x = y$

$= (y - 4)(y - 10) - 72$

$= y^2 - 10y - 4y - 40 - 72$

$= y^2 - 14y - 32$

$= y^2 + 2y - 16y - 32$

$= y(y + 2) - 16(y + 2)$

$= (y + 2)(y - 16)$

But $y = 9x^2 + 9x$

So

$= (9x^2 + 9x + 2)(9x^2 + 9x - 16)$

Q3 $\mathbf{(x + 2)(x + 4)(x + 6)(x + 8) - 9}$

Solution

$(x + 2)(x + 4)(x + 6)(x + 8) - 9$

Rearranging accordingly 4+6=2+8

$= (x + 2)(x + 8)(x + 4)(x + 6) - 9$

$= (x^2 + 8x + 2x + 16)(x^2 + 6x + 4x + 24) - 9$

$= (x^2 + 10x + 16)(x^2 + 10x + 24) - 9$

Let $x^2 + 10x = y$

$= (y + 16)(y + 24) - 9$

$= y^2 + 24y + 16y + 384 - 9$

$= y^2 + 40y + 375$

$= y^2 + 15y + 25y + 375$

$= y(y + 15) + 25(y + 15)$

$= (y + 15)(y + 25)$

But $y = x^2 + 10x$

So

$= (x^2 + 10x + 15)(x^2 + 10x + 25)$

Q4 $\boldsymbol{x(x+1)(x+2)(x+3)+1}$

Solution

$x(x+1)(x+2)(x+3)+1$

Rearranging accordingly 0+3=1+2

$= x(x+3)(x+1)(x+2)+1$

$= (x^2+3x)(x^2+2x+1x+2)+1$

$= (x^2+3x)(x^2+3x+2)+1$

Let $\boldsymbol{x^2+3x=y}$

$= (y)(y+2)+1$

$= y^2+2y+1$

$= (y)^2+(1)^2+2(y)(1)$

$= (y+1)^2$

But $\boldsymbol{y=x^2+3x}$

So

$= (x^2+3x+1)^2$

Q5 $\boldsymbol{(x+1)(x+2)(x+3)(x+6)-3x^2}$

Solution

$(x+1)(x+2)(x+3)(x+6)-3x^2$

Rearranging accordingly $1\times 6 = 2\times 3$

$= (x+1)(x+6)(x+2)(x+3)-3x^2$

$= (x^2+6x+1x+6)(x^2+3x+2x+6)-3x^2$

$= (x^2+7x+6)(x^2+5x+6)-3x^2$

$= (x^2+6+7x)(x^2+6+5x)-3x^2$

Let $\boldsymbol{x^2+6=y}$

$= (y+7x)(y+5x)-3x^2$

$= y^2+5xy+7xy+35x^2-3x^2$

$= y^2+12xy+32x^2$

$= y^2+4xy+8xy+32x^2$

$= y(y+4x)+8x(y+4x)$

$= (y+4x)(y+8x)$

But $\boldsymbol{y=x^2+6}$

$= (x^2+6+4x)(x^2+6+8x)$

$$= \left(\frac{x(x^2+6+4x)}{x}\right)\left(\frac{x(x^2+6+8x)}{x}\right)$$

$$= x.x\left(\frac{x^2}{x}+\frac{6}{x}+\frac{4x}{x}\right)\left(\frac{x^2}{x}+\frac{6}{x}+\frac{8x}{x}\right)$$

$$= x^2\left(x+\frac{6}{x}+4\right)\left(x+\frac{6}{x}+8\right)$$

Q6 $\boldsymbol{64x^3-144x^2y+108xy^2-27y^3}$

Solution

$64x^3-144x^2y+108xy^2-27y^3$

$= (4x)^3-3(4x)^2(3y)+3(4x)(3y)^2-(3y)^3$

As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$= (4x-3y)^3$

Q7 $\boldsymbol{\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}}$

Solution

$$\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}$$

$$= \left(\frac{a}{2}\right)^3-3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)+3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2-\left(\frac{b}{3}\right)^3$$

As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$$= \left(\frac{a}{2}-\frac{b}{3}\right)^3$$

Q9 $\boldsymbol{\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}}$

Solution

$$\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}$$

$$= \left(\frac{x}{a}\right)^3+3\left(\frac{x}{a}\right)^2\left(\frac{a}{x}\right)+3\left(\frac{x}{a}\right)\left(\frac{a}{x}\right)^2+\left(\frac{a}{x}\right)^3$$

As $a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$$= \left(\frac{x}{a}+\frac{a}{x}\right)^3$$

Q10 $\boldsymbol{27a^3+189a^2b+441ab^2+343b^3}$

Solution

$27a^3+189a^2b+441ab^2+343b^3$

$= (3a)^3+3(3a)^2(7b)+3(3a)(7b)^2+(7b)^3$

As $a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$= (3a+7b)^3$

Q11 $8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$

Solution

$$8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$$

$$= (2x)^3-3(2x)^2\left(\frac{1}{3x}\right)+3(2x)\left(\frac{1}{3x}\right)^2-\left(\frac{1}{3x}\right)^3$$

As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$$= \left(2x-\frac{1}{3x}\right)^3$$

Exercise# 5.5

Q1 $\boldsymbol{a^3 - 27}$

Solution

$a^3 - 27$

$= (a)^3 - (3)^3$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (a - 3)[(a)^2 + (a)(3) + (3)^2]$

$= (a - 3)(a^2 + 3a + 9)$

Q2 $\boldsymbol{a^6 + b^6}$

Solution

$a^6 + b^6$

$= (a^2)^3 + (b^2)^3$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a^2 + b^2)[(a^2)^2 - (a^2)(b^2) + (b^2)^2]$

$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

Q3 $\boldsymbol{24x^3 + 3}$

Solution

$24x^3 + 3$

$= 3(8x^3 + 1)$

$= 3[(2x)^3 + (1)^3]$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= 3\{(2x + 1)[(2x)^2 - (2x)(1) + (1)^2]\}$

$= 3(2x + 1)(4x^2 + 2x + 1)$

Q4 $\boldsymbol{1 - 27r^3}$

Solution

$1 - 27r^3$

$= (1)^3 - (3r)^3$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (1 - 3r)[(1)^2 + (1)(3r) + (3r)^2]$

$= (1 - 3r)(1 + 3r + 9r^2)$

Q5 $\boldsymbol{2x^3 - 128}$

Solution

$2x^3 - 128$

$2(x^3 - 64)$

$2[(x)^3 - (4)^3]$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$2\{(x - 4)[(x)^2 + (x)(4) + (4)^2]\}$

$2(x - 4)(x^2 + 4x + 16)$

Q6 $\boldsymbol{4x^5 - 256x^2}$

Solution

$4x^5 - 256x^2$

$= 4x^2(x^3 - 64)$

$= 4x^2[(x)^3 - (4)^3]$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= 4x^2\{(x - 4)[(x)^2 + (x)(4) + (4)^2]\}$

$= 4x^2(x - 4)(x^2 + 4x + 16)$

Q7 $\boldsymbol{18(x - y)^3 - 144(a - b)^3}$

Solution

$18(x - y)^3 - 144(a - b)^3$

$= 18[(x - y)^3 - 8(a - b)^3]$

$= 18[(x - y)^3 - (2(a - b))^3]$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$\boldsymbol{= 18[(x - y) - 2(a - b)]\left[(x - y)^2 + (x - y)(2(a - b)) + (2(a - b))^2\right]}$

$\boldsymbol{= 18(x - y - 2a + 2b)[(x - y)^2 + 2(x - y)(a - b) + 4(a - b)^2]}$

Q8 $\boldsymbol{x^9 + 1}$

Solution

$x^9 + 1$

$= (x^3)^3 + (1)^3$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (x^3 + 1)[(x^3)^2 - (x^3)(1) + (1)^2]$

$= (x + 1)[(x)^2 - (x)(1) + (1)^2](x^6 - x^3 + 1)$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (2x + 1)(x^2 - x + 1)(x^6 - x^3 + 1)$

Q9 $\boldsymbol{a^3 + (c + d)^3}$

Solution

$a^3 + (c + d)^3$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= [a + (c + d)][(a)^2 - (a)(c + d) + (c + d)^2]$

$= (a + c + d)[a^2 - a(c + d) + (c + d)^2]$

Q10 $\boldsymbol{27x^3 - y^3}$

Solution

$27x^3 - y^3$

$= (3x)^3 - (y)^3$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (3x - y)[(3x)^2 + (3x)(y) + (y)^2]$

$= (3x - y)(9x^2 + 3xy + y^2)$

Exercise# 5.6

Q1 **By using remainder theorem, find the remainder of the following polynomials when:**

(i) **$x^3+6x^2+8x-11$ is divided by $x-1$**

Solution

Here $P(x)=x^3+6x^2+8x-11$

$$x-r=x-1$$
$$-r=-1$$
$$r=1$$
$$R=P(r)=P(1)$$
$$=(1)^3+6(1)^2+8(1)-11$$
$$=1+6+8-11$$
$$=15-11$$
$$=4$$

So Remainder $=4$

(ii) **$2x^3+4x^2+7x-5$ is divided by $x+3$**

Solution

Here $P(x)=2x^3+4x^2+7x-5$

$$x-r=x+3$$
$$-r=3$$
$$r=-3$$
$$R=P(r)=P(-3)$$
$$=2(-3)^3+4(-3)^2+7(-3)-5$$
$$=2(-27)+4(9)-21-5$$
$$=-54+36-26$$
$$=-18-26$$
$$=-44$$

So Remainder $=-44$

(iii) **$3x^3+x-200$ is divided by $x-4$**

Solution

Here $P(x)=3x^3+x-200$

$$x-r=x-4$$
$$-r=-4$$
$$r=4$$
$$R=P(r)=P(4)$$
$$=3(4)^3+(4)-200$$

$=3(64)+4-200$

$=192-196$

$=-4$

So Remainder $=-4$

Q2 **Without performing division find the value of a, when $2x^3-ax^2-2ax+3x+2$ is exactly divisible by $x+1$**

Solution

Here $P(x)=2x^3-ax^2-2ax+3x+2$

$$x-r=x+1$$
$$-r=1$$
$$r=-1$$
$$R=P(r)=P(-1)=P(0)$$
$$2(-1)^3-a(-1)^2-2a(-1)+3(-1)+2=0$$
$$2(-1)-a(1)+2a-3+2=0$$
$$-2-a+2a-1=0$$
$$a-2-1=0$$
$$a-3=0$$
$$a=3$$

Q3 **Without performing division find the value of b, when x^3-4x^2+bx-2 is exactly divisible by $x-1$**

Solution

Here $P(x)=x^3-4x^2+bx-2$

$$x-r=x-1$$
$$-r=-1$$
$$r=1$$
$$R=P(r)=P(1)=P(0)$$
$$(1)^3-4(1)^2+b(1)-2=0$$
$$1-4(1)+b-2=0$$
$$1-4+b-2=0$$
$$b-3-2=0$$
$$b-5=0$$
$$b=5$$

Q4 **Using factor theorem, factorize the following.**

(i) x^3-2x^2-5x+6

Solution

$$P(x)=x^3-2x^2-5x+6$$
$$Let\ x=1$$
$$So\ P(1)=(1)^3-2(1)^2-5(1)+6$$
$$=1-2(1)-5+6$$
$$=1-2+1$$
$$=-1+1$$
$$=0$$

Since $P(x)=0$, So $x-1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x-1$

$$\begin{array}{r|l} & x^2-x-6 \\ \cline{2-2} x-1 & x^3-2x^2-5x+6 \\ & \pm x^3 \mp x^2 \\ \cline{2-2} & -x^2-5x+6 \\ & \mp x^2 \pm x \\ \cline{2-2} & -6x+6 \\ & \mp 6x \pm 6 \\ \cline{2-2} & \text{x} \end{array}$$

Here $Q(x) = (x^2 - x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 2x^2 - 5x + 6 = (x - 1)(x^2 - x - 6)$

$= (x - 1)(x^2 + 2x - 3x - 6)$

$= (x - 1)[x(x + 2) - 3(x + 2)]$

$= (x - 1)(x + 2)(x - 3)$

(ii) $\boldsymbol{x^3 + x^2 - 4x - 4}$

Solution

$P(x) = x^3 + x^2 - 4x - 4$

$Let\ x = -1$

$So\ P(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$

$= -1 + 1 + 4 - 4$

$= 0$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$\begin{array}{r|l} & x^2-4 \\ \cline{2-2} x+1 & x^3+x^2-4x-4 \\ & \pm x^3 \pm x^2 \\ \cline{2-2} & -4x-4 \\ & \mp 4x \mp 4 \\ \cline{2-2} & \text{X} \end{array}$$

Here $Q(x) = (x^2 - 4)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$

$= (x + 1)[(x)^2 - (2)^2]$

$= (x + 1)(x + 2)(x - 2)$

$\boldsymbol{x^3 - 7x + 6}$

Solution

$P(x) = x^3 - 7x + 6$

$Let\ x = 1$

$So\ P(1) = (1)^3 - 7(1) + 6$

$= 1 - 7 + 6$

$= -6 + 6$

$= 0$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

$$\begin{array}{r|l} & x^2+x-6 \\ \cline{2-2} x-1 & x^3-7x+6 \\ & \pm x^3 \qquad \mp x^2 \\ \cline{2-2} & x^2-7x+6 \\ & \pm x^2 \mp x \\ \cline{2-2} & -6x+6 \\ & \mp 6x \pm 6 \\ \cline{2-2} & \text{x} \end{array}$$

Here $Q(x) = (x^2 + x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 7x + 6 = (x - 1)(x^2 + x - 6)$

$= (x - 1)(x^2 - 2x + 3x - 6)$

$= (x - 1)[x(x - 2) + 3(x - 2)]$

$= (x - 1)(x - 2)(x + 3)$

(iv) $\boldsymbol{x^3 - 9x^2 + 23x - 15}$

Solution

$P(x) = x^3 - 9x^2 + 23x - 15$

$Let\ x = 1$

$So\ P(1) = (1)^3 - 9(1)^2 + 23(1) - 15$

$= 1 - 9(1) + 23 - 15$

$= 1 - 9 + 8$

$= -8 + 8$

$= 0$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

$$\begin{array}{r|l} & x^2-8x+15 \\ \hline x-1 & x^3-9x^2+23x-15 \\ & \pm x^3 \mp x^2 \\ \hline & -8x^2+23x \\ & \mp 8x^2 \pm 8x \\ \hline & 15x-15 \\ & \pm 15x \mp 15 \\ \hline & \text{x} \end{array}$$

Here $Q(x) = (x^2 - 8x + 15)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 7x + 6 = (x - 1)(x^2 - 8x + 15)$

$= (x - 1)(x^2 - 3x - 5x + 15)$

$= (x - 1)[x(x - 3) - 5(x - 3)]$

$= (x - 1)(x - 3)(x - 5)$

(v) $\boldsymbol{x^3 - 4x^2 - 3x + 18}$

Solution

$P(x) = x^3 - 4x^2 - 3x + 18$

$Let\ x = -2$

$So\ P(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18$

$= -8 - 4(4) + 6 + 18$

$= -8 - 16 + 24$

$= -24 + 24$

$= 0$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$\begin{array}{r|l} & x^2-6x+9 \\ \hline x+2 & x^3-4x^2-3x+18 \\ & \pm x^3 \pm 2x^2 \\ \hline & -6x^2 - 3x \\ & \mp 6x^2 \mp 12x \\ \hline & 9x+18 \\ & \pm 9x \pm 18 \\ \hline & \text{x} \end{array}$$

Here $Q(x) = (x^2 - 6x + 9)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 4x^2 - 3x + 18 = (x + 2)(x^2 - 6x + 9)$

$= (x + 2)[(x)^2 - 2(x)(3) + (3)^2]$

$= (x + 2)(x - 3)^2$

(vi) $\boldsymbol{x^3 + 2x^2 - 19x - 20}$

Solution

$P(x) = x^3 + 2x^2 - 19x - 20$

$Let\ x = -1$

$So\ P(-1) = (-1)^3 + 2(-1)^2 - 19(-1) - 20$

$= -1 + 2(1) + 19 - 20$

$= -1 + 2 - 1$

$= 1 - 1$

$= 0$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$\begin{array}{r|l} & x^2+x-20 \\ \hline x+1 & x^3+2x^2-19x-20 \\ & \pm x^3 \pm x^2 \\ \hline & x^2 - 19x \\ & \pm x^2 \pm x \\ \hline & -20x-20 \\ & \mp 20x \mp 20 \\ \hline & \text{x} \end{array}$$

Here $Q(x) = (x^2 + x - 20)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 + 2x^2 - 19x - 20 = (x + 1)(x^2 + x - 20)$

$= (x + 1)(x^2 - 4x + 5x - 20)$

$= (x + 1)[x(x - 4) + 5(x - 4)]$

$= (x + 1)(x - 4)(x + 5)$

(vii) $\boldsymbol{x^3 - x^2 - 14x + 24}$

Solution

$P(x) = x^3 - x^2 - 14x + 24$

$Let\ x = 2$

$So\ P(-2) = (2)^3 - (2)^2 - 14(2) + 24$

$= 8 - 4 - 28 + 24$

$= 4 - 4$

$= 0$

Since $P(x) = 0$, So $x - 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 2$

$$\begin{array}{r|l} & x^2 + x - 12 \\ \hline x - 2 & x^3 - x^2 - 14x + 24 \\ & \pm x^3 \mp 2x^2 \\ \hline & x^2 - 14x \\ & \pm x^2 \mp 2x \\ \hline & -12x + 24 \\ & \mp 12x \pm 24 \\ \hline & \text{x} \end{array}$$

Here $Q(x) = (x^2 + x - 12)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - x^2 - 14x + 24 = (x - 2)(x^2 + x - 12)$

$= (x - 2)(x^2 + 4x - 3x - 12)$

$= (x - 2)[x(x + 4) - 3(x + 4)]$

$= (x - 2)(x + 4)(x - 3)$

(viii) $\boldsymbol{x^3 - 6x^2 + 32}$

Solution

$P(x) = x^3 - 6x^2 + 32$

$Let\ x = -2$

$So\ P(-2) = (-2)^3 - 6(-2)^2 + 32$

$= -8 - 6(4) + 32$

$= -8 - 24 + 32$

$= -32 + 32$

$= 0$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$\begin{array}{r|l} & x^2 - 8x + 16 \\ \hline x + 2 & x^3 - 6x^2 + 32 \\ & \pm x^3 \pm 2x^2 \\ \hline & -8x^2 + 32 \\ & \mp 8x^2 \quad \mp 16x \\ \hline & 16x + 32 \\ & \pm 16x \pm 32 \\ \hline & \text{x} \end{array}$$

Here $Q(x) = (x^2 - 8x + 16)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 6x^2 + 32 = (x + 2)(x^2 - 8x + 16)$

$= (x + 2)[(x)^2 - 2(x)(4) + (4)^2]$

$= (x + 2)(x - 4)^2$

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	Example # 12 Page + 133
	Example # 17 Page # 136
	Example # 22, 23, 24, 25 Page # 140, 141

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Class 9th (KPK)

Chapter # 6 Algebraic Manipulations

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Chapter # 6

UNIT # 6

ALGEBRAIC MANIPULATIONS

Ex # 6.1

Highest Common Factor (H.C.F)

The highest number of factors common to all given expressions or polynomials is called Highest Common Factor (H.C.F)

In other words, H.C.F of two or more polynomials is a polynomial of the highest degree, which divides exactly the given polynomials.

There are two methods for finding H.C.F.

(i) H.C.F by Factorization

(ii) H.C.F by Division

H.C.F by Factorization

In this method, first factorize all the given expressions

Then we take all possible common factors which is the H.C.F of the given expression.

Example # 1

Find H.C.F of $x^2 - y^2,\ x^2 - xy$

Solution:

$x^2 - y^2,\ x^2 - xy$

$x^2 - y^2 = (x+y)(x-y)$

And

$x^2 - xy = x(x-y)$

Here $x - y$ is a common factor. Thus

$\mathrm{H.C.F} = \mathrm{x - y}$

Example # 2

Find H.C.F of $ax^2 + 5ax + 6a,$
$ax^3 + 9ax^2 + 14ax\ \ and\ \ 15a(x^2 - 4)$

Solution:

$ax^2 + 5ax + 6a,\ \ ax^3 + 9ax^2 + 14ax\ and$
$15a(x^2 - 4)$

$ax^2 + 5ax + 6a = a(x^2 + 5x + 6)$

$ax^2 + 5ax + 6a = a(x^2 + 2x + 3x + 6)$

$ax^2 + 5ax + 6a = a[x(x+2) + 3(x+2)]$

$ax^2 + 5ax + 6a = a(x+2)(x+3)$

And

$ax^3 + 9ax^2 + 14ax = ax(x^2 + 9x + 14)$

$ax^3 + 9ax^2 + 14ax = ax(x^2 + 2x + 7x + 14)$

$ax^3 + 9ax^2 + 14ax = ax[x(x+2) + 7(x+2)]$

$ax^3 + 9ax^2 + 14ax = ax(x+2)(x+7)$

Ex # 6.1

Now also

$15a(x^2 - 4) = 3 \times 5.a[(x)^2 - (2)^2]$

$15a(x^2 - 4) = 3 \times 5.a(x+2)(x-2)$

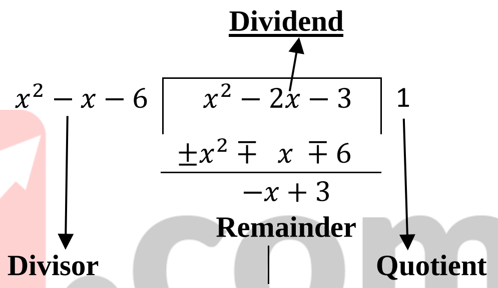
Here $a(x+2)$ is common in given three expressions.

$\mathrm{H.C.F} = \mathrm{a}(x+2)$

Note:

The H.C.F $\mathrm{a}(x+2)$ exactly divides all the given three expression

H.C.F by Division Method



Steps

1 Write the expressions in descending order

2 Take the common from the expressions if any.

3 Divide higher degree polynomial by the polynomial of lower degree

4 Divide to that time till the degree of remainder is less than the degree of divisor.

5 Now bring down the divisor and divide by remainder BUT before this take the common from the remainder if any.

6 Repeat the above steps till the remainder is zero.

7 Last divisor is the H.C.F of the given polynomials.

Note:

1 In H.C.F by division, if required, multiply the expression by a suitable integer to avoid fraction.

2 To find the H.C.F of three polynomials, first find H.C.F of any two of them, then find H.C.F of this H.C.F and the third polynomial.

$(x^2)(6) = 6x^2$

Add	Multiply
$+2x$	$+2x$
$+3x$	$+3x$
$\boldsymbol{+5x}$	$\boldsymbol{6x^2}$

$(x^2)(14) = 14x^2$

Add	Multiply
$+2x$	$+2x$
$+7x$	$+7x$
$\boldsymbol{+9x}$	$\boldsymbol{14x^2}$

Chapter # 6

Ex # 6.1

H.C.F by Division method in Urdu

1. تمام variables کو descending order میں لکھیں گے۔
2. اگر کوئی common ہو تو پہلے common لینگے۔
3. بڑے expression کو چھوٹے expression پر divide کریں گے۔
4. اس کو اس وقت تک divide کرتے رہیں گے جب تک remainder میں power ہمارے ساتھ divisor کے power سے کم نہ آئے
5. پھر divisor کو نیچے لائیں گے اور remainder پر divide کریں گے لیکن اس سے پہلے remainder میں common لیں گے اگر ہو۔
6. ان steps کو اس وقت تک کرو گے جب تک remainder میں zero نہ آئے۔
6. آخری divisor ہمارے ساتھ H.C.F ہوگا۔

Example # 3

Find H.C.F of $2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

Solution:

$2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

$$2x^3+7x^2+4x-4 \;\Big|\; 2x^3+9x^2+11x+2 \;\Big|\; 1$$

$$\pm 2x^3 \pm 7x^2 \pm 4x \mp 4$$

$$2x^2+7x+6 \;\Big|\; 2x^3+7x^2+4x-4 \;\Big|\; x$$

$$\pm 2x^3 \pm 7x^2 \pm 6x$$

$$-2 \;\Big|\; -2x-4 \qquad \textbf{Dividing by } -\mathbf{2}$$

$$x+2 \;\Big|\; 2x^2+7x+6 \;\Big|\; 2x+3$$

$$\pm 2x^2 \pm 4x$$

$$3x+6$$

$$\pm 3x \pm 6$$

$$\times$$

Hence H.C.F= $x+2$

Note:

H.C.F by Factorization

H.C.F of 24 and 32

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 24 = 1, 2, 4, 8, 16, 32

$\text{Common factors } = 1, 2, 4, 8$

$\text{H.C.F} = 8$

Chapter # 6

Ex # 6.1

Example # 4

Find H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

Solution:

$x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

$$3x^3-5x^2+6x-4\;\big)\;3x^3+5x^2-6x-2\;\big(\;1$$
$$\pm3x^3\mp5x^2\pm6x\mp4$$
$$2\;\big)\;10x^2-12x+2 \qquad \textbf{Dividing by 2}$$
$$5x^2-6x+1\;\big)\;3x^3-5x^2+6x-4\;\big(\;3x-7$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$15x^3-25x^2+30x-20$$
$$\pm15x^3\mp18x^2\pm3x$$
$$-7x^2+27x-20$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$-35x^2+135x-100$$
$$\mp35x^2\pm42x\mp7$$
$$93\;\big)\;93x-93 \qquad \textbf{Dividing by 93}$$
$$x-1\;\big)\;5x^2-6x+1\;\big(\;5x-1$$
$$\pm5x^2\mp5x$$
$$-x+1$$
$$\mp x\pm1$$
$$\times$$

Hence H.C.F= $x-1$

Now find the H.C.F of $x-1$ and $x^3-6x^2+11x-6$

$$x-1\;\big)\;x^3-6x^2+11x-6\;\big(\;x^2-5x+6$$
$$\pm x^3\mp x^2$$
$$-5x^2+11x-6$$
$$\mp5x^2\pm5x$$
$$6x-6$$
$$\pm6x\mp6$$
$$\times$$

Hence the required H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$ is $\boldsymbol{x-1}$

Least Common Multiple (L.C.M)

The polynomial of least degree which is divisible by the given polynomials.

There are two methods of finding L.C.M

(a) L.C.M by factorization

(b) L.C.M by formula

Chapter # 6

Ex # 6.1

(a) **L.C.M by factorization**

In this method, first factorize all the given expressions

Then find the L.C.M by given formula.

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

Example # 5

Find L.C.M of $\boldsymbol{x^2+4x+4}$ and $\boldsymbol{x^2+5x+6}$

Solution:

x^2+4x+4 and x^2+5x+6

$x^2+4x+4 = (x)^2+2(x)(2)+(2)^2$

$x^2+4x+4 = (x+2)^2$

$x^2+4x+4 = (x+2)(x+2)$

Now

$x^2+5x+6 = x^2+2x+3x+6$

$x^2+5x+6 = x(x+2)+3(x+2)$

$x^2+5x+6 = (x+2)(x+3)$

$Common\ Factor = x+2$

$Non - common\ factor = (x+2)(x+3)$

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x+2)(x+2)(x+3)$

$L.C.M = (x+2)^2(x+3)$

Example # 6

Find L.C.M of $\boldsymbol{x^2-4x+3}$, $\boldsymbol{x^2-3x+2}$ and $\boldsymbol{x^2-5x+6}$

Solution:

x^2-4x+3, x^2-3x+2 and x^2-5x+6

$x^2-4x+3 = x^2-x-3x+3$

$x^2-4x+3 = x(x-1)-3(x-1)$

$x^2-4x+3 = (x-1)(x-3)$….. (i)

Now

$x^2-3x+2 = x^2-x-2x+3$

$x^2-3x+2 = x(x-1)-2(x-1)$

$x^2-3x+2 = (x-1)(x-2)$….. (ii)

Now

$x^2-5x+6 = x^2-2x-3x+6$

$x^2-5x+6 = x(x-2)-3(x+2)$

$x^2-5x+6 = (x-2)(x-3)$….. (iii)

$x-1$ in expression (i)& (ii)

$x-2$ in expression (ii)& (iii)

$x-3$ in expression (i)& (iii)

Therefore:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x-1)(x-2)(x-3)\times 1$

$L.C.M = (x-1)(x-2)(x-3)$

Ex # 6.1

L.C.M Theorem:

If A and B are given polynomials and their H.C.F and L.C.M are represented by H and L respectively, then

$$A \times B = H \times L$$

Proof:

Since H is common factor of polynomial of A and B, then it divides exactly A and B. So

$\frac{A}{H} = a$

$A = Ha$… equ(i)

and

$\frac{B}{H} = b$

$B = Hb$… equ(ii)

As a and b have no common factor.

As we know that:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L = H \times a \times b$

Multiply B.S by H

$L \times H = H \times a \times b \times H$

$L \times H = (Ha)\times(Hb)$

Put equ(i) and equ(ii), we get

$L \times H = A \times B$

Or

$H \times L = Product\ of\ two\ polynomials$

Formula for L.C.M

As $L \times H = A \times B$

$L = \frac{A\times B}{H}$

$L.C.M = \frac{Product\ of\ two\ polynomials}{H.C.F}$

Chapter # 6

Ex # 6.1

Example # 7

Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3

$x^3-6x^2+11x-6$ and x^3-4x+3

Solution:

$Let\ A=x^3-6x^2+11x-6$

$and\ B=x^3-4x+3$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$x^3-4x+3 \,\big)\, x^3-6x^2+11x-6 \,\big(\, 1$$
$$\pm x^3 \quad \mp 4x \pm 3$$
$$-3 \,\big)\, -6x^2+15x-9$$
$$2x^2-5x+3 \,\big)\, x^3-4x+3 \,\big(\, x+5$$
$$\times 2$$
$$2x^3-8x+6$$
$$\pm 2x^3 \pm 3x \quad \mp 5x^2$$
$$5x^2-11x+6$$
$$\times 2$$
$$10x^2-22x+12$$
$$\pm 10x^2 \mp 25x \pm 15$$
$$3 \,\big)\, 3x-3$$
$$x-1 \,\big)\, 2x^2-5x+3 \,\big(\, 2x-3$$
$$\pm 2x^2 \mp 2x$$
$$-3x+3$$
$$\mp 3x \pm 3$$
$$\times$$

$H.C.F=x-1$

Now put the values in equ (i)

$$L.C.M=\frac{(x^3-6x^2+11x-6\,)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$x^2-5x+6$$
$$x-1 \,\big)\, x^3-6x^2+11x-6$$
$$\pm x^3 \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

$So\ L.C.M=(x^2-5x+6)(x^3-4x+3)$

Chapter # 6

Ex # 6.1

Example # 8

Find H.C.F and L.C.M of $3x^3-2x^2-3x+2$ and $6x^3-7x^2-x+2$

$3x^3-2x^2-3x+2$ and $6x^3-7x^2-x+2$

Solution:

$Let\ A=3x^3-2x^2-3x+2$

$and\ B=6x^3-7x^2-x+2$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
3x^3-2x^2-3x+2 & 6x^3-7x^2-x+2 & 2 \\
 & \pm 6x^3\mp 4x^2\mp 6x\pm 4 & \\
\hline
-1 & -3x^2+5x-2 & \\
\hline
3x^2-5x+2 & 3x^3-2x^2-3x+2 & x+1 \\
 & \pm 3x^3\mp 5x^2\pm 2x & \\
\hline
 & 3x^2-5x+2 & \\
 & \pm 3x^2\mp 5x\pm 2 & \\
\hline
 & \times & \\
\end{array}$$

$$H.C.F=3x^2-5x+2$$

Now put the values in equ (i)

$$L.C.M=\frac{(3x^3-2x^2-3x+2)(6x^3-7x^2-x+2)}{3x^2-5x+2}$$

Now by Simple Division

$$\begin{array}{r|l}
 & x+1 \\
\hline
3x^2-5x+2 & 3x^3-2x^2-3x+2 \\
 & \pm 3x^3\mp 5x^2\pm 2x \\
\hline
 & 3x^2-5x+2 \\
 & \pm 3x^2\mp 5x\pm 2 \\
\hline
 & \times \\
\end{array}$$

$So\ L.C.M=(x+1)(6x^3-7x^2-x+2)$

Example # 9

If H.C.F and L.C.M of two polynomials are $x-3$ and $x^3-9x^2+26x-24$ respectively. Find the second polynomial when one polynomial is x^2-5x+6.

Solution:

$H.C.F=x-3$

$L.C.M=x^3-9x^2\_26x-24$

$Let\ First\ polynomial=A=x^2-5x+6$

$Second\ polynomial=B=?$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}$$

$A\times B=L.C.M\times H.C.F$

Chapter # 6

Ex # 6.1

$$B=\frac{L.C.M\times H.C.F}{A}$$

Put the values

$$B=\frac{(x^3-9x^2\_26x-24)(x-3)}{x^2-5x+6}$$

Now by simple Division

$$\begin{array}{r|l} & x-4 \\ \hline x^2-5x+6 & x^3-9x^2+26\,x-24 \\ & \pm x^3 \mp 5x^2 \pm 6x \\ \hline & -4x^2+20x-24 \\ & \mp 4x^2 \pm 20x \mp 24 \\ \hline & \times \end{array}$$

$So\ B=(x-4)(x-3)$

$B=x^2-3x-4x+12$

$B=x^2-7x+12$

Hence the second polynomial is $x^2-7x+12$

Example # 10

If H.C.F and L.C.M of two polynomials are $x-1$ and x^3+4x^2+x-6 respectively. Find the polynomials of degree 2.

Solution:

$H.C.F=x-1$

$L.C.M=x^3+4x^2+x-6$

$First\ polynomial=A=?$

$Second\ polynomial=B=?$

$As\ H.C.F=x-1$

$then\ it\ is\ also\ the\ factor\ of\ L.C.M$

Now

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x-1 & x^3+4x^2+x-6 \\ & \pm x^3 \mp x^2 \\ \hline & 5x^2+x-6 \\ & \pm 5x^2 \mp 5x \\ \hline & 6x-6 \\ & \pm 6x \mp 6 \\ \hline & \times \end{array}$$

$L.C.M=x^3+4x^2+x-6$

$L.C.M=(x-1)(x^2+5x+6)$

$L.C.M=(x-1)(x^2+3x+2x+6)$

$L.C.M=(x-1)[x(x+3)+2(x+3)]$

$L.C.M=(x-1)(x+3)(x+2)$

$As\ x-1\ is\ common\ factor.So$

$A=(x-1)(x+3)$

Ex # 6.1

$A=x^2+2x-3$

And

$B=(x-1)(x+2)$

$B=x^2+2x-1x-2$

$B=x^2+x-2$

Example # 11

The sum of two numbers is 120 and their H.C.F is 12. Find the numbers.

Solution:

Let x and y be the two numbers.

As H.C.F is 12, means 12 is common factor.

So, it becomes

$12x+12y=120$

$12(x+y)=120$

Divide B.S by 12, we get

$x+y=12$

As the sum of two numbers is 10, so the possible pairs of numbers are (1,9), (2,8), (3,7), (4,6), (5,5)

As (1,9), (3,7) are non commo factors

Then the required numbers are:

$1\times 12=12$ and $9\times 12=108$

OR

$3\times 12=36$ and $7\times 12=84$

Chapter # 6

Exercise# 6.1

Page # 159-160

Q1: 159 **Find H.C.F of the following expression by factorization method.**

(i) $(x+y)^2$ **and** x^2-36

Solution:

$(x+y)^2$ and x^2-36

$(x+y)^2 = (x+y)(x+y)$

And

$x^2-36 = (x)^2-(6)^2$

$= (x+6)(x-6)$

$H.C.F = x-6$

(iii) $x-3, x^2-9, (x-3)^2$

Solution:

$x-3, x^2-9, (x-3)^2$

$x-3 = x-3$

And

$x^2-9 = (x)^2-(3)^2$

$= (x+3)(x-3)$

And

$(x-3)^2 = (x-3)(x-3)$

$H.C.F = \ x-3$

Ex # 6.1

(ii) x^4-y^4 **and** $x^4+2x^2y^2+y^4$

Solution:

x^4-y^4 and $x^4+2x^2y^2+y^4$

$x^4-y^4 = (x^2)^2-(y^2)^2$

$= (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

And

$x^4+2x^2y^2+y^4 = (x^2)^2+2(x^2)(y^2)+(y^2)^2$

$= (x^2+y^2)^2$

$= (x^2+y^2)(x^2+y^2)$

$H.C.F = x^2+y^2$

(v) $2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

Solution:

$2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

$2x^4-2y^4 = 2[(x^2)^2-(y^2)^2]$

$= 2(x^2+y^2)(x^2-y^2)$

$= 2(x^2+y^2)(x+y)(x-y)$

And

$6x^2+12xy+6y^2 = 6(x^2+2xy+y^2)$

$= 2\times3(x+y)^2$

$= 2\times3(x+y)(x+y)$

And

$9x^3+9y^3 = 9(x^3+y^3)$

$= 9(x+y)(x^2-xy+y^2)$

$H.C.F = x+y$

(iv) $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2 = 2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3 = 2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y) = 3.3(x-y)(x-y)(x+2y)$

$H.C.F = 3.3(x-y)(x-y)(x+2y)$

$H.C.F = 3^2(x-y)^2(x+2y)$

Chapter # 6

Ex # 6.1

Q2: Find H.C.F by division method.

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(i) $x^2 - x - 6$ and $x^2 - 2x - 3$

Solution:

$x^2 - x - 6$ and $x^2 - 2x - 3$

$$\begin{array}{r|l|l}
x^2 - x - 6 & x^2 - 2x - 3 & 1 \\
 & \pm x^2 \mp \ \ x \mp 6 & \\
\hline
-1 & -x + 3 & \\
\hline
x - 3 & x^2 - x - 6 & x + 2 \\
 & \pm x^2 \mp 3x & \\
\hline
 & 2x - 6 & \\
 & \pm 2x \mp 6 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = x - 3$

(ii) $y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

Solution:

$y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

$$\begin{array}{r|l|l}
y^3 - 3y + 2 & y^3 - 5y^2 + 7y - 3 & 1 \\
 & \pm y^3 \qquad \mp 3y \pm 2 & \\
\hline
-5 & -5y^2 + 10y - 5 & \\
\hline
y^2 - 2y + 1 & y^3 - 3y + 2 & y + 2 \\
 & \pm y^3 \pm 1y \qquad \mp 2y^2 & \\
\hline
 & 2y^2 - 4y + 2 & \\
 & \pm 2y^2 \mp 4y \pm 2 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = \ y^2 - 2y + 1$

Chapter # 6

Ex # 6.1

(iii) **$2x^5-4x^4-6x$ and $x^5+x^4-3x^3-3x^2$**

Solution:

$2x^5-4x^4-6x$ and $x^5+x^4-3x^3-3x^2$

$2x^5-4x^4-6x = 2x(x^4-2x^3-3)$

$x^5+x^4-3x^3-3x^2 = x^2(x^3+x^2-3x-3)$

$= x.x(x^3+x^2-3x-3)$

$x^3+x^2-3x-3 \;\big|\; x^4-2x^3-3 \;\big|\; x$

$\pm x^4 \pm x^3 \quad \mp 3x^2 \mp 3x$

$-3 \;\big|\; -3x^3+3x^2+3x-3$

$x^3-x^2-x+1 \;\big|\; x^3+x^2-3x-3 \;\big|\; 1$

$\pm x^3 \mp x^2 \mp x \pm 1$

$2 \;\big|\; 2x^2-2x-4$

$x^2-x-2 \;\big|\; x^3-x^2-x+1 \;\big|\; x$

$\pm x^3 \mp x^2 \mp 2x$

$x+1 \;\big|\; x^2-x-2 \;\big|\; x-2$

$\pm x^2 \pm x$

$-2x-2$

$\mp 2x \mp 2$

$\times$

$H.C.F = x(x+1)$

(iv) **$2x^3+10x^2+5x+25$ and x^3+5x^2-x-5**

Solution:

$2x^3+10x^2+5x+25$ and x^3+5x^2-x-5

$x^3+5x^2-x-5 \;\big|\; 2x^3+10x^2+5x+25 \;\big|\; 2$

$\pm 2x^3 \pm 10x^2 \mp 2x \mp 10$

$7 \;\big|\; 7x+35$

$x+5 \;\big|\; x^3+5x^2-x-5 \;\big|\; x^2-1$

$\pm x^3 \mp 5x^2$

$-x-5$

$\mp x \mp 5$

$\times$

$H.C.F = x+5$

Chapter # 6

Ex # 6.1

Q3: Find L.C.M by factorization.

(i) $\boldsymbol{x+y, \quad x^2-y^2}$

Solution:

$x+y, \quad x^2-y^2$

$x+y = x+y$

And

$x^2-y^2 = (x+y)(x-y)$

$Common\ Factor = x+y$

$Non-common\ factor = x-y$

$\boldsymbol{L.C.M = common\ factor \times non-common\ factor}$

$L.C.M = (x+y)(x-y)$

$L.C.M = x^2-y^2$

(ii) $\boldsymbol{x^3-y^3, x-y}$

Solution:

$x^3-y^3, x-y$

$x^3-y^3 = (x-y)(x^2+xy+y^2)$

And

$x-y = x-y$

$Common\ Factor = x-y$

$Non-common\ factor = x^2+xy+y^2$

$\boldsymbol{L.C.M = common\ factor \times non-common\ factor}$

$L.C.M = (x-y)(x^2+xy+y^2)$

$L.C.M = x^3-y^3$

Ex # 6.1

(iii) $\boldsymbol{x^5-x, x^5-x^2\ and\ x^5-x^3}$

Solution:

$x^5-x, x^5-x^2\ and\ x^5-x^3$

$x^5-x = x(x^4-1)$

$= x[(x^2)^2-(1)^1]$

$= x(x^2+1)(x^2-1)$

$= x(x^2+1)(x+1)(x-1)$

And

$x^5-x^2 = x^2(x^3-1)$

$= x.x[(x)^3-(1)^3]$

$= x.x(x-1)(x^2+(x)(1)+1^2)$

$= x.x(x-1)(x^2+x+1)$

And

$x^5-x^3 = x^3(x^2-1)$

$= x.x.x[(x)^2-(1)^2]$

$= x.x.x(x+1)(x-1)$

$Common\ Factor = x(x-1)$

$Non-common\ factor = x.x(x^2+1)(x+1)(x^2+x+1)$

$\boldsymbol{L.C.M = common\ factor \times non-common\ factor}$

$L.C.M = x(x-1) \times x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M = x^3(x-1)(x+1)(x^2+1)(x^2+x+1)$

(iv) $\boldsymbol{2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)}$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2 = 2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3 = 2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y) = 3.3(x-y)(x-y)(x+2y)$

$Common\ Factor = 3.3(x-y)(x-y)(x+2y)$

$Non-common\ factor = 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M = common\ factor \times non-common\ factor$

$L.C.M = 3.3(x-y)(x-y)(x+2y) \times 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M = 2^3 3^2(x-y)^3(x+2y)^3$

Chapter # 6

Ex # 6.1

Q4: Find H.C.F and L.C.M of the following expression.

160

(i) $\boldsymbol{x^3 - 2x^2 - 13x - 10}$ **and** $\boldsymbol{x^3 - x^2 - 10x - 8}$

Solution:

$x^3 - 2x^2 - 13x - 10$ and $x^3 - x^2 - 10x - 8$

$Let\ A = x^3 - 2x^2 - 13x - 10$

$and\ B = x^3 - x^2 - 10x - 8$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l} x^3 - x^2 - 10x - 8 & x^3 - 2x^2 - 13x - 10 & 1 \\ & \pm x^3 \mp\ x^2 \mp 10x \mp 8 & \\ \hline -1 & -x^2 - 3x - 2 & \end{array}$$

$$\begin{array}{r|l|l} x^2 + 3x + 2 & x^3 - x^2 - 10x - 8 & x - 4 \\ & \pm x^3 \pm 3x^2 \pm 2x & \\ \hline & -4x^2 - 12x - 8 & \\ & \mp 4x^2 \mp 12x \mp 8 & \\ \hline & \times & \end{array}$$

$H.C.F = x^2 + 3x + 2$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3 - 2x^2 - 13x - 10)(x^3 - x^2 - 10x - 8)}{x^2 + 3x + 2}$$

Now by Simple Division

$$\begin{array}{r|l} & x - 5 \\ \hline x^2 + 3x + 2 & x^3 - 2x^2 - 13x - 10 \\ & \pm x^3 \pm 3x^2 \pm 2x \\ \hline & -5x^2 - 15x - 10 \\ & \mp 5x^2 \mp 15x \mp 10 \\ \hline & \times \end{array}$$

$So\ L.C.M = (x - 5)(x^3 - x^2 - 10x - 8)$

Chapter # 6

Ex # 6.1

(ii) $\boldsymbol{2x^4-2x^3+x^2+3x-6}$ **and** $\boldsymbol{4x^4-2x^3+3x-9}$

Solution:

$2x^4-2x^3+x^2+3x-6\ and\,4x^4-2x^3+3x-9$

$Let\ A=2x^4-2x^3+x^2+3x-6$

$and\ B=4x^4-2x^3+3x-9$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$\begin{array}{rl}
2x^4-2x^3+x^2+3x-6 & \big) \; 4x^4-2x^3+3x-9 \; \big(\; 2 \\
 & \pm4x^4\mp4x^3\pm6x\mp12\pm2x^2 \\ \hline
2x^3-2x^2-3x+3 & \big) \; 2x^4-2x^3+x^2+3x-6 \; \big(\; x \\
 & \pm2x^4\mp2x^3\mp3x^2\pm3x \\ \hline
2 & \big) \; 4x^2-6 \\ \hline
2x^2-3 & \big) \; 2x^3-2x^2-3x+3 \; \big(\; x-1 \\
 & \pm2x^3 \qquad \mp3x \\ \hline
 & -2x^2+3 \\
 & \mp2x^2\pm3 \\ \hline
 & \times
\end{array}$$

$H.C.F=2x^2-3$

Now put the values in equ (i)

$$L.C.M=\frac{(2x^4-2x^3+x^2+3x-6)(4x^4-2x^3+3x-9)}{2x^2-3}$$

Now by Simple Division

$$\begin{array}{rl}
 & x^2-x+2 \\ \hline
2x^2-3 & \big) \; 2x^4-2x^3+\;x^2+3x-6 \\
 & \pm2x^4 \qquad \mp3x^2 \\ \hline
 & -2x^3+4x^2+3x-6 \\
 & \mp2x^3 \qquad \pm3x \\ \hline
 & 4x^2-6 \\
 & \pm4x^2\mp6 \\ \hline
 & \times
\end{array}$$

$So\ L.C.M=(x^2-x+2)(4x^4-2x^3+3x-9)$

(iii) $\boldsymbol{a^4-a^3-a+1}$ **and** $\boldsymbol{a^4+a^2+1}$

Solution:

a^4-a^3-a+1 and a^4+a^2+1

$Let\ A=a^4-a^3-a+1$

$and\ B=a^4+a^2+1$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l} a^4+a^2+1 & a^4-a^3-a+1 & 1 \\ & \pm a^4 \qquad \pm 1 \pm a^2 & \\ \hline -a & -a^3-a^2-a & \end{array}$$

$$\begin{array}{r|l|l} a^2+a+1 & a^4+a^2+1 & a^2-a+1 \\ & \pm a^4 \pm a^2 \qquad \pm a^3 & \\ \hline & -a^3+1 & \\ & \mp a^3 \qquad \mp a^2 \mp a & \\ \hline & a^2+a+1 & \\ & \pm a^2 \pm a \pm 1 & \\ \hline & \times & \end{array}$$

$H.C.F=a^2+a+1$

Now put the values in equ (i)

$$L.C.M=\frac{(a^4-a^3-a+1)(a^4+a^2+1)}{a^2+a+1}$$

Now by Simple Division

$$\begin{array}{r|l} & a^2-2a+1 \\ \hline a^2+a+1 & a^4-a^3-a+1 \\ & \pm a^4 \pm a^3 \qquad \pm a^2 \\ \hline & -2a^3-a^2-\ a+1 \\ & \mp 2a^3 \mp 2a^2 \mp 2a \\ \hline & a^2+a+1 \\ & \pm a^2 \pm a \pm 1 \\ \hline & \times \end{array}$$

$So\ L.C.M=(a^2-2a+1)(a^4+a^2+1)$

Chapter # 6

Ex # 6.1

(iv) $\mathbf{1 - x^2 - x^4 + x^5}$ **and** $\mathbf{1 + 2x + x^2 - x^4 - x^5}$

Solution:

$1 - x^2 - x^4 + x^5$ and $1 + 2x + x^2 - x^4 - x^5$

$x^5 - x^4 - x^2 + 1$ and $-x^5 - x^4 + x^2 + 2x + 1$

$Let\ A = x^5 - x^4 - x^2 + 1$

$and\ B = -x^5 - x^4 + x^2 + 2x + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
x^5 - x^4 - x^2 + 1 & -x^5 - x^4 + x^2 + 2x + 1 & -1 \\
 & \mp x^5 \pm x^4 \pm x^2 \qquad \mp 1 & \\
\hline
-2 & -2x^4 + 2x + 2 & \\
\hline
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 & x - 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x & \\
\hline
 & -x^4 + x + 1 & \\
 & \mp x^4 \pm x \pm 1 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = \quad x^4 - x - 1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^5 - x^4 - x^2 + 1)(-x^5 - x^4 + x^2 + 2x + 1)}{x^4 - x - 1}$$

Now by Simple Division

$$\begin{array}{r|l}
 & \quad x - 1 \\
\hline
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x \\
\hline
 & -x^4 + x + 1 \\
 & \mp x^4 \pm x \pm 1 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (x + 2)(-x^5 - x^4 + x^2 + 2x + 1)$

$So\ L.C.M = (x + 2)(1 + 2x + x^2 - x^4 - x^5)$

Q5: 160

H.C.F and L.C.M of two polynomials are $x-2$ and x^3+3x^2-6x-8 respectively. If one polynomial is x^2+2x-8, find the second polynomial.

Solution:

$$H.C.F = x-2$$
$$L.C.M = x^3+3x^2-6x-8$$
$$First\ polynomial = A = x^2+2x-8$$
$$Second\ polynomial = B = ?$$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$
$$L.C.M \times H.C.F = A \times B$$
$$\frac{L.C.M \times H.C.F}{A} = B$$
$$B = \frac{L.C.M \times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3+3x^2-6x-8)(x-2)}{x^2+2x-8}$$

Now by simple Division

$$\begin{array}{r|l} & x+1 \\ \hline x^2+2x-8, & x^3+3x^2-6x-8 \\ & \pm x^3 \pm 2x^2 \mp 8x \\ \hline & x^2+2x-8 \\ & \pm x^2 \pm 2x \mp 8 \\ \hline & \times \end{array}$$

$$So\ B = (x+1)(x-2)$$
$$B = x^2-2x+1x-2$$
$$B = x^2-x-2$$

Q6: 160

If product of two polynomials is $x^4+5x^3-6x^2-2x-28$ and their H.C.F is $x-2$. Find their L.C.M.

Solution:

Let Product of two polynomials $= A \times B$

$$Then\ A \times B = x^4+5x^3-6x^2-2x-28$$
$$H.C.F = x-2$$
$$L.C.M = ?$$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

Put the values

$$L.C.M = \frac{x^4+5x^3-6x^2-2x-28}{x-2}$$

$$\begin{array}{r|l} & x^3+7x^2+8x+14 \\ \hline x-2 & x^4+5x^3-6x^2-2x-28 \\ & \pm x^4 \mp 2x^3 \\ \hline & 7x^3-6x^2-2x-28 \\ & \pm 7x^3 \mp 14x^2 \\ \hline & 8x^2-2x-28 \\ & \pm 8x^2 \mp 16x \\ \hline & 14x-28 \\ & \pm 14 \mp 28 \\ \hline & \times \end{array}$$

$$L.C.M = x^3+7x^2+8x+14$$

Q7: 160

H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find the polynomials of degree 2.

Solution:

$$H.C.F = x+5$$
$$L.C.M = 2x^3+11x^2+2x-15$$
$$First\ polynomial = A = ?$$
$$Second\ polynomial = B = ?$$
$$As\ H.C.F = x+5$$
$$then\ it\ is\ also\ the\ factor\ of\ L.C.M$$

Now

$$\begin{array}{r|l} & 2x^2+x-3 \\ \hline x+5 & 2x^3+11x^2+2x-15 \\ & \pm 2x^3 \pm 10x^2 \\ \hline & x^2+2x-15 \\ & \pm x^2 \pm 5x \\ \hline & -3x-15 \\ & \mp 3x \mp 15 \\ \hline & \times \end{array}$$

$$L.C.M = 2x^3+11x^2+2x-15$$
$$L.C.M = (x+5)(2x^2+x-3)$$
$$L.C.M = (x+5)(2x^2+3x-2x-3)$$
$$L.C.M = (x+5)[x(2x+3)-1(2x+3)]$$
$$L.C.M = (x+5)(2x+3)(x-1)$$
$$As\ x+5\ is\ common\ factor. So$$
$$A = (x+5)(2x+3)$$
$$A = 2x^2+3x+10x+15$$
$$A = 2x^2+13x+15$$

And

$$B = (x+5)(x-1)$$
$$B = x^2-1x+5x-5$$
$$B = x^2+4x-5$$

Ex # 6.1

Q8: 160 **If product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and their L.C.M is $x^3+2x^2-11x-12$. Find their H.C.F.**

Solution:

Let Product of two polynomials $= A\times B$

$Then\ A\times B = x^4+6x^3-3x^2-56x-48$

$L.C.M = x^3+2x^2-11x-12$

$H.C.F = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

$$H.C.F = \frac{A\times B}{L.C.M}$$

Put the values

$$H.C.F = \frac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$$

Now by Simple Division

$$\begin{array}{r|l} & \quad x+4 \\ \hline x^3+2x^2-11x-12 & x^4+6x^3-\ 3x^2-\ 56x-48 \\ & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\ \hline & \quad 4x^3+8x^2-44x-48 \\ & \quad \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\ \hline & \qquad \times \end{array}$$

So $H.C.F = x+4$

Q9: 160 **Waqar wishes to distribute 128 bananas and also 176 apples equally among certain number of children. Find the highest number of children who can get the fruit in this way.**

Solution:

Bananas = 128

Apples = 176

Highest number of children = ?

Now

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	176
2	88
2	44
2	22
11	11
	1

$128 = 2\times2\times2\times2\times2\times2\times2$

$176 = 2\times2\times2\times2\times11$

$H.C.F = 2\times2\times2\times2$

$= 16$

So highest number of children = 16

Ex # 6.2

Algebraic fractions

An algebraic fraction is the quotient of two algebraic expressions.

Example:

$$\frac{x-y}{y^2-4x^2}$$

Example # 12

Simplify $\boldsymbol{\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}}$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$

$$=\frac{x+y+x-y}{3x+2y}$$

$$=\frac{x+x+y-y}{3x+2y}$$

$$=\frac{2x}{3x+2y}$$

Example # 13

Simplify $\boldsymbol{\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}}$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$

$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$

$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$

$$=\frac{3y^2-2xy}{x^2-y^2}$$

Chapter # 6

Ex # 6.2

Example # 14

Simplify $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$$

$$=\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$=\frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$=\frac{x+x-y+y-1}{x^2-y^2}$$

$$=\frac{2x-1}{x^2-y^2}$$

Example # 15

Simplify $\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$

Solution:

$$\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$$

$$=\frac{y}{y^2-2y+y-2}-\frac{1}{y^2-2y+7y-14}-\frac{2}{y^2+1y+7y+7}$$

$$=\frac{y}{y(y-2)+1(y-2)}-\frac{1}{y(y-2)+7(y-2)}-\frac{2}{y(y+1)+7(y+1)}$$

$$=\frac{y}{(y-2)(y+1)}-\frac{1}{(y-2)(y+7)}-\frac{2}{(y+1)(y+7)}$$

$$=\frac{y(y+7)-1(y+1)-2(y-2)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+7y-1y-1-2y+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+6y-2y-1+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+4y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+1y+3y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y(y+1)+3(y+1)}{(y-2)(y+1)(y+7)}$$

$$=\frac{(y+1)(y+3)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y+3}{(y-2)(y+7)}$$

Ex # 6.2

Example # 16

Simplify $\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$

Solution:

$$\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$$

$$=\frac{x+4}{x-3}\times\frac{x^2-3^2}{x^2-2x+1x-2}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{x(x-2)+1(x-2)}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{(x-2)(x+1)}$$

$$=\frac{x+4}{1}\times\frac{(x+3)}{(x-2)(x+1)}$$

$$=\frac{(x+4)(x+3)}{(x-2)(x+1)}$$

Example # 17

Multiply $\frac{x^2-2x}{2x^2+5x+3}$ **by** $\frac{2x^2-3x-9}{x^2-9}$

Solution:

$$\frac{x^2-2x}{2x^2+5x+3}\times\frac{2x^2-3x-9}{x^2-9}$$

$$=\frac{x(x-2)}{2x^2+2x+3x+3}\times\frac{2x^2+3x-6x-9}{x^2-9^2}$$

$$=\frac{x(x-2)}{2x(x+1)+3(x+1)}\times\frac{x(2x+3)-3(2x+3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)(2x+3)}\times\frac{(2x+3)(x-3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)}\times\frac{1}{(x+3)}$$

$$=\frac{x(x-2)}{(x+1)(x+3)}$$

Example # 18

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

Solution:

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

$$=\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\times\frac{y^2}{x^2+xy+y^2}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{y.y.y}\times\frac{y}{x-y}\times\frac{y.y}{x^2+xy+y^2}$$

$$=1$$

Chapter # 6

Ex # 6.2

Q1: Simplify:

$$(i)\quad \frac{x}{x+y}+\frac{2y}{x+y}$$

Solution:

$$\frac{x}{x+y}+\frac{2y}{x+y}$$
$$=\frac{x+2y}{x+y}$$

$$(ii)\quad \frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$
$$=\frac{x+y+x-y}{3x+2y}$$
$$=\frac{x+x+y-y}{3x+2y}$$
$$=\frac{2x}{3x+2y}$$

$$(iii)\quad \frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$$

Solution:

$$\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-(2)^2}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{(y+2)(y-2)}$$
$$=\frac{3(y+2)-2(y-2)-y}{(y+2)(y-2)}$$
$$=\frac{3y+6-2y+4-y}{(y+2)(y-2)}$$
$$=\frac{3y-2y-y+6+4}{(y+2)(y-2)}$$
$$=\frac{3y-3y+10}{y^2-(2)^2}$$
$$=\frac{10}{y^2-4}$$

$$(iv)\quad \frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

Ex # 6.2

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$
$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$
$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$
$$=\frac{3y^2-2xy}{x^2-y^2}$$

$$(v)\quad \frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$$

Solution:

$$\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$$
$$=\frac{x}{2x^2+2xy+1xy+y^2}-\frac{x-y}{-4x^2+y^2}+\frac{y}{2x^2+2xy-1xy-y^2}$$
$$=\frac{x}{2x(x+y)+y(x+y)}-\frac{x-y}{-(4x^2-y^2)}+\frac{y}{2x(x+y)-y(x+y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x)^2-y^2}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x+y)(2x-y)}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2-xy+x^2-y^2+2xy+y^2}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2+x^2-xy+2xy-y^2+y^2}{(x+y)((2x)^2-y^2)}$$
$$=\frac{3x^2+xy}{(x+y)(4x^2-y^2)}$$

$$(vi)\quad \frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$$

Solution:

$$\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$$
$$=\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{(3x)^2-y^2}$$

$$=\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{(3x+y)(3x-y)}$$

$$=\frac{a(3x+y)+a(3x-y)-6ax}{(3x+y)(3x-y)}$$

$$=\frac{3ax+ay+3ax-ay-6ax}{(3x+y)(3x-y)}$$

$$=\frac{3ax+3ax-6ax+ay-ay}{(3x+y)(3x-y)}$$

$$=\frac{6ax-6ax}{(3x+y)(3x-y)}$$

$$=\frac{0}{(3x+y)(3x-y)}$$

$$=0$$

(vii) $\boldsymbol{\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}}$

Solution:

$$\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{y(x+y)+y(x-y)}{(x-y)(x+y)}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{xy+y^2+xy-y^2}{(x-y)(x+y)}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{xy+xy+y^2-y^2}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2xy(x^2+y^2)+2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2x^3y+2xy^3+2x^3y-2xy^3}{(x^2)^2-(y^2)^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2x^3y+2x^3y+2xy^3-2xy^3}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{4x^3y}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{4x^3y(x^4+y^4)+4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

$$=\frac{4x^7y+4x^3y^5+4x^7y-4x^3y^5}{(x^4)^2-(y^4)^2}$$

$$=\frac{4x^7y+4x^7y+4x^3y^5-4x^3y^5}{x^8-y^8}$$

$$=\frac{8x^7y}{x^8-y^8}$$

(viii) $\boldsymbol{\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}}$

Solution:

$$\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$$

$$=\frac{1}{a^2+2a+5a+10}+\frac{1}{a^2+2a+8a+16}$$

$$=\frac{1}{a(a+2)+5(a+2)}+\frac{1}{a(a+2)+8(a+2)}$$

$$=\frac{1}{(a+2)(a+5)}+\frac{1}{(a+2)(a+8)}$$

$$=\frac{1(a+8)+1(a+5)}{(a+2)(a+5)(a+8)}$$

$$=\frac{a+8+a+5}{(a+2)(a+5)(a+8)}$$

$$=\frac{a+a+8+5}{(a+2)(a+5)(a+8)}$$

$$=\frac{2a+13}{(a+2)(a+5)(a+8)}$$

(ix) $\boldsymbol{\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}}$

Solution:

$$\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{1(a+b)+1(a-b)}{(a-b)(a+b)}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{a+b+a-b}{(a-b)(a+b)}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{a+a+b-b}{a^2-b^2}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{2a}{a^2-b^2}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

Ex # 6.2

$$= \frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2ab^2+2a^3-2ab^2}{(a^2)^2-(b^2)^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2a^3+2ab^2-2ab^2}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

$$= \frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{(a^4)^2-(b^4)^2}$$

$$= \frac{4a^7+4a^7+4a^3b^4-4a^3b^4}{a^8-b^8}$$

$$= \frac{8a^7}{a^8-b^8}$$

(x) $\dfrac{x^2-xy+y^2}{x^3+y^3} + \dfrac{x^2+xy+y^2}{x^3-y^3} - \dfrac{1}{x^2-y^2}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}$$

$$= \frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)} + \frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1}{x+y} + \frac{1}{x-y} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$= \frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$= \frac{x+x-y+y-1}{x^2-y^2}$$

$$= \frac{2x-1}{x^2-y^2}$$

Ex # 6.2

Q2: Simplify

(i) $\dfrac{x^2-25}{5-x}$

Solution:

$$\frac{x^2-25}{5-x}$$

$$= \frac{x^2-(5)^2}{-x+5}$$

$$= \frac{(x+5)(x-5)}{-(x-5)}$$

$$= -(x+5)$$

(ii) $\dfrac{x^2+5x+4}{4y^3} \times \dfrac{2y^2}{x^2+3x+2}$

Solution:

$$\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}$$

$$= \frac{x^2+4x+1x+4}{4y.y.y} \times \frac{2y.y}{x^2+2x+1x+2}$$

$$= \frac{x(x+4)+1(x+4)}{2y} \times \frac{1}{x(x+2)+1(x+2)}$$

$$= \frac{(x+4)(x+1)}{2y} \times \frac{1}{(x+2)(x+1)}$$

$$= \frac{x+4}{2y} \times \frac{1}{x+2}$$

$$= \frac{x+4}{2y(x+2)}$$

(iii) $\dfrac{x^2-5x+4}{x^3-3x-4} \div \dfrac{x^3-4x^2+x-4}{2x-1}$

Solution:

$$\frac{x^2-5x+4}{x^2-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}$$

$$= \frac{x^2-5x+4}{x^2-3x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x^2-4x-1x+4}{x^2-4x+1x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)} \times \frac{2x-1}{x^2(x-4)+1(x-4)}$$

$$= \frac{(x-4)(x-1)}{(x-4)(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

$$= \frac{(x-1)}{(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

$$= \frac{(x-1)(2x-1)}{(x+1)(x-4)(x^2+1)}$$

(iv) $\boldsymbol{\frac{a(a+b)}{a^3-b^3} \times \frac{a^2+ab+b^2}{a^2+b^2}}$

Solution:

$$\frac{a(a+b)}{a^3-b^3} \times \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)(a^2+ab+b^2)} \times \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)} \times \frac{1}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)(a^2+b^2)}$$

(v) $\boldsymbol{\frac{7}{x^2-4} \div \frac{xy}{x+2}}$

Solution:

$$\frac{7}{x^2-4} \div \frac{xy}{x+2}$$

$$= \frac{7}{x^2-2^2} \times \frac{x+2}{xy}$$

$$= \frac{7}{(x+2)(x-2)} \times \frac{x+2}{xy}$$

$$= \frac{7}{x-2} \times \frac{1}{xy}$$

$$= \frac{7}{xy(x-2)}$$

(vi) $\boldsymbol{\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2}}$

Solution:

$$\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a^3-b^3}{a^4-b^4} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

$$= \frac{1}{(a+b)} \times \frac{1}{1}$$

$$= \frac{1}{(a+b)}$$

(vii) $\boldsymbol{\frac{2x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8}}$

Solution:

$$\frac{2x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8}$$

$$= \frac{2x}{3x-12} \times \frac{x^2-6x+8}{x^2-2x}$$

$$= \frac{2x}{3(x-4)} \times \frac{x^2-2x-4x+8}{x(x-2)}$$

$$= \frac{2x}{3(x-4)} \times \frac{x(x-2)-4(x-2)}{x(x-2)}$$

$$= \frac{2x}{3(x-4)} \times \frac{(x-2)(x-4)}{x(x-2)}$$

$$= \frac{2}{3} \times \frac{1}{1}$$

$$= \frac{2}{3}$$

$(viii)$ $\boldsymbol{\frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \div \frac{a^2-2a}{a+3}}$

Solution:

$$\frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \div \frac{a^2-2a}{a+3}$$

$$= \frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a^2-2a}$$

$$= \frac{a(a^3-8)}{2a^2+6a-1a-3} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= \frac{a(a^3-2^3)}{2a(a+3)-1(a+3)} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= \frac{a(a-2)(a^2+2a+4)}{(a+3)(2a-1)} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= 1$$

Ex # 6.2

$$\textbf{(ix)}\quad \frac{9-x^2}{x^4+6x^3}\div\frac{x^3-2x^2-3x}{x^2+7x+6}$$

Solution:

$$\frac{9-x^2}{x^4+6x^3}\div\frac{x^3-2x^2-3x}{x^2+7x+6}$$

$$=\frac{-x^2+9}{x^4+6x^3}\times\frac{x^2+7x+6}{x^3-2x^2-3x}$$

$$=\frac{-(x^2-9)}{x^3(x+6)}\times\frac{x^2+1x+6x+6}{x(x^2-2x-3)}$$

$$=\frac{-(x^2-3^2)}{x^3(x+6)}\times\frac{x(x+1)+6(x+1)}{x(x^2-3x+1x-3)}$$

$$=\frac{-(x+3)(x-3)}{x^3(x+6)}\times\frac{(x+1)(x+6)}{x[x(x-3)+1(x-3)]}$$

$$=\frac{-(x+3)(x-3)}{x^3(x+6)}\times\frac{(x+1)(x+6)}{x[(x-3)(x+1)]}$$

$$=\frac{-(x+3)}{x^3}\times\frac{1}{x}$$

$$=\frac{-(x+3)}{x^4}$$

$$\textbf{(x)}\quad \frac{ax+ab+cx+bc}{a^2-x^2}\times\frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$$

Solution:

$$\frac{ax+ab+cx+bc}{a^2-x^2}\times\frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$$

$$=\frac{ax+ab+cx+bc}{-x^2+a^2}\times\frac{x^2-2ax+a^2}{x^2+bx+ax+ab}$$

$$=\frac{a(x+b)+c(x+b)}{-(x^2-a^2)}\times\frac{(x-a)^2}{x(x+b)+a(x+b)}$$

$$=-\frac{(x+b)(a+c)}{(x+a)(x-a)}\times\frac{(x-a)(x-a)}{(x+b)(x+a)}$$

$$=-\frac{(a+c)}{(x+a)}\times\frac{(x-a)}{(x+a)}$$

$$=-\frac{(a+c)(x-a)}{(x+a)^2}$$

Ex # 6.3

Square root

Square root of a number is a number that can be multiplied by itself to produce the original

Square root of an algebraic expression can be found out by the following two methods.

(i) Factorization Method
(ii) Division Method

Square root by Factorization

In this method make the expression a perfect square then finds square root.

Example # 20

Find the square root of $\boldsymbol{x^2+ax+\frac{1}{4}a^2}$ ***by factorization***

Solution:

$$x^2+ax+\frac{1}{4}a^2$$

$$x^2+ax+\frac{1}{4}a^2=(x)^2+2(x)\left(\frac{1}{2}a\right)+\left(\frac{1}{2}a\right)^2$$

$$x^2+ax+\frac{1}{4}a^2=\left(x+\frac{1}{2}a\right)^2$$

Now take square root on B.S

$$\sqrt{x^2+ax+\frac{1}{4}a^2}=\sqrt{\left(x+\frac{1}{2}a\right)^2}$$

$$\sqrt{x^2+ax+\frac{1}{4}a^2}=\pm\left(x+\frac{1}{2}a\right)$$

Example # 21

Find the square root of $\boldsymbol{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}$

Solution:

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27=x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+25+2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27=x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+25$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(5)+(5)^2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27=\left(x+\frac{1}{x}-5\right)^2$$

Chapter # 6

Ex # 6.3

Taking square root on B.S

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left(x+\frac{1}{x}-5\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\left(x+\frac{1}{x}-5\right)$$

Square root by Division

طریقہ:

Expression کو Descending ترتیب میں لکھیں۔

پہلے expression کا Square root لینگے پھر Divisor اور Quotient میں لکھیں گے۔

Divisor اور Quotient کو آپس میں Multiply کریں اور پہلے expression کے نیچے لکھیں پھر Subtract کریں تو Remainder حاصل ہو جائے گا

Divisor کو ڈبل کر دے اور Remainder کو اس پر Divide کر دے اور جو Term آئے گا تو Divisor اور Quotient میں اس کو لکھیں۔

اب اس Quotient کو پورے Divisor کے ساتھ Multiply کرے پھر Subtract کرے

اب Divisor کے دوسرے Term کو ڈبل کرے اور اوپر کا طریقہ دوبارہ کریں۔

Find the square root of $16x^4-24x^3+25x^2-12x+4$

Solution:

Write the expression in descending order

$16x^4-24x^3+25x^2-12x+4$

Take the square root of first element of expression.

$$\sqrt{16x^4}=4x^2$$

Write $4x^2$ in divisor and quotient

$$\begin{array}{l|l} & 4x^2 \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \end{array}$$

Multiply the divisor and quotient and write it under first expression then subtract from given expression to get the remainder.

$$\begin{array}{l|l} & 4x^2 \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} & -24x^3+25x^2-12x+4 \end{array}$$

Now twice the divisor

$$\begin{array}{l|l} & 4x^2 \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2 & -24x^3+25x^2-12x+4 \end{array}$$

Divide the 2nd expression by this divisor then write that term in quotient and with this divisor.

$$\frac{-24x^3}{8x^2}=-3x$$

$$\begin{array}{l|l} & 4x^2-3x \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2-3x & -24x^3+25x^2-12x+4 \end{array}$$

Multiply this quotient with entire divisor

$$-3x(8x^2-3x)=-24x^3+9x^2$$

Write $-24x^3+9x^2$ under given expression then subtract it.

$$\begin{array}{l|l} & 4x^2-3x \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2-3x & -24x^3+25x^2-12x+4 \\ & \mp24x^3\pm9x^2 \\ \cline{2-2} & 16x^2-12x+4 \end{array}$$

Now twice the 2nd term of the divisor

$$\begin{array}{l|l} & 4x^2-3x \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2-3x & -24x^3+25x^2-12x+4 \\ & \mp24x^3\pm9x^2 \\ \cline{2-2} 8x^2-6x & 16x^2-12x+4 \end{array}$$

Repeat the above procedure.

Divide $16x^2$ by divisor $8x^2$ then write that term in quotient and with this divisor.

$$\frac{16x^2}{8x^2}=2$$

$$\begin{array}{l|l} & 4x^2-3x+2 \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2-3x & -24x^3+25x^2-12x+4 \\ & \mp24x^3\pm9x^2 \\ \cline{2-2} 8x^2-6x+2 & 16x^2-12x+4 \end{array}$$

Multiply this quotient with entire divisor

$$2(8x^2-6x+2)=16x^2-12x+4$$

Write $16x^2-12x+4$ under given expression then subtract it.

$$\begin{array}{l|l} & 4x^2-3x+2 \\ \cline{2-2} 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \cline{2-2} 8x^2-3x & -24x^3+25x^2-12x+4 \\ & \mp24x^3\pm9x^2 \\ \cline{2-2} 8x^2-6x+2 & 16x^2-12x+4 \\ & \pm16x^2\mp12x\pm4 \\ \cline{2-2} & 0 \end{array}$$

Chapter # 6

Ex # 6.3

Example # 22

Find the square root of $16x^4-24x^3+25x^2-12x+4$

Solution:

Now

$$\begin{array}{r|l}
 & \quad 4x^2-3x+2 \\
\hline
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\
\hline
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\
\hline
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm16x^2\mp12x\pm4 \\
\hline
 & 0
\end{array}$$

So

$$\sqrt{16x^4-24x^3+25x^2-12x+4}=\pm(4x^2-3x+2)$$

Example # 20

Find the square root of $\frac{x^2}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$

Solution:

$$\frac{x^2}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$$

The descending order of the expression are:

$$\frac{x^2}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}$$

Now

$$\begin{array}{r|l}
 & \quad \frac{x^2}{2}-2x+\frac{a}{3} \\
\hline
\frac{x^2}{2} & \frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\
 & \pm\frac{x^4}{4} \\
\hline
x^2-2x & -2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\
 & \mp2x^3\pm4x^2 \\
\hline
x^2-4x+\frac{a}{3} & \frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\
 & \pm\frac{ax^2}{3}\mp\frac{4ax}{3}x\pm\frac{a^2}{9} \\
\hline
 & 0
\end{array}$$

So

$$\sqrt{\frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}}=\pm\left(\frac{x^2}{2}-2x+\frac{a}{3}\right)$$

Ex # 6.3

Example # 24

What should be added to

What should be subtracted from

For what value of x

The expression $9x^4-12x^3+10x^2-3x-3$

to make the perfect square

Solution:

$$9x^4-12x^3+10x^2-3x-3$$

$$\begin{array}{r|l}
 & \quad 3x^2-2x+1 \\
\hline
3x^2 & 9x^4-12x^3+10x^2-3x-3 \\
 & \pm9x^4 \\
\hline
6x^2-2x & -12x^3+10x^2-3x-3 \\
 & \mp12x^3\pm4x^2 \\
\hline
6x^2-4x+1 & 6x^2-3x-3 \\
 & \pm6x^2\mp4x\pm1 \\
\hline
 & x-4
\end{array}$$

As for perfect square, Remainder $=0$

$-x+4$ should be Added to $9x^4-12x^3+10x^2-3x-3$ will become perfect square.

$-x+4+(x-4)=-x+4+x-4$

$-x+4+(x-4)=0$

$x-4$ should be Subtracted to $9x^4-12x^3+10x^2-3x-3$ will become perfect square.

$x-4-(x-4)=x-4-x+4$

$x-4-(x-4)=0$

For x

$x-4=0$

$x=4$

Chapter # 6

Exercise# 6.3

Page # 169

Q1: Find the square root by factorization method.

$(i)\ \boldsymbol{x^2+4x+4}$

Solution:

x^2+4x+4

$x^2+4x+4=x^2+2(x)(2)+2^2$

$x^2+4x+4=(x+2)^2$

Taking Square on B.S

$\sqrt{x^2+4x+4}=\pm\sqrt{(x+2)^2}$

$\sqrt{x^2+4x+4}=\pm(x+2)$

$(ii)\ \boldsymbol{(x-y)^2+6(x-y)+9}$

Solution:

$(x-y)^2+6(x-y)+9$

$(x-y)^2+6(x-y)+9=(x-y)^2+2(x-y)(3)+3^2$

$(x-y)^2+6(x-y)+9=(x-y+3)^2$

Taking Square on B.S

$\sqrt{(x-y)^2+6(x-y)+9}=\pm\sqrt{(x-y+3)^2}$

$\sqrt{(x-y)^2+6(x-y)+9}=\pm(x-y+3)$

$(iii)\ \boldsymbol{x^2y^2-8xy+16}$

Solution:

$x^2y^2-8xy+16$

$x^2y^2-8xy+16=(xy)^2+2(xy)(4)+4^2$

$x^2y^2-8xy+16=(xy+4)^2$

Taking Square on B.S

$\sqrt{x^2y^2-8xy+16}=\pm\sqrt{(xy+4)^2}$

$\sqrt{x^2y^2-8xy+16}=\pm(xy+4)$

$$(iv)\ \boldsymbol{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}$$

Solution:

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$$

$$=x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+2+16$$

$$=x^2+\frac{1}{x^2}+2-8\left(x+\frac{1}{x}\right)+16$$

$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(4)+(4)^2$$

$$=\left(x-\frac{1}{x}+4\right)^2$$

Now

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18=\left(x-\frac{1}{x}+4\right)^2$$

Taking Square on B.S

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}=\pm\sqrt{\left(x-\frac{1}{x}+4\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}=\pm\left(x-\frac{1}{x}+4\right)$$

$(v)\ \boldsymbol{(x+1)(x+2)(x+3)+1}$

Solution:

$x(x+1)(x+2)(x+3)+1$

Rearranging accordingly $0+3=1+2$

$=x(x+3)(x+1)(x+2)+1$

$=(x^2+3x)(x^2+2x+1x+2)+1$

$=(x^2+3x)(x^2+3x+2)+1$

$Let\ \ x^2+3x=y$

$=y^2+2y+1$

$=(y)^2+2(y)(1)+(1)^2$

$=(y+1)^2$

But $y=x^2+3x$

$=(x^2+3x+1)^2$

Now

$x(x+1)(x+2)(x+3)+1=(x^2+3x+1)^2$

Taking Square on B.S

$\sqrt{x(x+1)(x+2)(x+3)+1}=\pm\sqrt{(x^2+3x+1)^2}$

$\sqrt{x(x+1)(x+2)(x+3)+1}=\pm(x^2+3x+1)$

Ex # 6.3

(vi) $\left(\boldsymbol{x}+\frac{\mathbf{1}}{\boldsymbol{x}}\right)^{\mathbf{2}}-\mathbf{5}\left(\boldsymbol{x}-\frac{\mathbf{1}}{\boldsymbol{x}}\right)+\frac{\mathbf{9}}{\mathbf{4}}$

Solution:

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=x^2+\frac{1}{x^2}+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

Subtract and Add 2

$$=x^2+\frac{1}{x^2}-2+2+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2+4-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+4$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9+16}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{25}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2$$

$$=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Now

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\sqrt{\left(x-\frac{1}{x}-\frac{5}{2}\right)^2}$$

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\left(x-\frac{1}{x}-\frac{5}{2}\right)$$

(vii) $\left(\boldsymbol{x}^{\mathbf{2}}+\frac{\mathbf{1}}{\boldsymbol{x}^{\mathbf{2}}}\right)^{\mathbf{2}}-\mathbf{4}\left(\boldsymbol{x}+\frac{\mathbf{1}}{\boldsymbol{x}}\right)^{\mathbf{2}}+\mathbf{12}$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$$

Ex # 6.3

$$=\left(x^2+\frac{1}{x^2}\right)^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(4)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Now

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2}$$

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

(viii) $\frac{\mathbf{4}\boldsymbol{x}^{\mathbf{6}}-\mathbf{12}\boldsymbol{x}^{\mathbf{3}}\boldsymbol{y}^{\mathbf{3}}+\mathbf{9}\boldsymbol{y}^{\mathbf{6}}}{\mathbf{9}\boldsymbol{x}^{\mathbf{4}}+\mathbf{24}\boldsymbol{x}^{\mathbf{2}}\boldsymbol{y}^{\mathbf{2}}+\mathbf{16}\boldsymbol{y}^{\mathbf{6}}}$

Solution:

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Now

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Taking square root on B.S

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\sqrt{\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2}$$

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

Ex # 6.3

Q2: Find the square root of the following by Division method.

(i) $4x^4 - 4x^3 + 13x^2 - 6x + 9$

Solution:

$$4x^4 - 4x^3 + 13x^2 - 6x + 9$$

$$\begin{array}{r|l} & 2x^2 - x + 3 \\ \hline 2x^2 & 4x^4 - 4x^3 + 13x^2 - 6x + 9 \\ & \pm 4x^4 \\ \hline 4x^2 - x & -4x^3 + 13x^2 - 6x + 9 \\ & \mp 4x^3 \pm x^2 \\ \hline 4x^2 - 2x + 3 & 12x^2 - 6x + 9 \\ & \pm 12x^2 \mp 6x \pm 9 \\ \hline & 0 \end{array}$$

So

$$\sqrt{4x^4 - 4x^3 + 13x^2 - 6x + 9} = \pm(2x^2 - x + 3)$$

(ii) $x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$

Solution:

$$x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$$

$$\begin{array}{r|l} & x^2 + \frac{x}{2} - 4 \\ \hline x^2 & x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16 \\ & \pm x^4 \\ \hline 2x^2 + \frac{x}{2} & x^3 - \frac{31}{4}x^2 - 4x + 16 \\ & \pm x^3 \pm \frac{x^2}{4} \\ \hline 2x^2 + x - 4 & -8x^2 - 4x + 16 \\ & \mp 8x^2 \mp 4x \pm 16 \\ \hline & 0 \end{array}$$

So

$$\sqrt{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16} = \pm\left(x^2 + \frac{x}{2} - 4\right)$$

(iii) $x^2 - 2x + 1 + 2xy - 2y + y^2$

Solution:

$$x^2 - 2x + 1 + 2xy - 2y + y^2$$

Ex # 6.3

$$\begin{array}{r|l} & x - 1 + y \\ \hline x & x^2 - 2x + 1 + 2xy - 2y + y^2 \\ & \pm x^2 \\ \hline 2x - 1 & -2x + 1 + 2xy - 2y + y^2 \\ & \mp 2x \pm 1 \\ \hline 2x - 2 + y & 2xy - 2y + y^2 \\ & \pm 2xy \mp 2y \pm y^2 \\ \hline & 0 \end{array}$$

So

$$\sqrt{x^2 - 2x + 1 + 2xy - 2y + y^2} = \pm(x - 1 + y)$$

(iv) $\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$

Solution:

$$\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) - 12x^2 + \frac{12}{x^2} + 36$$

$$= x^4 + \frac{1}{x^4} - 2 - 12x^2 + \frac{12}{x^2} + 36$$

Arrange it in ascending order

$$= x^4 - 12x^2 - 2 + 36 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$= x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$\begin{array}{r|l} & x^2 - 6 - \frac{1}{x^2} \\ \hline x^2 & x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \pm x^4 \\ \hline 2x^2 - 6 & -12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \mp 12x^2 \pm 36 \\ \hline 2x^2 - 12 - \frac{1}{x^2} & -2 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \mp 2 \pm \frac{12}{x^2} \pm \frac{1}{x^4} \\ \hline & 0 \end{array}$$

So

$$\sqrt{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36} = \pm\left(x^2 - 6 - \frac{1}{x^2}\right)$$

Ex # 6.3

Q3 (i): For what value of k the expression

$$4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4}$$

will become perfect square.

Solution:

$$4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4}$$

$$\begin{array}{r|l}
 & 2x^2 + 8 + \frac{8}{x^2} \\ \hline
2x^2 & 4x^4 + 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 4x^4 \\ \hline
4x^2 + 8 & 32x^2 + 96 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 32x^2 \pm 64 \\ \hline
4x^2 + 16 + \frac{8}{x^2} & 32 + \frac{128}{x^2} + \frac{k}{x^4} \\
 & \pm 32 \pm \frac{128}{x^2} \pm \frac{64}{x^4} \\ \hline
 & \frac{k}{x^4} - \frac{64}{x^4}
\end{array}$$

As for perfect square, Remainder $= 0$

$$\frac{k}{x^4} - \frac{64}{x^4} = 0$$

$$\frac{k - 64}{x^4} = 0$$

$$k - 64 = 0 \times x^4$$

$$k - 64 = 0$$

$$k = 64$$

Q3 (ii):

(i) What should be added to

(ii) What should be subtracted to

(iii) For what value of x the expression

$4x^4 - 12x^3 + 17x^2 - 13x + 6$ so that it becomes perfect square

Solution:

$$4x^4 - 12x^3 + 17x^2 - 13x + 6$$

$$\begin{array}{r|l}
 & 2x^2 - 3x + 2 \\ \hline
2x^2 & 4x^4 - 12x^3 + 17x^2 - 13x + 6 \\
 & \pm 4x^4 \\ \hline
4x^2 - 3x & -12x^3 + 17x^2 - 13x + 6 \\
 & \mp 12x^3 \pm 9x^2 \\ \hline
4x^2 - 6x + 2 & 8x^2 - 13x + 6 \\
 & \pm 8x^2 \mp 12x \pm 4 \\ \hline
 & -x + 2
\end{array}$$

As for perfect square, Remainder $= 0$

Ex # 6.3

$x - 2$ should be Added to $4x^4 - 12x^3 + 17x^2 - 13x + 6$ will become perfect square.

$$-x + 2 + (x - 2) = -x + 2 + x - 2$$

$$-x + 2 + (x - 2) = 0$$

$-x + 2$ should be Subtracted to $4x^4 - 12x^3 + 17x^2 - 13x + 6$ will become perfect square.

$$-x + 2 - (-x + 2) = -x + 2 + x - 2$$

$$-x + 2 - (-x + 2) = 0$$

For x

$$-x + 2 = 0$$

$$-x = -2$$

$$x = 2$$

Q4: What should be subtracted and added to the expression $x^4 - 4x^3 + 10x + 7$ so that the expression is made perfect square?

Solution:

$$x^4 - 4x^3 + 10x + 7$$

$$\begin{array}{r|l}
 & x^2 - 2x - 2 \\ \hline
x^2 & x^4 - 4x^3 + 10x + 7 \\
 & \pm x^4 \\ \hline
2x^2 - 2x & -4x^3 + 10x + 7 \\
 & \mp 4x^3 \pm 4x^2 \\ \hline
2x^2 - 4x - 2 & -4x^2 + 10x + 7 \\
 & \mp 4x^2 \pm 8x \pm 4 \\ \hline
 & 2x + 3
\end{array}$$

As for perfect square, Remainder $= 0$

$-2x - 3$ should be Added to $x^4 - 4x^3 + 10x + 7$ will become perfect square.

$$-2x - 3 + (2x + 3) = 2x + 3 - 2x - 3$$

$$-2x - 3 + (2x + 3) = 0$$

$2x + 3$ should be Subtracted to $x^4 - 4x^3 + 10x + 7$ will become perfect square.

$$2x + 3 - (2x + 3) = 2x + 3 - 2x - 3$$

$$2x + 3 - (2x + 3) = 0$$

Chapter # 6

Ex # 6.3

Q5 (i): Find the value of l and m for which expression will become perfect square

$$\boldsymbol{x^4 + 4x^3 + 16x^2 + lx + m}$$

Solution:

$$x^4 + 4x^3 + 16x^2 + lx + m$$

$$\begin{array}{r|l}
 & x^2 + 2x + 6 \\
\hline
x^2 & x^4 + 4x^3 + 16x^2 + lx + m \\
 & \pm x^4 \\
\hline
2x^2 + 2x & 4x^3 + 16x^2 + lx + m \\
 & \pm 4x^3 \pm 4x^2 \\
\hline
2x^2 + 4x + 6 & 12x^2 + lx + m \\
 & \pm 12x^2 \pm 24x \pm 36 \\
\hline
 & lx - 24x + m - 36
\end{array}$$

As for perfect square, Remainder $= 0$

$$lx - 24x + m - 36 = 0$$

$$(l - 24)x + (m - 36) = 0$$

This $(l - 24)x + (m - 36) = 0$ when

$$(l - 24)x + (m - 36) = 0x + 0$$

By compare the co-efficient of x and constant

$$l - 24 = 0$$

$$l = 24$$

And $m - 36 = 0$

$$m = 36$$

Hence

$$l = 24 \text{ and } m = 36$$

Q5 (ii): Find the value of l and m for which expression will become perfect square

$$\boldsymbol{49x^4 - 70x^3 + 109x^2 + lx - m}$$

Solution:

$$49x^4 - 70x^3 + 109x^2 + lx - m$$

$$\begin{array}{r|l}
 & 7x^2 - 5x + 6 \\
\hline
7x^2 & 49x^4 - 70x^3 + 109x^2 + lx - m \\
 & \pm 49x^4 \\
\hline
14x^2 - 5x & -70x^3 + 109x^2 + lx - m \\
 & \mp 70x^3 \pm 25x^2 \\
\hline
14x^2 - 10x + 6 & 84x^2 + lx - m \\
 & \pm 84x^2 \mp 60x \pm 36 \\
\hline
 & lx + 60x - m - 36
\end{array}$$

As for perfect square, Remainder $= 0$

$$lx + 60x - m - 36 = 0$$

Ex # 6.3

$$(l + 60)x + (-m - 36) = 0$$

This $(l + 60)x + (-m - 36) = 0$ when

$$(l + 60)x + (-m - 36) = 0x + 0$$

By compare the co-efficient of x and constant

$$l + 60 = 0$$

$$l = -60$$

And $-m - 36 = 0$

$$-m = 36$$

$$m = -36$$

Hence

$$l = -60 \text{ and } m = -36$$

Review Exercise # 6

Page # 171

Q2: Simplify the following.

(i): $\boldsymbol{\dfrac{5}{2s+4} - \dfrac{3}{s^2+3s+2} + \dfrac{s}{s^2-s-2}}$

Solution:

$$\frac{5}{2s+4} - \frac{3}{s^2+3s+2} + \frac{s}{s^2-s-2}$$

$$= \frac{5}{2(s+2)} - \frac{3}{s^2+2s+1s+2} + \frac{s}{s^2-2s+1s-2}$$

$$= \frac{5}{2(s+2)} - \frac{3}{s(s+2)+1(s+2)} + \frac{s}{s(s-2)+1(s-2)}$$

$$= \frac{5}{2(s+2)} - \frac{3}{(s+2)(s+1)} + \frac{s}{(s-2)(s+1)}$$

$$= \frac{5(s+1)(s-2) - 3 \times 2(s-2) + s \times 2(s+2)}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5(s^2-2s+1s-2) - 6(s-2) + 2s(s+2)}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5(s^2-1s-2) - 6s + 12 + 2s^2 + 4s}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5s^2 - 5s - 10 - 6s + 12 + 2s^2 + 4s}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5s^2 + 2s^2 - 5s - 6s + 4s - 10 + 12}{2(s+2)(s+1)(s-2)}$$

$$= \frac{7s^2 - 11s + 4s - 2}{2(s+2)(s+1)(s-2)}$$

$$= \frac{7s^2 - 7s - 2}{2(s+2)(s+1)(s-2)}$$

Chapter # 6

Review Ex # 6

$$(\boldsymbol{ii}).\frac{\boldsymbol{a}}{(\boldsymbol{c}-\boldsymbol{a})(\boldsymbol{a}-\boldsymbol{b})}+\frac{\boldsymbol{b}}{(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{b}-\boldsymbol{c})}+\frac{\boldsymbol{c}}{(\boldsymbol{b}-\boldsymbol{c})(\boldsymbol{c}-\boldsymbol{a})}$$

Solution:

$$\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$$

$$=\frac{a(b-c)+b(c-a)+c(a-b)}{(c-a)(a-b)(b-c)}$$

$$=\frac{ab-ac+bc-ab+ac-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{ab-ab-ac+ac+bc-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{0}{(a-b)(b-c)(c-a)}$$

$$=0$$

$$(\mathbf{iii}):\ \frac{\boldsymbol{x^2-4}}{\boldsymbol{xy^2}}\cdot\frac{\boldsymbol{2xy}}{\boldsymbol{x^2-4x+4}}$$

Solution:

$$\frac{x^2-4}{xy^2}\cdot\frac{2xy}{x^2-4x+4}$$

$$=\frac{x^2-2^2}{xyy}\cdot\frac{2xy}{x^2-2(x)(2)+2^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)(x+2)}$$

$$=\frac{(x-2)}{y}\cdot\frac{2}{(x+2)}$$

$$=\frac{2(x-2)}{y(x+2)}$$

$$(\mathrm{iv}):\ \frac{\boldsymbol{a^3-b^3}}{\boldsymbol{a^4-b^4}}\div\frac{\boldsymbol{a^2+ab+b^2}}{\boldsymbol{a^2+b^2}}$$

Solution:

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

Review Ex # 6

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{a+b}\times\frac{1}{1}$$

$$=\frac{1}{a+b}$$

Review Ex # 6

Q3: **Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3**

$x^3-6x^2+11x-6$ and x^3-4x+3

Solution:

$Let\ A = x^3-6x^2+11x-6$

$and\ B = x^3-4x+3$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$x^3-4x+3\ \overline{)\ x^3-6x^2+11x-6\ }\ 1$$
$$\pm x^3 \quad \mp 4x \pm 3$$
$$-3\ \overline{)\ -6x^2+15x-9}$$
$$2x^2-5x+3\ \overline{)\ x^3-4x+3\ }\ x+5$$
$$\times 2$$
$$2x^3-8x+6$$
$$\pm 2x^3 \pm 3x \quad \mp 5x^2$$
$$5x^2-11x+6$$
$$\times 2$$
$$10x^2-22x+12$$
$$\pm 10x^2 \mp 25x \pm 15$$
$$3\ \overline{)\ 3x-3}$$
$$x-1\ \overline{)\ 2x^2-5x+3\ }\ 2x-3$$
$$\pm 2x^2 \mp 2x$$
$$-3x+3$$
$$\mp 3x \pm 3$$
$$\times$$

$H.C.F = x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-6x^2+11x-6\,)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$x^2-5x+6$$
$$x-1\ \overline{)\ x^3-6x^2+11x-6}$$
$$\pm x^3 \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

$So\ L.C.M = (x^2-5x+6)(x^3-4x+3)$

Chapter # 6

Review Ex # 6

Q4: Find the square root of :

(i): $4x^2 - 12x + 9$

Solution:

$4x^2 - 12x + 9$

$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + (3)^2$

$4x^2 - 12x + 9 = (2x - 3)^2$

Taking Square on B.S

$$\sqrt{4x^2 - 12x + 9} = \pm\sqrt{(2x-3)^2}$$

$$\sqrt{4x^2 - 12x + 9} = \pm(2x - 3)$$

Review Ex # 6

(ii): $x^4 + 4x^3 + 6x^2 + 4x + 1$

Solution:

$x^4 + 4x^3 + 6x^2 + 4x + 1$

$$\begin{array}{r|r}
 & x^2 + 2x + 1 \\
\hline
x^2 & x^4 + 4x^3 + 6x^2 + 4x + 1 \\
 & \pm x^4 \qquad\qquad\qquad\qquad \\
\hline
2x^2 + 2x & 4x^3 + 6x^2 + 4x + 1 \\
 & \pm 4x^3 \pm 4x^2 \qquad\qquad \\
\hline
2x^2 + 4x + 1 & 2x^2 + 4x + 1 \\
 & \pm 2x^2 \pm 4x \pm 1 \\
\hline
 & 0
\end{array}$$

So

$$\sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1} = \pm(x^2 + 2x + 1)$$

Think

$Q5$: Simplify $\dfrac{\boldsymbol{x^3 - y^3}}{\boldsymbol{x^3 - z^3}} \times \dfrac{\boldsymbol{x^2 + xy + xz + yz}}{\boldsymbol{x^4 + x^2y^2 + y^4}} \times \dfrac{\boldsymbol{x^3 + y^3}}{\boldsymbol{x^2 - y^2}}$

Solution:

$$\frac{x^3 - y^3}{x^3 + z^3} \times \frac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \frac{x^3 + y^3}{x^2 - y^2}$$

$$= \frac{(x-y)(x^2 + xy + y^2)}{(x+z)(x^2 - xz + z^2)} \times \frac{x(x+y) + z(x+y)}{x^4 + y^4 + x^2y^2} \times \frac{(x+y)(x^2 - xy + y^2)}{(x+y)(x-y)}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)}{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2 + x^2y^2} \times \frac{(x^2 - xy + y^2)}{1}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - x^2y^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - (xy)^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2 + xy)(x^2 + y^2 - xy)}$$

$$= \frac{1}{(x^2 - xz + z^2)} \times \frac{(x+y)}{1}$$

$$= \frac{(x+y)}{(x-z)(x^2 + xz + z^2)}$$

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MATHEMATICS

Class 9th (KPK)

Chapter # 7 Linear Equations & Inequealites

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Chapter # 7

UNIT # 7

LINEAR EQUATIONS AND INEQULITIES

Ex # 7.1

Linear equation

An equation the highest degree or exponent of a variable is one is called linear equation.

Linear equation in one variable

A linear equation in which one variable is used is called linear equation in one variable.

General form

$$ax + b = 0$$

Example:

$2x + 3 = 0$

$\frac{5}{2}y - 4 = 0$

$5x - 15 = 2x + 3$

Solution of Linear Equation

To solve the linear equation, follow the following steps.

First solve the brackets if any

Now shift the constant term to other side of equation by adding or subtracting to B.S

Transfer all terms containing variable on one side and simplify them if any.

Divide or multiply both sides of the equation by the co – efficient of the variable.

At last, sing numerical value is obtained.

Verify by putting the value in original equation.

پہلے Brackets کو Solve کریں۔

پھر constant term کو دوسرے طرف Shift کریں Add یا Subtract کر کے

Variable والے Term کو بھی ایک طرف Shift کریں

Equation کے دونوں طرف Variable کے Co – efficient کے ساتھ Multiply یا Divide کریں

Example # 2

Solve $2x + 3 = 1 - (x - 1)$

Solution:

$2x + 3 = 1 - 6(x - 1) \dots\dots equ(i)$

$2x + 3 = 1 - 6x + 6$

$2x + 3 = -6x + 1 + 6$

Ex # 7.1

$2x + 3 = -6x + 7$

Subtract 3 from B.S

$2x + 3 - 3 = -6x + 7 - 3$

$2x = -6x + 4$

$Add\ 6x\ on\ B.S$

$2x + 6x = -6x + 6x + 4$

$8x = 4$

Divide B.S by 8

$\frac{8x}{8} = \frac{4}{8}$

$x = \frac{1}{2}$

Verification

Put $x = \frac{1}{2}$ in equ (i)

$2\left(\frac{1}{2}\right) + 3 = 1 - 6\left(\frac{1}{2} - 1\right)$

$1 + 3 = 1 - 6\left(\frac{1-2}{2}\right)$

$4 = 1 - 6\left(\frac{-1}{2}\right)$

$4 = 1 - 3(-1)$

$4 = 1 + 3$

$4 = 4$

Thus Solution Set $= \left\{\frac{1}{2}\right\}$

Example # 3

$Solve\ 3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x$

Solution:

$3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x \dots\dots equ(i)$

Separate the variable and constant

$3x + \frac{x}{5} - 5x = \frac{1}{5} + 5$

$3x - 5x + \frac{x}{5} = \frac{1}{5} + 5$

$\frac{x}{5} + 3x - 5x = \frac{1}{5} + 5$

$\frac{x}{5} - 2x = \frac{1}{5} + 5$

$$\frac{x-10x}{5}=\frac{1+25}{5}$$

$$\frac{-9x}{5}=\frac{26}{5}$$

Multiply B.S by 5

$$5\times\frac{-9x}{5}=5\times\frac{26}{5}$$

$$-9x=26$$

Divide B.S by -9

$$\frac{-9x}{-9}=\frac{26}{-9}$$

$$x=-\frac{26}{9}$$

Verification

Put $x=-\frac{26}{9}$ in equ (i)

$$3\left(-\frac{26}{9}\right)+\frac{-\frac{26}{9}}{5}-5=\frac{1}{5}+5\left(-\frac{26}{9}\right)$$

$$-\frac{26}{3}+\left(-\frac{26}{9}\right)\div 5-5=\frac{1}{5}-\frac{130}{9}$$

$$-\frac{26}{3}-\frac{26}{9}\times\frac{1}{5}-5=\frac{1}{5}-\frac{130}{9}$$

$$-\frac{26}{3}-\frac{26}{45}-5=\frac{1}{5}-\frac{130}{9}$$

$$\frac{-390-26-225}{45}=\frac{9-650}{45}$$

$$\frac{-641}{45}=\frac{-641}{45}$$

Thus Solution Set $=\left\{-\frac{26}{9}\right\}$

Example # 4

Age of mother is 13 time the age of her daughter. It will be only five times after four years. Find their present ages.

Solution:

Let the present age of daughter $= x$ years

So the present age of mother $= 13x$ years

After four years

Age of daughter $=(x+4)years$

and age of mother $=(13x+4)years$

According to condition

Age of mother $=5$(Age of daughter)

$$13x+4=5(x+4)$$

$$13x+4=5x+20$$

Now shift the variable and constant

$$13x-5x=20-4$$

$$8x=16$$

Divide B.S by 8

$$\frac{8x}{8}=\frac{16}{8}$$

$$x=2$$

Thus present age of daughter $= x = 2years$

And present age of mother $=13\times 2$

$= 26years$

Example # 5

A number consist of two digits. The sum of digits is 8. If digits are interchanged, then new number becomes 36 less than the original numbers. Find the number.

Solution:

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ the\ original\ number\ = 10\times y+1\times x$

$= 10y + x$

$If\ place\ of\ digits\ are\ interchanged$

$New\ number = 10\times x+1\times y$

$= 10x+y$

According to given conditions

Sum of digits is 8

So,

$x+y=8\ldots\ldots equ(i)$

And

$New\ number = Original\ number - 36$

$$10x+y=10y+x-36$$

$$10x-x=10y-y-36$$

$$9x=9y-36$$

$$9x=9(y-4)$$

Divide B.S by 9

$$\frac{9x}{9}=\frac{9(y-4)}{9}$$

$x=y-4\ldots\ldots equ(ii)$

Put $x=y-4$in equ (i)

$$y-4+y=8$$

Add 4 on B.S

$$y-4+4+y=8+4$$

$$y+y=12$$

$$2y=12$$

Ex # 7.1

Divide B.S by 2

$$\frac{2y}{2} = \frac{12}{2}$$

$y = 6$

Put $y = 6$in equ (ii)

$x = 6 - 4$

$x = 2$

$$\begin{aligned} As\ the\ Original\ number &= 10y + x \\ &= 10(6) + 2 \\ &= 60 + 2 \\ &= 62 \end{aligned}$$

Exercise # 7.1

Page # 177

Q1: **Find the solution sets of the following equations and verify the answer.**

(i) $\mathbf{5x + 8 = 23}$

Solution:

$5x + 8 = 23 \dots \dots equ(i)$

Subtract 8 from B.S

$5x + 8 - 8 = 23 - 8$

$5x = 15$

Divide 5 on B.S

$$\frac{5x}{5} = \frac{15}{5}$$

$x = 3$

Verification

Put $x = 3$ in equ (i)

$5(3) + 8 = 23$

$15 + 8 = 23$

$23 = 23$

Thus Solution Set $= \{\ 3\ \}$

(ii) $\mathbf{\frac{3}{5}x - \frac{2}{3} = 2}$

Solution:

$\frac{3}{5}x - \frac{2}{3} = 2 \dots \dots equ(i)$

$$\frac{9x - 10}{15} = 2$$

Multiply 15 on B.S

$$\frac{9x - 10}{15} \times 15 = 2 \times 15$$

$9x - 10 = 30$

Ex # 7.1

Add 10 on B.S

$9x - 10 + 10 = 30 + 10$

$9x = 40$

Divide 9 on B.S

$$\frac{9x}{9} = \frac{40}{9}$$

$$x = \frac{40}{9}$$

Verification

Put $x = \frac{40}{9}$in equ (i)

$$\frac{3}{5} \times \frac{40}{9} - \frac{2}{3} = 2$$

$$\frac{1}{1} \times \frac{8}{3} - \frac{2}{3} = 2$$

$$\frac{8}{3} - \frac{2}{3} = 2$$

$$\frac{8 - 2}{3} = 2$$

$$\frac{6}{3} = 2$$

$2 = 2$

Thus Solution Set $= \left\{\frac{40}{9}\right\}$

(iii) $\mathbf{6x - 5 = 2x + 9}$

Solution:

$6x - 5 = 2x + 9 \dots \dots equ(i)$

Add 5 on B.S

$6x - 5 + 5 = 2x + 9 + 5$

$6x = 2x + 14$

Subtract $2x$ from B.S

$6x - 2x = 2x - 2x + 14$

$4x = 14$

Divide B.S by 4

$$\frac{4x}{4} = \frac{14}{4}$$

$$x = \frac{7}{2}$$

Verification

Put $x = \frac{7}{2}$ in equ (i)

$$6\left(\frac{7}{2}\right) - 5 = 2\left(\frac{7}{2}\right) + 9$$

$3(7) - 5 = 7 + 9$

$21 - 5 = 16$

$16 = 16$

Ex # 7.1

Thus Solution Set $= \left\{\frac{7}{2}\right\}$

(iv) $\boldsymbol{\frac{2}{x-1}=\frac{1}{x-2}}$

Solution:

$$\frac{2}{x-1}=\frac{1}{x-2} \dots\dots equ(i)$$

By Cross Multiplication

$$2(x-2)=1(x-1)$$

$$2x-4=x-1$$

Add 4 on B.S

$$2x-4+4=x-1+4$$

$$2x=x+3$$

Subtract x from B.S

$$2x-x=x-x+3$$

$$x=3$$

Verification

Put $x=3$ in equ (i)

$$\frac{2}{3-1}=\frac{1}{3-2}$$

$$\frac{2}{2}=\frac{1}{1}$$

$$1=1$$

Solution Set $=\{3\}$

(v) $\boldsymbol{\frac{1}{7x+13}=\frac{2}{9}}$

Solution:

$$\frac{1}{7x+13}=\frac{2}{9} \dots\dots equ(i)$$

By Cross Multiplication

$$1\times 9=2(7x+13)$$

$$9=14x+26$$

Subtract 26 from B.S

$$9-26=14x-26$$

$$-17=14x$$

Divide B.S by 14

$$\frac{-17}{14}=\frac{14x}{14}$$

$$\frac{-17}{14}=x$$

$$x=\frac{-17}{14}$$

Verification

Ex # 7.1

Put $x=\frac{-17}{14}$ in equ (i)

$$\frac{1}{7\left(\frac{-17}{14}\right)+13}=\frac{2}{9}$$

$$\frac{1}{\frac{-17}{2}+13}=\frac{2}{9}$$

$$\frac{1}{\frac{-17+26}{2}}=\frac{2}{9}$$

$$\frac{1}{\frac{9}{2}}=\frac{2}{9}$$

$$1\div\frac{9}{2}=\frac{2}{9}$$

$$1\times\frac{2}{9}=\frac{2}{9}$$

$$\frac{2}{9}=\frac{2}{9}$$

Solution Set $=\left\{\frac{-17}{14}\right\}$

(vi) $\boldsymbol{10(x-4)=4(2x-1)+5}$

Solution:

$$10(x-4)=4(2x-1)+5 \dots\dots equ(i)$$

$$10x-40=8x-4+5$$

$$10x-40=8x+1$$

$$10x-40=8x+1$$

Add 40 on B.S

$$10x-40+40=8x+1+40$$

$$10x=8x+41$$

Subtract $8x$ from B.S

$$10x-8x=8x-8x+41$$

$$2x=41$$

Divide B.S by 2

$$\frac{2x}{2}=\frac{41}{2}$$

$$x=\left\{\frac{41}{2}\right\}$$

Verification

Put $x=\frac{41}{2}$ in equ (i)

$$10\left(\frac{41}{2}-4\right)=4\left(2\times\frac{41}{2}-1\right)+5$$

$$10\left(\frac{41-8}{2}\right)=4(41-1)+5$$

$$10\left(\frac{33}{2}\right)=4(40)+5$$

Ex # 7.1

$5(33) = 160 + 5$

$165 = 165$

Solution Set $= \frac{41}{2}$

Q2: Awais thought of a number, add 3 with it. Then he doubled the sum. He got 40. What was the original number?

Solution:

Let the number $= x$

As the given condition is defined as

Add 3 and double the sum got 40

So, we get

$2(x+3) = 40$

Divide B.S by 2

$\frac{2(x+3)}{2} = \frac{40}{2}$

$x + 3 = 20$

Subtract 3 from B.S

$x + 3 - 3 = 20 - 3$

$x = 17$

Thus, the original number $= 17$

Q3: The sum of two numbers is -4 and their difference is 6. What are the numbers?

Solution:

Let the two numbers are x and y

According to first condition

The sum of two numbers is -4

So,

$x + y = -4 \dots\dots equ(i)$

According to second condition

The difference of two numbers is 6

So,

$x - y = 6 \dots\dots equ(ii)$

Now add equ(i) and equ (ii)

$x + y + x - y = -4 + 6$

$x + x + y - y = 2$

$2x = 2$

Divide B.S by 2

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

Put $x = 1$ in equ (i)

$1 + y = -4$

Ex # 7.1

Subtract 1 from B.S

$1 - 1 + y = -4 - 1$

$y = -5$

Thus the two numbers are 1 and -5

Q4: The sum of three consecutive odd integers is 81. Find the numbers.

Solution:

As the difference is 2 between two consecutive odd integers

Let first odd integer $= x$

Second odd integer $= x + 2$

And third odd integer $= x + 4$

According to given condition

The sum of three consecutive odd integers is 81

So,

$x + x + 2 + x + 4 = 81$

$x + x + x + 2 + 4 = 81$

$3x + 6 = 81$

Subtract 6 from B.S

$3x + 6 - 6 = 81 - 6$

$3x = 75$

Divide B.S by 3

$\frac{3x}{3} = \frac{75}{3}$

$x = 25$

Let first odd integer $= x = 25$

Second odd integer $= x + 2$

$= 25 + 2$

$= 27$

And third odd integer $= x + 4$

$= 25 + 4$

$= 29$

So the consecutive odd integers are 25, 27 and 29

Chapter # 7

Ex # 7.1

Q5: **A man is 41 year old and his son is 9 year old. In how many years will the father be three times as old as the son?**

Solution:

$let\ father's\ age = 41\ years$

$and\ son'sage = 9\ years$

$Let\ the\ required\ years = x$

$So\ after\ x\ years$

$Father's\ age = 41 + x$

$Son's\ age = 9 + x$

According to given condition

$Age\ of\ father = 3(Age\ of\ son)$

$41 + x = 3(9 + x)$

$41 + x = 27 + 3x$

$41 - 27 = 3x - x$

$14 = 2x$

$2x = 14$

Divide B.S by 2

$$\frac{2x}{2} = \frac{14}{2}$$

$x = 7$

So the required number of years=7

Thus after 7 years father's age will be three times as his son

Q6: **The tens digit of a certain two – digitnumber exceeds the unit digit by 4 and is 1 less than twice the ones digit. Find the number.**

Solution:

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ two\ digit\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

According to given conditions

Tens digit exceeds the unit digit by 1

So,

$Tens\ digit = Ones\ digit + 4$

$y = x + 4 \dots \dots equ(i)$

And

Tens digit is 1 less than twice the ones digits

So,

$Tens\ digit = twice\ the\ one\ digit - 1$

$y = 2x - 1 \dots \dots equ(ii)$

Ex # 7.1

Compare equ (i) and (ii), we get

$x + 4 = 2x - 1$

$4 + 1 = 2x - x$

$5 = x$

$x = 5$

Put $x = 5$in equ (i)

$y = 5 + 4$

$y = 9$

$Thus\ the\ two\ digit = 10y + x$

$= 10(9) + 5$

$= 90 + 5$

$= 95$

Q7: **The sum of two digits is 10. It the place of digits are changed then the new number is decreased by 18. Find the numbers.**

Solution:

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ the\ original\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

If place of digits are interchanged

New number $= 10 \times x + 1 \times y$

$= 10x + y$

According to given conditions

Sum of digits is 10

So,

$x + y = 10 \dots \dots equ(i)$

And

$New\ number = Original\ number - 18$

$10x + y = 10y + x - 18$

$10x - x = 10y - y - 18$

$9x = 9y - 18$

$9x = 9(y - 2)$

Divide B.S by 9

$$\frac{9x}{9} = \frac{9(y-2)}{9}$$

$x = y - 2 \dots \dots equ(ii)$

Put $x = y - 2$in equ (i)

$y - 2 + y = 10$

Add 2 on B.S

$y - 2 + 2 + y = 10 + 2$

$y + y = 12$

$2y = 12$

Ex # 7.1

Divide B.S by 2

$$\frac{2y}{2}=\frac{12}{2}$$

$y=6$

Put $y=6$ in equ (ii)

$x=6-2$

$x=4$

$$\begin{aligned}As\ the\ Original\ number &= 10y+x\\ &= 10(6)+4\\ &= 60+4\\ &= 64\end{aligned}$$

Q8: **It the breadth of the room is one fourth of its length and the perimeter of the room is 20m. Find length and breadth of the room.**

Solution:

$Let\ length\ of\ room = x\ m$

As breadth is one fourth of its length

$Then\ breadth\ of\ room = \frac{x}{4}\ m$

As Perimeter of room=20 m

As we know that

$$P=2(l+2)$$

Put the values

$$20=2\left(x+\frac{x}{4}\right)$$

$$20=2\left(\frac{4x+x}{4}\right)$$

$$20=\frac{5x}{2}$$

$Multiply\ B.S\ by\ \frac{2}{5}$

$$20\times\frac{2}{5}=\frac{5x}{2}\times\frac{2}{5}$$

$4\times 2=x$

$8=x$

$x=8$

Thus

$Let\ length\ of\ room = x\ m = 8m$

$$\begin{aligned}breadth\ of\ room &= \frac{x}{4}\ m\\ &= \frac{8}{4}\ m\\ &= 2\ m\end{aligned}$$

Ex # 7.2

Radical equation

An equation in which the variable occurs under a radical is called radical equation.

Note:

The radicand should be a variable (unknown).

$\sqrt{x}+5=9$ is a radical equation but $2x+\sqrt{5}=9$ is not a radical equation.

The radical equation will be considered as positive numbers.

$\sqrt{x+6}=-11$ has no real solution and is not true for any value of x.

Example # 5

$\boldsymbol{Solve\ \sqrt{2x}+5=9}$

Solution:

$\sqrt{2x}+5=9\ \dots\dots equ(i)$

Subtract 5 from B.S

$\sqrt{2x}+5-5=9-5$

$\sqrt{2x}=4$

Taking square on B.S

$$\left(\sqrt{2x}\right)^2=(4)^2$$

$2x=16$

Divide B.S by 2

$$\frac{2x}{2}=\frac{16}{2}$$

$x=8$

Verification

Put $x=8$ in equ (i)

$\sqrt{2(8)}+5=9$

$\sqrt{16}+5=9$

$4+5=9$

$9=9$

Thus Solution Set $=\{8\}$

Example # 7

$\boldsymbol{\sqrt{3x-2}=\sqrt{5x+4}}$

Solution:

$\sqrt{3x-2}=\sqrt{5x+4}$

$\sqrt{3x-2}=\sqrt{5x+4}\ \dots\dots equ(i)$

Take square root on B.S

$$\left(\sqrt{3x-2}\right)^2=\left(\sqrt{5x+4}\right)^2$$

$3x-2=5x+4$

Subtract $5x$ from B.S

$3x-5x-2=5x-5x+4$

$-2x-2=4$

Add 2 on B.S

$-2x-2+2=4+2$

$-2x=6$

Divide B.S by -2

$\frac{-2x}{-2}=\frac{6}{-2}$

$x=-3$

Verification

Put $x=-3$ in equ (i)

$\sqrt{3(-3)-2}=\sqrt{5(-3)+4}$

$\sqrt{-9-2}=\sqrt{-15+4}$

$\sqrt{-11}=\sqrt{-11}$

Thus Solution Set $=\{-3\}$

Example # 8

$\boldsymbol{\sqrt{3x+2}+6=2}$

Solution:

$\sqrt{3x+2}+6=2 \ldots\ldots equ(i)$

Subtract 6 from B.S

$\sqrt{3x+2}+6-6=2-6$

$\sqrt{3x+2}=-4$

Taking square on B.S

$\left(\sqrt{3x+2}\right)^2=(-4)^2$

$3x+2=16$

Subtract 2 from B.S

$3x+2-2=16-2$

$3x=14$

Divide B.S by 3

$\frac{3x}{3}=\frac{14}{3}$

$x=\frac{14}{3}$

Solution Set $=\{\ \}$

Verification

Put $x=\frac{14}{3}$ in equ (i)

$\sqrt{3\left(\frac{14}{3}\right)+2}+6=2$

$\sqrt{14+2}+6=2$

$\sqrt{16}+6=2$

$4+6=2$

$10=2$

Hence

$10\neq 2$

Thus the given equation has no solution.

Solution Set $=\{\ \}$

Exercise # 7.2

Page # 180

Q: Solve the following radical equation.

1. $\boldsymbol{2\sqrt{a}-3=7}$

Solution:

$2\sqrt{a}-3=7 \ldots\ldots equ(i)$

Add 3 on B.S

$2\sqrt{a}-3+3=7+3$

$2\sqrt{a}=10$

Divide B.S by 2

$\frac{2\sqrt{a}}{2}=\frac{10}{2}$

$\sqrt{a}=5$

Taking square on B.S

$\left(\sqrt{a}\right)^2=(5)^2$

$a=25$

Verification

Put $a=25$ in equ (i)

$2\sqrt{25}-3=7$

$2(5)-3=7$

$10-3=7$

$7=7$

Thus Solution Set $=\{25\}$

2. $\boldsymbol{8+3\sqrt{b}=20}$

Solution:

$8+3\sqrt{b}=20 \ldots\ldots equ(i)$

Subtract 8 from B.S

$8-8+3\sqrt{b}=20-8$

$3\sqrt{b}=12$

Divide B.S by 3

$\frac{3\sqrt{b}}{3}=\frac{12}{3}$

$\sqrt{b}=4$

Taking square on B.S

$\left(\sqrt{b}\right)^2 = (4)^2$

$b = 16$

Verification

Put $b = 16$ in equ (i)

$8 + 3\sqrt{16} = 20$

$8 + 3(4) = 20$

$8 + 12 = 20$

$20 = 20$

Thus Solution Set $= \{16\}$

3. $\mathbf{7 - \sqrt{2b} = 3}$

Solution:

$7 - \sqrt{2b} = 3 \dots\dots equ(i)$

Subtract 7 from B.S

$7 - 7 - \sqrt{2b} = 3 - 7$

$-\sqrt{2b} = -4$

$\sqrt{2b} = 4$

Taking square on B.S

$\left(\sqrt{2b}\right)^2 = (4)^2$

$2b = 16$

Divide B.S by 2

$\frac{2b}{2} = \frac{16}{2}$

$b = 8$

Verification

Put $b = 8$ in equ (i)

$7 - \sqrt{2(8)} = 3$

$7 - \sqrt{16} = 3$

$7 - 4 = 3$

$3 = 3$

Thus Solution Set $= \{8\}$

4. $\mathbf{\sqrt{r} - 5 = \sqrt{r} + 9}$

Solution:

$8\sqrt{r} - 5 = \sqrt{r} + 9 \dots\dots equ(i)$

Add 5 on B.S

$8\sqrt{r} - 5 + 5 = \sqrt{r} + 9 + 5$

$8\sqrt{r} = \sqrt{r} + 14$

Subtract $\sqrt{r}$ from B.S

$8\sqrt{r} - \sqrt{r} = \sqrt{r} - \sqrt{r} + 14$

$7\sqrt{r} = 14$

Divide B.S by 7

$\frac{7\sqrt{r}}{7} = \frac{14}{7}$

$\sqrt{r} = 2$

Taking square on B.S

$\left(\sqrt{r}\right)^2 = (2)^2$

$r = 4$

Verification

Put r $= 4$ in equ (i)

$7\sqrt{4} - 5 = \sqrt{4} + 9$

$7(2) - 5 = 2 + 9$

$14 - 5 = 11$

$11 = 11$

Thus Solution Set $= \{4\}$

5. $\mathbf{20 - 3\sqrt{t} = \sqrt{t} - 4}$

Solution:

$20 - 3\sqrt{t} = \sqrt{t} - 4 \dots\dots equ(i)$

Subtract 20 from B.S

$20 - 20 - 3\sqrt{t} = \sqrt{t} - 4 - 20$

$-3\sqrt{t} = \sqrt{t} - 24$

Subtract $\sqrt{t}$ from B.S

$-3\sqrt{t} - \sqrt{t} = \sqrt{t} - \sqrt{t} - 24$

$-4\sqrt{t} = -24$

$4\sqrt{t} = 24$

Divide B.S by 4

$\frac{4\sqrt{t}}{4} = \frac{24}{4}$

$\sqrt{t} = 6$

Taking square on B.S

$\left(\sqrt{t}\right)^2 = (6)^2$

$t = 36$

Verification

Put $t = 36$ in equ (i)

$20 - 3\sqrt{36} = \sqrt{36} - 4$

$20 - 3(6) = 6 - 4$

$20 - 18 = 2$

$2 = 2$

Thus Solution Set $= \{36\}$

Ex # 7.2

6. $\mathbf{2\sqrt{5x} - 3 = 7}$

Solution:

$2\sqrt{5x} - 3 = 7 \ldots \ldots equ(i)$

Add 3 on B.S

$2\sqrt{5x} - 3 + 3 = 7 + 3$

$2\sqrt{5x} = 10$

Divide B.S by 2

$\frac{2\sqrt{5x}}{2} = \frac{10}{2}$

$\sqrt{5x} = 5$

Taking square on B.S

$\left(\sqrt{5x}\right)^2 = (5)^2$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

Verification

Put $x = 5$ in equ (i)

$2\sqrt{5(5)} - 3 = 7$

$2\sqrt{25} - 3 = 7$

$2(5) - 3 = 7$

$10 - 3 = 7$

$7 = 7$

Thus Solution Set $= \{5\}$

7. $\mathbf{\sqrt{2x - 7} + 8 = 11}$

Solution:

$\sqrt{2x - 7} + 8 = 11 \ldots \ldots equ(i)$

Subtract 8 from B.S

$\sqrt{2x - 7} + 8 - 8 = 11 - 8$

$\sqrt{2x - 7} = 3$

Taking square on B.S

$\left(\sqrt{2x - 7}\right)^2 = (3)^2$

$2x - 7 = 9$

Add 7 on B.S

$2x - 7 + 7 = 9 + 7$

$2x = 16$

Divide B.S by 2

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

Ex # 7.2

Verification

Put $x = 8$ in equ (i)

$\sqrt{2(8) - 7} + 8 = 11$

$\sqrt{16 - 7} + 8 = 11$

$\sqrt{9} + 8 = 11$

$3 + 8 = 11$

$11 = 11$

Thus Solution Set $= \{8\}$

8. $\mathbf{22 = 17 + \sqrt{40 - 3y}}$

Solution:

$22 = 17 + \sqrt{40 - 3y} \ldots \ldots equ(i)$

Subtract 17 from B.S

$22 - 17 = 17 - 17 + \sqrt{40 - 3y}$

$5 = \sqrt{40 - 3y}$

$\sqrt{40 - 3y} = 5$

Taking square on B.S

$\left(\sqrt{40 - 3y}\right)^2 = (5)^2$

$40 - 3y = 25$

Subtract 40 from B.S

$40 - 40 - 3y = 25 - 40$

$-3y = -15$

$3y = 15$

Divide B.S by 3

$\frac{3y}{3} = \frac{15}{3}$

$y = 5$

Verification

Put $x = 5$ in equ (i)

$22 = 17 + \sqrt{40 - 3(5)}$

$22 = 17 + \sqrt{40 - 15}$

$22 = 17 + \sqrt{25}$

$22 = 17 + 5$

$22 = 22$

Thus Solution Set $= \{5\}$

Chapter # 7

Ex # 7.3

Absolute value

The absolute value of a number is always be non–negative.

Example

$$|5| = 5$$

And also

$$|-5| = 5$$

Note:

It should be noted that $|x|$ can never be negative, that is $|x| \geq 0$

$|0| = 0$

Solution of Absolute value equation

To solve equations involving absolute value in one variable, we have to consider both the possible values of the variable.

Example

$$|x| = 2$$

Then there is two possibilities

$$x = 2$$

Or

$$x = -2$$

Example # 9

$$\mathbf{|x-1| = 7}$$

Solution:

$|x-1| = 7$

$There\ are\ two\ possibilities$

$Either$

$x - 1 = 7\ \ \ldots\ldots equ(i)$

or

$x - 1 = -7\ \ \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$x - 1 = 7$

Add 1 on B.S

$x - 1 + 1 = 7 + 1$

$x = 8$

$Now\ equ(ii) \Longrightarrow$

$x - 1 = -7$

Add 1 on B.S

$x - 1 + 1 = -7 + 1$

$x = -6$

Solution Set $= \{8, -6\}$

Ex # 7.3

Example # 10

$$\mathbf{|3x-5| + 7 = 11}$$

Solution:

$|3x - 5| + 7 = 11$

Subtract 7 from B.S

$|3x - 5| + 7 - 7 = 11 - 7$

$|3x - 5| = 4$

$There\ are\ two\ possibilities$

$Either$

$3x - 5 = 4\ \ \ldots\ldots equ(i)$

or

$3x - 5 = -4\ \ \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$3x - 5 = 4$

Add 5 on B.S

$3x - 5 + 5 = 4 + 5$

$3x = 9$

$Divide\ B.S\ by\ 3$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

$Now\ equ(ii) \Longrightarrow$

$3x - 5 = -4$

Add 5 on B.S

$3x - 5 + 5 = -4 + 5$

$3x = 1$

$Divide\ B.S\ by\ 3$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

Solution Set $= \left\{3, \frac{1}{3}\right\}$

Exercise # 7.3

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Q: 1. **Solve for x**

$$\mathbf{|x+3| = 5}$$

Solution:

$|x + 3| = 5$

$There\ are\ two\ possibilities$

$Either$

$x + 3 = 5\ \ \ldots\ldots equ(i)$

or

$x + 3 = -5\ \ \ldots\ldots equ(ii)$

Ex # 7.3

$Now\ equ(i) \Longrightarrow$

$x + 3 = 5$

Subtract 3 from B.S

$x + 3 - 3 - = 5 - 3$

$x = 2$

$Now\ equ(ii) \Longrightarrow$

$x + 3 = -5$

Subtract 3 from B.S

$x + 3 - 3 - = -5 - 3$

$x = -8$

Solution Set $= \{2, -8\}$

2. $\boldsymbol{|-5x + 1| = 6}$

Solution:

$|-5x + 1| = 6$

There are two possibilities

Either

$-5x + 1 = 6 \ \ldots\ldots equ(i)$

or

$-5x + 1 = -6 \ \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$-5x + 1 = 6$

Subtract 1 from B.S

$-5x + 1 - 1 = 6 - 1$

$-5x = 5$

$Divide\ B.S\ by\ -5$

$$\frac{-5x}{-5} = \frac{5}{-5}$$

$x = -1$

$Now\ equ(ii) \Longrightarrow$

$-5x + 1 = -6$

Subtract 1 from B.S

$-5x + 1 - 1 = -6 - 1$

$-5x = -7$

$5x = 7$

$Divide\ B.S\ by\ 5$

$$\frac{5x}{5} = \frac{7}{5}$$

$$x = \frac{7}{5}$$

Solution Set $= \left\{-1, \frac{7}{5}\right\}$

Ex # 7.3

3. $\boldsymbol{\left|\frac{3}{4}x - 8\right| = 1}$

Solution:

$$\left|\frac{3}{4}x - 8\right| = 1$$

There are two possibilities

Either

$\frac{3}{4}x - 8 = 1 \ \ldots\ldots equ(i)$

or

$\frac{3}{4}x - 8 = -16 \ \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$\frac{3}{4}x - 8 = 1$

Add 8 on B.S

$\frac{3}{4}x - 8 + 8 = 1 + 8$

$\frac{3}{4}x = 9$

$Multiply\ B.S\ by\ \frac{4}{3}$

$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times 9$

$x = 4 \times 3$

$x = 12$

$Now\ equ(ii) \Longrightarrow$

$\frac{3}{4}x - 8 = -1$

Add 8 on B.S

$\frac{3}{4}x - 8 + 8 = -1 + 8$

$\frac{3}{4}x = 7$

$Multiply\ B.S\ by\ \frac{4}{3}$

$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times 7$

$$x = \frac{28}{3}$$

Solution Set $= \left\{12, \frac{28}{3}\right\}$

Ex # 7.3

4. $\boldsymbol{|x-4|=3}$

Solution:

$|x-4|=3$

There are two possibilities

Either

$x-4=3 \quad \dots\dots equ(i)$

or

$x-4=-3 \quad \dots\dots equ(ii)$

Now $equ(i) \Longrightarrow$

$x-4=3$

Add 4 on B.S

$x-4+4=3+4$

$x=7$

Now $equ(ii) \Longrightarrow$

$x-4=-3$

Add 4 on B.S

$x-4+4=-3+4$

$x=1$

Solution Set $=\{7,1\}$

5. $\boldsymbol{|3x+4|=-2}$

Solution:

$|3x+4|=-2$

As there is no such a number whose absolute value is negative

Thus Solution Set $=\{\ \}$

6. $\boldsymbol{|2x-9|=0}$

Solution:

$|2x-9|=0$

$|x|=0 \Longrightarrow x=0$

So

$2x-9=0$

Add 9 on B.S

$2x-9+9=0+9$

$2x=9$

Divide B.S by 2

$$\frac{2x}{2}=\frac{9}{2}$$

$$x=\frac{9}{2}$$

Solution Set $=\left\{\frac{9}{2}\right\}$

Ex # 7.3

7. $\boldsymbol{\left|\frac{3x-2}{5}\right|=7}$

Solution:

$$\left|\frac{3x-2}{5}\right|=7$$

There are two possibilities

Either

$$\frac{3x-2}{5}=7 \quad \dots\dots equ(i)$$

or

$$\frac{3x-2}{5}=-7 \quad \dots\dots equ(ii)$$

Now $equ(i) \Longrightarrow$

$$\frac{3x-2}{5}=7$$

Multiply B.S by 5

$$5\times\frac{3x-2}{5}=5\times 7$$

$3x-2=35$

Add 2 on B.S

$3x-2+2=35+2$

$3x=37$

Divide B.S by 3

$$\frac{3x}{3}=\frac{37}{3}$$

$$x=\frac{37}{3}$$

Now $equ(ii) \Longrightarrow$

$$\frac{3x-2}{5}=-7$$

Multiply B.S by 5

$$5\times\frac{3x-2}{5}=-7\times 5$$

$3x-2=-35$

Add 2 on B.S

$3x-2+2=-35+2$

$3x=-33$

Divide B.S by 3

$$\frac{3x}{3}=\frac{-33}{3}$$

$x=-11$

Solution Set $=\left\{\frac{37}{3},-11\right\}$

Ex # 7.3

8. $\mathbf{4|5x-2|+3=11}$

Solution:

$4|5x-2|+3=11$

Subtract 3 from B.S

$4|5x-2|+3-3=11-3$

$4|5x-2|=8$

Divide B.S by 4

$\frac{4|5x-2|}{4}=\frac{8}{4}$

$|5x-2|=2$

There are two possibilities

Either

$5x-2=2\ \dots\dots equ(i)$

or

$5x-2=-2\ \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=2+2$

$5x=4$

Divide B.S by 5

$\frac{5x}{5}=\frac{4}{5}$

$x=\frac{4}{5}$

$Now\ equ(ii) \Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=-2+2$

$5x=0$

Divide B.S by 5

$\frac{5x}{5}=\frac{0}{5}$

$x=0$

Solution Set $=\left\{\frac{4}{5},0\right\}$

9. $\mathbf{\frac{2}{5}|4x-3|-9=-1}$

Solution:

$\frac{2}{5}|4x-3|-9=-1$

Add 9 on B.S

$\frac{2}{5}|4x-3|-9+9=-1+9$

Ex # 7.3

$\frac{2}{5}|4x-3|=8$

Multiply B.S by $\frac{5}{2}$

$\frac{5}{2}\times\frac{2}{5}|4x-3|=\frac{5}{2}\times 8$

$|4x-3|=5\times 4$

$|4x-3|=20$

There are two possibilities

Either

$4x-3=20\ \dots\dots equ(i)$

or

$4x-3=-20\ \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$4x-3=20$

Add 3 on B.S

$4x-3+3=20+3$

$4x=23$

Divide B.S by 4

$\frac{4x}{4}=\frac{23}{4}$

$x=\frac{23}{4}$

$Now\ equ(ii) \Longrightarrow$

$4x-3=-20$

Add 3 on B.S

$4x-3+3=-20+3$

$4x=-17$

Divide B.S by 4

$\frac{4x}{4}=\frac{-17}{4}$

$x=\frac{-17}{4}$

Solution Set $=\left\{\frac{23}{4},\frac{-17}{4}\right\}$

Chapter # 7

Ex # 7.4

Linear Inequality

Inequality

The relation which compares two real numbers e.g. x and y but $x \neq y$.

Following symbols of inequality as under:

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

We have the following possibilities

$x < y$ *means that x less than y*

$x > y$ *means that x greater than y*

$x \leq y$ *means that x less than or equal to y*

$x \geq y$ *means that x is grater than or equal to y*

Solution of Linear Inequalities

The set of all possible values of the variable which makes the inequality a true statement is called solution set of the inequality.

It is simple to represent the solution of an inequality with the help on real number line.

Real Number Line

A line whose points are represented by real number is called real number line.

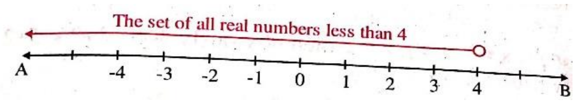
Geometrical representation with examples

Example: $x < 4$

$x < 4$, it means that all real numbers less than 4.

Geometrically all real numbers lying to the left of 4 but 4 is not included.

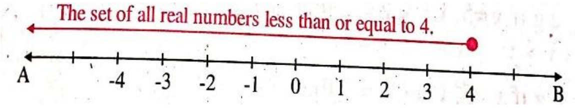
This is represented by using hollow circle around 4.



Example: $x \leq 4$

$x \leq 4$, it means that all real numbers less than or equal to 4. Geometrically all real numbers lying to the left of 4 and also including 4.

This is represented by using thick, filled or solid circle around 4.



Ex # 7.4

Example # 11

Show $-2 < x < 5$ on a number line.

Solution:

$$-2 < x < 5$$

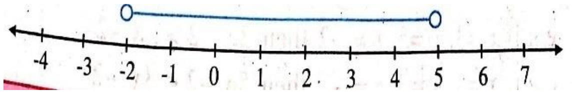
$-2 < x < 5$ means the set of real numbers which are greater than -2 but less than 5.

$-2 < x < 5$ means the set of real numbers which are between -2 and 5

Geometrical $-2 < x < 5$ means the set of real numbers lying to the right of -2 and left to 5.

Note:

Here -2 and 5 are not included.



Properties of Inequality of Real Numbers

Trichotomy Property

Trichotomy property means when comparing two numbers, one of the following must be true:

(a) $a = b$

(b) $a < b$

(c) $a > b$

Examples:

(i) $5 = 5$

(ii) $3 < 5$

(iii) $3 > 5$

Transitive Property

(a) If $a > b$ and $b > c$ then $a > c$

Example:

If $7 > 5$ and $5 > 3$ then $\mathbf{7} > 3$

(b) If $a < b$ and $b < c$ then $a < c$

Example:

If $3 < 5$ and $5 < 7$ then $\mathbf{3} < 7$

Additive Property

(a) If $a < b$ then $a + c < b + c$

(b) If $a < b$ then $a - c < b - c$

Examples:

(i) $3 < 5$ then $3 + 2 < 5 + 2$

(ii) $3 < 5$ then $3 - 2 < 5 - 2$

(iii) $\boldsymbol{x - 3 > 5}$

Add 3 on B.S

$x - 3 + 3 = 5 + 3$

$x = 8$

Ex # 7.4

(c) If $a > b$ then $a + c > b + c$

(d) If $a > b$ then $a - c > b - c$

Example:

(i) $5 > 3$ then $5 + 2 > 3 + 2$

(ii) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(iii) $\boldsymbol{x + 3 > 5}$

Subtract 3 from B.S

$x + 3 - 3 = 5 - 3$

$x = 2$

Multiplicative Property

1. When $c > 0$:

(a) If $a < b$ then $ac < bc$

(b) If $a > b$ then $ac > bc$

Example:

(i) $5 > 3$ then $5 \times 2 > 3 \times 2$

(ii) $\boldsymbol{\frac{x}{3} > 5}$

Multiply B.S by 3

$\frac{x}{3} \times 3 > 5 \times 3$

$x > 15$

(iii) $\boldsymbol{2x > 24}$

$\frac{2x}{2} > \frac{24}{2}$

Divide B.S by 2

$x > 12$

2. When $c < 0$:

(a) If $a < b$ then $ac > bc$

(b) If $a > b$ then $ac < bc$

Example:

(i) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

(ii) $\frac{x}{-3} < 5$

Multiply B.S by -3

$\frac{x}{-3} \times -3 > 5 \times -3$

$x > -15$

Example # 12

Write the names of properties used in the following statements.

$21 < 31 \quad \Rightarrow \quad 31 < 41$

$21 < 31 \quad \Rightarrow \quad 21 + 10 < 31 + 10$

Hence $21 < 31 \quad \Rightarrow \quad 31 < 41$

Additive Property

Ex # 7.4

$\boldsymbol{15 > 8 \quad \Rightarrow \quad 22 > 15}$

Solution:

$15 > 8 \quad \Rightarrow \quad 15 + 7 > 8 + 7$

Hence $15 > 8 \quad \Rightarrow \quad 22 > 15$

Additive Property

$\boldsymbol{10 < 20 \quad \Rightarrow \quad 30 < 60}$

Solution:

$10 < 20 \quad \Rightarrow \quad 10 \times 3 < 20 \times 3$

Hence $10 < 20 \quad \Rightarrow \quad 30 < 60$

Multiplicative Property

$\boldsymbol{-12 > -15 \quad \Rightarrow \quad 24 < 30}$

Solution:

$-12 > -15 \quad \Rightarrow \quad -12 \times -2 < -15 \times -2$

Hence $-12 > -15 \quad \Rightarrow \quad 24 < 30$

Multiplicative Property

If $x > 4$ and $4 > z$ then $x > z$

Solution:

$x > 4$ and $4 > z \Rightarrow x > z$

Transitive Property

Solution of Linear Inequalities

Linear inequalities are solved in almost the same way as linear equations.

Principles in Inequalities

(i) If $a > b$, then

$a + c > b + c\,,\; a - c > b - c\,,\; a - b > 0$

(ii) If $a > b$ and $k > 0$, then

$ka > kb \;\; and \;\; \frac{a}{k} > \frac{b}{k}$

(iii) If $a > b$ and $k < 0$, then

$ka < kb \;\; and \;\; \frac{a}{k} < \frac{b}{k}$

Example # 13

You are checking a bag at an airport. Bags can weigh no more than 50 Kgs. Your bag weighs 16.8 kg. Find the possible weight w (in Kg) that you can add to the bag.

Solution:

$Bag's\ weight + weight\ you\ can\ add \leq weight\ limit$

$16.8 + W \leq 50$

Subtract 16.8 from B.S

$16.8 - 16.8 + W \leq 50 - 16.8$

$W \leq 33.2$

So we can add upto 33.2 Kg

Chapter # 7

Ex # 7.4

<u>Example # 14</u> (i)

Solve the inequality $\boldsymbol{2\left(\frac{x}{4}+1\right)<\frac{3}{2}}$

where x is a natural number

<u>Solution</u>:

$$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$$

$$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$$

$$\frac{x+4}{2}<\frac{3}{2}$$

Multiply B.S by 2

$$2\times\frac{x+4}{2}<2\times\frac{3}{2}$$

$$x+4<3$$

Subtract 4 from B.S

$$x+4-4<3-4$$

$$x<-1$$

As natural number cannot be less than $-1,$

then it has no solution

Thus, Solution Set $=\{\ \}$

<u>Example # 14</u> (ii)

Solve the inequality $\boldsymbol{2\left(\frac{x}{4}+1\right)<\frac{3}{2}}$

where x is a real number

<u>Solution</u>:

$$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$$

$$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$$

$$\frac{x+4}{2}<\frac{3}{2}$$

Multiply B.S by 2

$$2\times\frac{x+4}{2}<2\times\frac{3}{2}$$

$$x+4<3$$

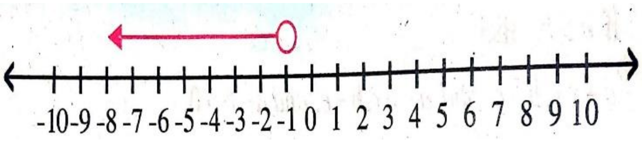
Subtract 4 from B.S

$$x+4-4<3-4$$

$$x<-1$$

Thus it consists of all real numbers less than -1

Thus Solution Set $=\{x : x\,\epsilon\,R\wedge x<-1\}$



Ex # 7.4

<u>Example # 15</u> (i)

Solve the inequality $\boldsymbol{x-\frac{5}{7}\le\frac{15+2x}{7}}$

where x is a natural number

<u>Solution</u>:

$$x-\frac{5}{7}\le\frac{15+2x}{7}$$

$$\frac{7x-5}{7}\le\frac{15+2x}{7}$$

Multiply B.S by 7

$$7\times\frac{7x-5}{7}\le7\times\frac{15+2x}{7}$$

$$7x-5\le15+2x$$

Add 5 on B.S

$$7x-5+5\le15+5+2x$$

$$7x\le20+2x$$

Subtract $2x$ from B.S

$$7x-2x\le20+2x-2x$$

$$5x\le20$$

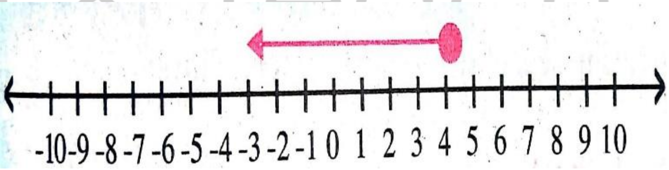
Divide B.S by 5

$$\frac{5x}{5}\le\frac{20}{5}$$

$$x\le4$$

As x is natural number and less than or equal to 4

Thus Solution Set $=\{1,2,3,4\}$



<u>Example # 15</u> (ii)

Solve the inequality $\boldsymbol{x-\frac{5}{7}\le\frac{15+2x}{7}}$

where x is a real number

<u>Solution</u>:

$$x-\frac{5}{7}\le\frac{15+2x}{7}$$

$$\frac{7x-5}{7}\le\frac{15+2x}{7}$$

Multiply B.S by 7

$$7\times\frac{7x-5}{7}\le7\times\frac{15+2x}{7}$$

$$7x-5\le15+2x$$

Add 5 on B.S

$$7x-5+5\le15+5+2x$$

$$7x\le20+2x$$

Ex # 7.4

Subtract $2x$ from B.S

$7x - 2x \leq 20 + 2x - 2x$

$5x \leq 20$

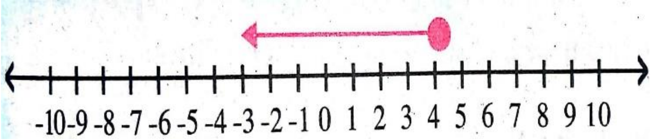
Divide B.S by 5

$\frac{5x}{5} \leq \frac{20}{5}$

$x \leq 4$

Thus it consists of all real numbers less than or equal to 4

Thus Solution Set $= \{x : x \in R \land x \leq 4\}$



Example # 16

Solve the inequality $\frac{x+3}{2} \leq \frac{x-5}{3}$ ***where*** $x \in R$

Solution:

$\frac{x+3}{2} \leq \frac{x-5}{3}$

Multiply B.S by 6

$6 \times \frac{x+3}{2} \leq 6 \times \frac{x-5}{3}$

$3(x+3) \leq 2(x-5)$

$3x + 9 \leq 2x - 10$

Subtract 9 from B.S

$3x + 9 - 9 \leq 2x - 10 - 9$

$3x \leq 2x - 19$

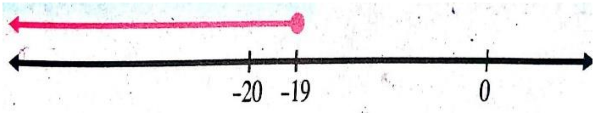
Subtract $2x$ from B.S

$3x - 2x \leq 2x - 2x - 19$

$x \leq -19$

Thus it consists of all real numbers less than or equal to -19

Thus Solution Set $= \{x : x \in R \land x \leq -19\}$



Exercise # 7.4

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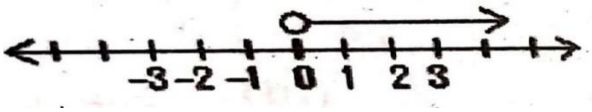
Q1: **Show the following inequalities on number line.**

(i) $x > 0$

Solution:

$x > 0$



(ii) $x < 0$

Solution:

$x < 0$

(iii) $\frac{x-3}{2} \leq -1$

Solution:

$\frac{x-3}{2} \leq -1$

Multiply B.S by 2

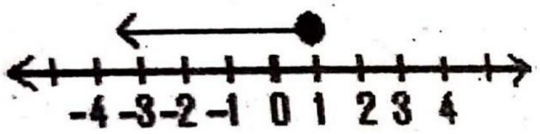
$2 \times \frac{x-3}{2} \leq -1 \times 2$

$x - 3 \leq -2$

Add 3 on B.S

$x - 3 + 3 \leq -2 + 3$

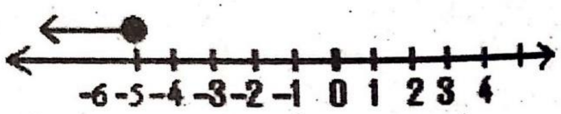
$x \leq 1$



(v) $x \leq -5$

Solution:

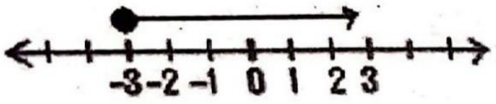
$x \leq -5$



$x \geq -3$

Solution:

$x \geq -3$



Ex # 7.4

(vi) $\mathbf{\frac{3x-2}{6} > \frac{5}{2}}$

Solution:

$\frac{3x-2}{6} > \frac{5}{2}$

Multiply B.S by 6

$6 \times \frac{3x-2}{6} > 6 \times \frac{5}{2}$

$3x - 2 > 3 \times 5$

$3x - 2 > 15$

Add 2 on B.S

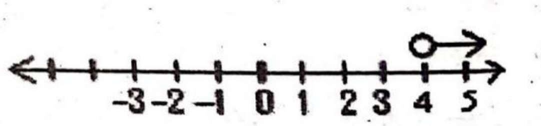
$3x - 2 + 2 > 15 + 2$

$3x > 17$

Divide B.S by 3

$\frac{3x}{3} > \frac{17}{3}$

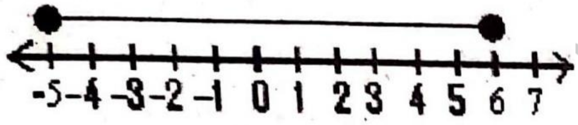
$x > 5.67$



(vii) $\mathbf{-5 \le x \le 6}$

Solution:

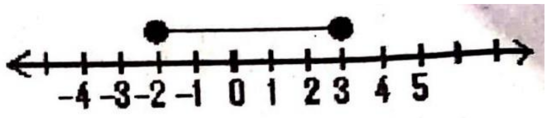
$-5 \le x \le 6$



(viii) $\mathbf{3 \ge x \ge -2}$

Solution:

$3 \ge x \ge -2$



(ix) $\mathbf{0 < \frac{x}{4} - 1 < \frac{1}{2}}$

Solution:

$0 < \frac{x}{4} - 1 < \frac{1}{2}$

Multiply by 4

$4 \times 0 < 4\left(\frac{x}{4} - 1\right) < 4 \times \frac{1}{2}$

$0 < 4 \times \frac{x}{4} - 4 \times 1 < 2 \times 1$

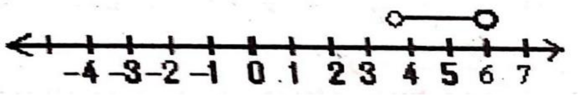
$0 < x - 4 < 2$

Ex # 7.4

Add 4

$0 + 4 < x - 4 + 4 < 2 + 4$

$4 < x < 6$



(x) $\mathbf{0 < \frac{x+3}{2} < \frac{3}{2}}$

Solution:

$0 < \frac{x+3}{2} < \frac{3}{2}$

Multiply by 2

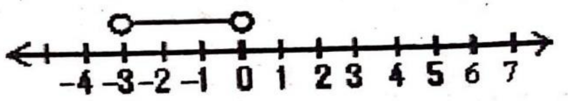
$2 \times 0 < 2 \times \frac{x+3}{2} < 2 \times \frac{3}{2}$

$0 < x + 3 < 3$

Subtract 3

$0 - 3 < x + 3 - 3 < 3 - 3$

$-3 < x < 0$



Q2: **Find the solution set of the following inequalities.**

(i) $\mathbf{7 - 2x \ge 1}$, $\quad \boldsymbol{x \in N}$

Solution:

$7 - 2x \ge 1$, $\quad x \in N$

Now

$7 - 2x \ge 1$

Subtract 7 from B.S

$7 - 7 - 2x \ge 1 - 7$

$-2x \ge -6$

Divide B.S by -2

$\frac{-2x}{-2} \le \frac{-6}{-2}$

$x \le 3$

As $x \in N$ *and* $x \le 3$

Thus Solution Set $= \{1, 2, 3\}$

(ii) $\mathbf{5x + 4} < 34$, $\quad x \in N$

Solution:

$5x + 4 < 34$, $\quad x \in N$

Now

$5x + 4 < 34$

Subtract 4 from B.S

$5x + 4 - 4 < 34 - 4$

$5x < 30$

Ex # 7.4

Divide B.S by 5

$$\frac{5x}{5} < \frac{30}{5}$$

$x < 6$

As $x \epsilon N$ *and* $x < 6$

Thus Solution Set $= \{1, 2, 3, 4, 5\}$

(iii) $\boldsymbol{\frac{8x+1}{2} < 2x - 1.5\,, \qquad x \epsilon R}$

Solution:

$$\frac{8x+1}{2} < 2x - 1.5\,, \qquad x \epsilon R$$

Now

$$\frac{8x+1}{2} < 2x - 1.5$$

Multiply B.S by 2

$$2 \times \frac{8x+1}{2} < 2(2x - 1.5)$$

$8x + 1 < 4x - 3$

Now

$8x - 4x < -3 - 1$

$4x < -4$

Divide B.S by 4

$$\frac{4x}{4} < \frac{-4}{4}$$

$x < -1$

As $x \epsilon R$ *and* $x < -1$

Thus Solution Set $= \{x : x \epsilon R \wedge x < -1\}$

(iv) $\boldsymbol{(4x + 3) \geq 23\,, \qquad x \epsilon \{1, 2, 3, 4, 5, 6\}}$

Solution:

$(4x + 3) \geq 23\,, \qquad x \epsilon \{1, 2, 3, 4, 5, 6\}$

Now

$4x + 3 \geq 23$

Subtract 3 from B.S

$4x + 3 - 3 \geq 23 - 3$

$4x \geq 20$

Divide B.S by 4

$$\frac{4x}{4} \geq \frac{20}{4}$$

$x \geq 5$

As $x \epsilon \{1, 2, 3, 4, 5, 6\}$ *and* $x \geq 5$

Thus Solution Set $= \{5, 6\}$

Ex # 7.4

(v) $\boldsymbol{5x + 1}$

$\boldsymbol{\geq 13 - x\,, \quad x \epsilon \{-2, -1, 0, 1, 2, 3, 4, 5\}}$

Solution:

$5x + 1 \geq 13 - x\,, \qquad x \epsilon \{-2, -1, 0, 1, 2, 3, 4, 5\}$

Now

$5x + 1 \geq 13 - x$

Now

$5x + x \geq 13 - 1$

$6x \geq 12$

Divide B.S by 6

$$\frac{6x}{6} \geq \frac{12}{6}$$

$x \geq 2$

As $x \epsilon \{-2, -1, 0, 1, 2, 3, 4, 5\}$ *and* $x \geq 2$

Thus Solution Set $= \{2, 3, 4, 5\}$

(vi) $\boldsymbol{\frac{2x+6}{2} \leq \frac{x-9}{3}\,, \qquad x \epsilon R}$

Solution:

$$\frac{2x+6}{2} \leq \frac{x-9}{3}\,, \qquad x \epsilon R$$

Now

$$\frac{2x+6}{2} \leq \frac{x-9}{3}$$

Multiply B.S by 6

$$6 \times \frac{2x+6}{2} \leq 6 \times \frac{x-9}{3}$$

$3(2x + 6) \leq 2(x - 9)$

$6x + 18 \leq 2x - 18$

Now

$6x - 2x \leq -18 - 18$

$4x \leq -36$

Divide B.S by 4

$$\frac{4x}{4} \leq \frac{-36}{4}$$

$x \leq -9$

As $x \epsilon R$ *and* $x \leq -9$

Thus Solution Set $= \{x : x \epsilon R \wedge x \leq -9\}$

(vii) $\boldsymbol{\frac{x-1}{3} \leq \frac{1-x}{2}\,, \qquad x \epsilon Z}$

Solution:

$$\frac{x-1}{3} \leq \frac{1-x}{2}\,, \qquad x \epsilon Z$$

Now

$$\frac{x-1}{3} \leq \frac{1-x}{2}$$

Multiply B.S by 6

$6 \times \frac{x-1}{3} \leq 6 \times \frac{1-x}{2}$

$2(x-1) \leq 3(1-x)$

$2x - 2 \leq 3 - 3x$

Now

$2x + 3x \leq 3 + 2$

$5x \leq 5$

Divide B.S by 5

$\frac{5x}{5} \leq \frac{5}{5}$

$x \leq 1$

$As \;\; x \,\epsilon\, Z \; and \; x \leq 1$

Thus Solution Set $= \{1, 0, -1, -2, -3 \dots \dots\}$

Q3: **Solve the following inequalities and plot the solution on the number line.**

(i) $\boldsymbol{\frac{x}{12} \leq \frac{1}{4}}$

Solution:

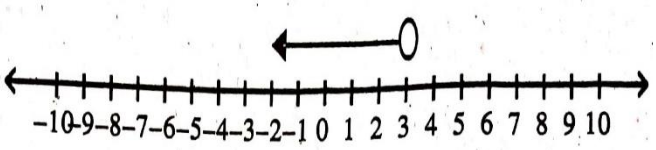
$\frac{x}{12} \leq \frac{1}{4}$

Multiply B.S by 12

$12 \times \frac{x}{12} \leq 12 \times \frac{1}{4}$

$x \leq 3 \times 1$

$x \leq 3$



(ii) $\boldsymbol{x + 7 \geq 2}$

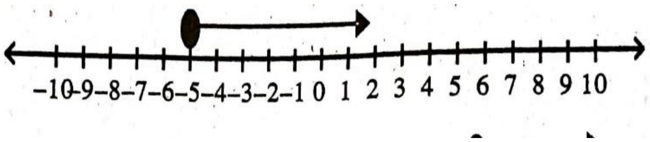
Solution:

$x + 7 \geq 2$

Subtract 7 from B.S

$x + 7 - 7 \geq 2 - 7$

$x \geq -5$



(iii) $\boldsymbol{3(x-2)} > 15$

Solution:

$3(x-2) > 15$

$3x - 6 > 15$

Add 6 on B.S

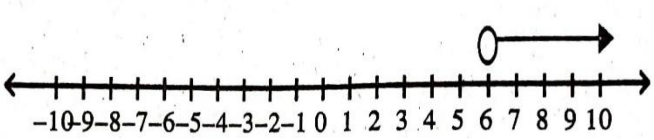
$3x - 6 + 6 > 15 + 6$

$3x > 21$

Divide B.S by 3

$\frac{3x}{3} > \frac{21}{3}$

$x > 7$



(iv) $\boldsymbol{\frac{1}{2} > \frac{x}{4}} > -2$

Solution:

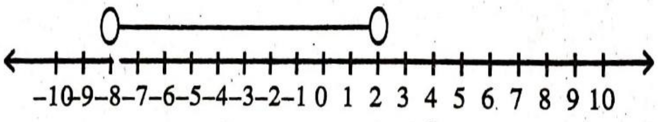
$\frac{1}{2} > \frac{x}{4} > -2$

Multiply by 4

$4 \times \frac{1}{2} > 4 \times \frac{x}{4} > -2 \times 4$

$2 \times 1 > x > -8$

$2 > x > -8$



(v) $\boldsymbol{2.5 \leq \frac{x}{2} + 1 \leq 4.5}$

Solution:

$2.5 \leq \frac{x}{2} + 1 \leq 4.5$

Multiply B.S by 2

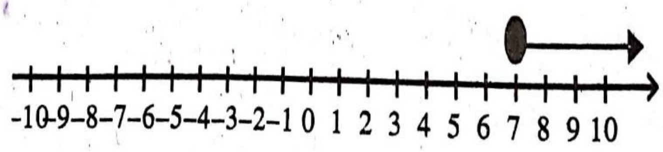
$2 \times 2.5 \leq 2\left(\frac{x}{2} + 1\right) \leq 2 \times 4.5$

$5 \leq x + 2 \leq 9$

Subtract 2 from them

$5 - 2 \leq x + 2 - 2 \leq 9 - 2$

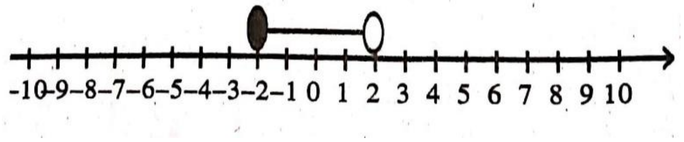
$3 \leq x \leq 7$



(vi) $-2 \le x < 2$

Solution:

$-2 \le x < 2$



Review Ex # 7

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Q2: **Solve the following equation for x**

(i) $5(3x+1) = 2(x-4)$

Solution:

$5(3x+1) = 2(x-4) \ldots\ldots equ(i)$

$15x + 5 = 2x - 8$

Subtract 5 from B.S

$15x + 5 - 5 = 2x - 8 - 5$

$15x = 2x - 13$

Subtract $2x$ from B.S

$15x - 2x = 2x - 2x - 13$

$13x = -13$

Divide B.S by 13

$\frac{13x}{13} = \frac{-13}{13}$

$x = -1$

Verification

Put $x = -1$ in equ (i)

$5(3(-1)+1) = 2(-1-4)$

$5(-3+1) = 2(-5)$

$5(-2) = -10$

$-10 = -10$

Solution Set $= \{-1\}$

(ii) $\frac{x-8}{3} + \frac{x-3}{2} = 0$

Solution:

$\frac{x-8}{3} + \frac{x-3}{2} = 0 \ldots\ldots equ(i)$

Multiply all terms by 6

$6 \times \frac{x-8}{3} + 6 \times \frac{x-3}{2} = 6 \times 0$

$2(x-8) + 3(x-3) = 0$

$2x - 16 + 3x - 9 = 0$

$2x + 3x - 16 - 9 = 0$

$5x - 25 = 0$

Add 25 on B.S

$5x - 25 + 25 = 0 + 25$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

Verification

Put $x = 5$ in equ (i)

$\frac{5-8}{3} + \frac{5-3}{2} = 0$

$\frac{-3}{3} + \frac{2}{2} = 0$

$-1 + 1 = 0$

$0 = 0$

Solution Set $= \{5\}$

(iii) $\sqrt{2(5x-1)} = \sqrt{2x+14}$

Solution:

$\sqrt{2(5x-1)} = \sqrt{2x+14}$

$\sqrt{2(5x-1)} = \sqrt{2x+14} \ldots\ldots equ(i)$

Take square root on B.S

$\left(\sqrt{2(5x-1)}\right)^2 = \left(\sqrt{2x+14}\right)^2$

$2(5x-1) = 2x + 14$

$10x - 2 = 2x + 14$

Now

$10x - 2x = 14 + 2$

$8x = 16$

Divide B.S by 4

$\frac{8\sqrt{x}}{8} = \frac{16}{8}$

$\sqrt{x} = 2$

Taking square on B.S

$\left(\sqrt{x}\right)^2 = (2)^2$

$x = 4$

Verification

Put $x = 2$ in equ (i)

$\sqrt{2(5(2)-1)} = \sqrt{2(2)+14}$

$\sqrt{2(10-1)} = \sqrt{4+14}$

$\sqrt{2(9)} = \sqrt{18}$

$\sqrt{18} = \sqrt{18}$

$\sqrt{9 \times 2} = \sqrt{9 \times 2}$

$3\sqrt{2} = 3\sqrt{2}$

Solution Set $= \{36\}$

Review Ex # 7

(iv) $|2x+7|=9$

Solution:

$|2x+7|=9$

There are two possibilities

Either

$2x+7=9 \quad \dots\dots equ(i)$

or

$2x+7=-9 \quad \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$2x+7=9$

Subtract 7 from B.S

$2x+7-7=9-7$

$2x=2$

Divide B.S by 2

$\frac{2x}{2}=\frac{2}{2}$

$x=1$

$Now\ equ(ii) \Longrightarrow$

$2x+7=-9$

Subtract 7 from B.S

$2x+7-7=-9-7$

$2x=-16$

Divide B.S by 2

$\frac{2x}{2}=\frac{-16}{2}$

$x=-8$

Solution Set $=\{1,-8\}$

Q3: **Solve the following inequalities and graph the solution on the number line.**

(i) $-1<\frac{x-3}{2}<0$

Solution:

$-1<\frac{x-3}{2}<0$

Multiply by 2

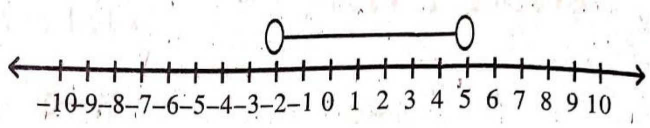
$-1\times 2<2\times\frac{x-3}{2}<2\times 0$

$-2<x-3<0$

Add 3

$-2+3<x-3+3<0+3$

$1<x<3$



Review Ex # 7

(ii) $-1<\frac{x-4}{5}<0$

Solution:

$-1<\frac{x-4}{5}<0$

Multiply by 5

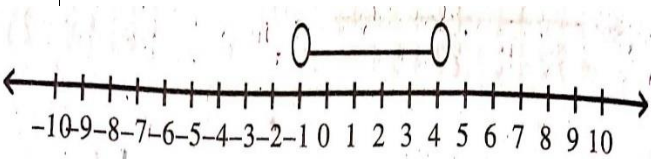
$-1\times 5<5\times\frac{x-4}{5}<5\times 0$

$-5<x-4<0$

Add 4

$-5+4<x-4+4<0+4$

$-1<x<4$



(iii) $7<-3x+1\le 13$

Solution:

$7<-3x+1\le 13$

Subtract 1

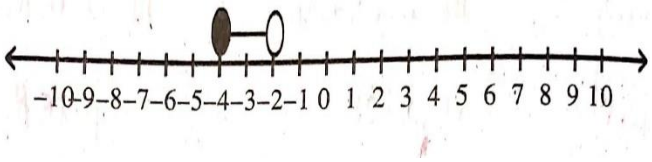
$7-1<-3x+1-1\le 13-1$

$6<-3x\le 12$

Divide B.S by 3

$\frac{6}{-3}>\frac{-3x}{-3}\ge\frac{12}{-3}$

$-2>x\le -4$



Review Ex # 7

Q4: **A father is 4 times older than his son. In 20 years, he will be twice as old as his son. What ages have they now?**

Solution:

Let the present age of son $= x$ years

So the present age of father $= 4x$ years

After twenty years

Age of son $= (x + 20) years$

and age of son $= (4x + 20) years$

According to condition

Age of father $= 2$(Age of son)

$4x + 20 = 2(x + 20)$

$4x + 20 = 2x + 40$

Now shift the variable and constant

$4x - 2x = 40 - 20$

$2x = 20$

Divide B.S by 2

$$\frac{2x}{2} = \frac{20}{2}$$

$x = 10$

Thus present age of son $= x = 10\ years$

And present age of father $= 4x\ years$

$= 4 \times 10\ years$

$= 40\ years$

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MATHEMATICS

Class 9th (KPK)

Chapter # 9 Introduction to Coordinate Geometry

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UNIT # 9

INTRODUCTION TO COORDINATE GEOMETRY

Ex # 9.1

Introduction

The relationship between algebra and geometry was given by a French Philosopher and Mathematician Rene Descartes in 1637 when his book *La Geometrie* was published.

Coordinate Geometry

The study of geometric properties of figures by the study of their equations is called coordinated or analytic geometry.

Coordinates

Coordinates are a set of values which helps to show the exact position of a point in the coordinate plane.

Coordinate plane

A coordinate plane is formed by intersection of two perpendicular lines known as x – axis and y – axis at origin. These two perpendicular lines is divided into four quadrants.

Distance between points on real line

Suppose we are given two distinct points a & b on the real line then

The directed distance from a to b is $b - a$

The directed distance from b to a is $a - b$

Note:

The distance between two points on the real line can never be negative.

The distance between a and b is $|a - b|$ or $|b - a|$

Or

The distance d between points x_1 & x_2 on the real line is given by $d = |x_2 - x_1| = (x_2 - x_1)^2$

Note:

The order of subtraction with x_1 & x_2 does not matter in finding the distance between them since

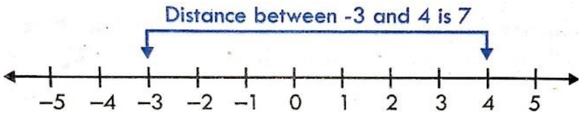
$|x_1 - x_2| = |x_2 - x_1|$ and $(x_1 - x_2)^2 = (x_2 - x_1)^2$

Example # 1

Determine the distance between -3 and 4 on the real line. What is the directed distance from -3 *to* 4 and from 4 *to* -3 ?

Ex # 9.1

Solution:



The distance between -3 *and* 4 is given by:

$|4 - (-3)| = |4 + 3| = |7| = 7$

Or

$|-3 - 4| = |-7| = 7$

The directed distance form -3 *to* 4 is

$4 - (-3) = 4 + 3 = 7$

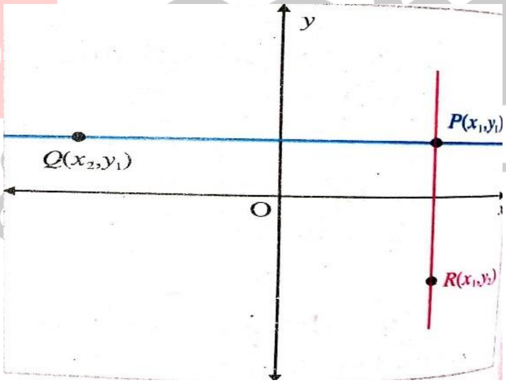
The directed distance from 4 *to* -3 is:

$-3 - 4 = -7 = 7$

As distance can never be negative

Distance between two points in a plane

Suppose two points on the same horizontal line or the same vertical line in the plane, then the distance between them is given by:



Distance of x – coordinates

Let the two points on $x - coordinates$ are $P(x_1, y_1)$ *and* $Q(x_2,\ y_2)$, the distance of $x - coordinates$ is $|x_2 - x_1|$

Distance of y – coordinates

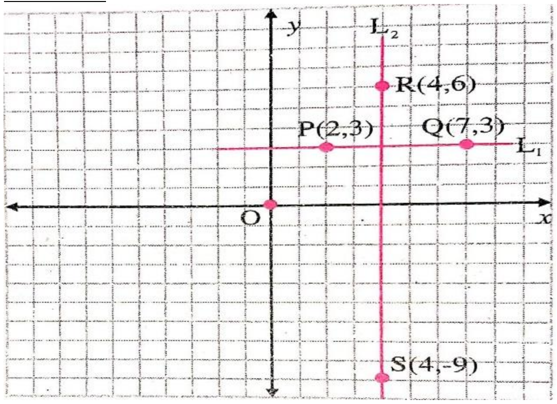
Let the two points on $y - coordinates$ are $P(x_1, y_1)$ *and* $R(x_1,\ y_2)$, the distance of $y - coordinates$ is $|y_2 - y_1|$

Ex # 9.1

Example # 2

Find the distance between (2, 3) & (7, 3) and the distance between (4, 6) & $(4, -9)$

Solution:



In the figure:

Let $\mathrm{P}(2,3)$ & $\mathrm{Q}(7,3)$ lie on the same horizantal line so the distance is:

$L_1 = |7-2|$

$L_1 = |5|$

$L_1 = 5$

And also

Let $\mathrm{R}(4,6)$ & $\mathrm{S}(4,-9)$ lie on the same vertical line so the distance is:

$L_2 = |6-(-9)|$

$L_2 = |6+9|$

$L_2 = |15|$

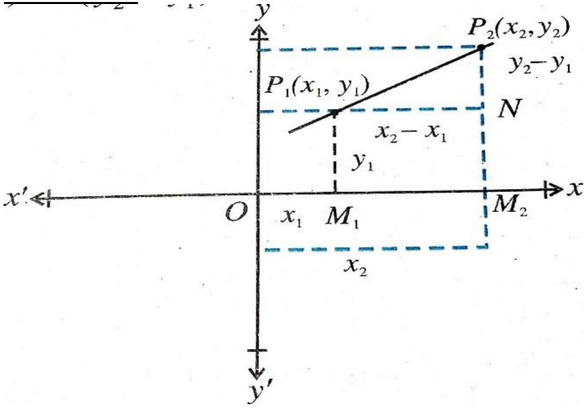
$L_2 = 15$

Distance formula

the distance d between two points $(x_1\,, y_1)$ and $(x_2\,, y_2)$ is

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Derivation



Ex # 9.1

In the given figure

The two points are $P_1(x_1\,, y_1)$ and $P_2(x_2\,, y_2)$

Let $\overline{OM_1} = x_1$ and $\overline{OM_2} = x_2$

Now

$\overline{M_1M_2} = \overline{OM_2} - \overline{OM_1}$

$\overline{M_1M_2} = x_2 - x_1$

$\overline{M_1M_2} = \overline{P_1N}$

$\overline{P_1N} = x_2 - x_1$

Let $\overline{M_1P_1} = \overline{M_2N} = y_1$ and $\overline{M_2P_2} = y_2$

Now

$\overline{NP_2} = \overline{M_2P_2} - \overline{M_2N}$

$\overline{NP_2} = y_2 - y_1$

As P_1NP_2 is a right-angled triangle,

So, by Pythagoras theorem

$|P_1P_2|^2 = |P_1N|^2 + |NP_2|^2$

$|P_1P_2|^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

Taking square root on B.S

$\sqrt{|P_1P_2|^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Example # 3

Find the distance between points $A(-5,1)$ and $B(3,1)$.

Solution:

$A(-5,1)$ and $B(3,1)$

Let $x_1 = -5,\; y_1 = 1$

And $x_2 = 3,\; y_2 = 1$

As distance formula is:

$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB| = \sqrt{\left(3-(-5)\right)^2 + (1-1)^2}$

$|AB| = \sqrt{(3+5)^2+(0)^2}$

$|AB| = \sqrt{(8)^2+0}$

$|AB| = \sqrt{64}$

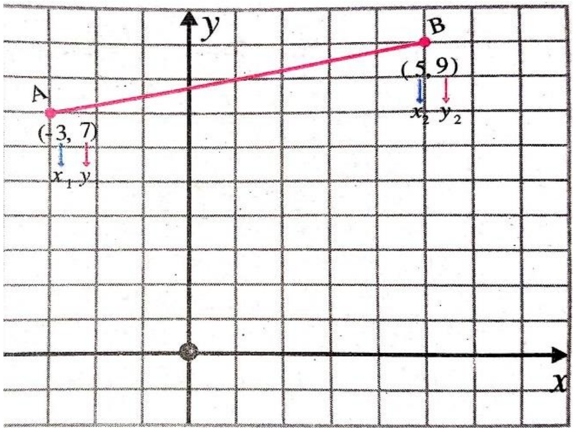
$|AB| = 8$

Ex # 9.1

Example # 4

Plot the points $(-3,7)$ & $(5,9)$ and find the distance between them.

Solution:



$(-3,7)$ & $(5,9)$

Let $x_1 = -3,\ y_1 = 7$

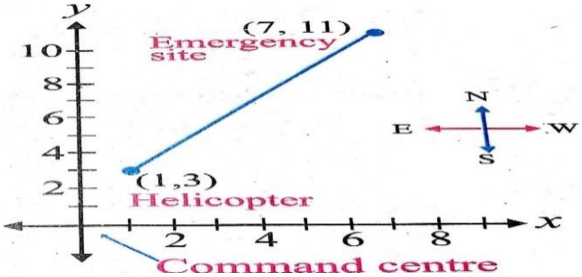
And $x_2 = 5,\ y_2 = 9$

As distance formula is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{\left(5 - (-3)\right)^2 + (9 - 7)^2}$$
$$d = \sqrt{(5+3)^2 + (2)^2}$$
$$d = \sqrt{(8)^2 + 4}$$
$$d = \sqrt{64 + 4}$$
$$d = \sqrt{68}$$
$$d = \sqrt{4 \times 17}$$
$$d = \sqrt{4} \times \sqrt{17}$$
$$d = 2\sqrt{17}$$

Example # 5

A helicopter pilot located 1 mile west and 3 miles north of the command centre must respond to an emergency located 7 miles west and 11 miles north of the centre. How far must the helicopter travel to get to the emergency site?



Ex # 9.1

Solution:

$As\ west\ direction\ represents\ x - axis$
$and\ nort\ direction\ represents\ y - axix$
$Let\ coordinates\ of\ command\ centre = O(0,0)$
$Coordinates\ of\ Helicopter = A(1,3)$
$Coordinates\ of\ emergency\ site = B(7,11)$

Let $x_1 = 1,\ y_1 = 3$

And $x_2 = 7,\ y_2 = 11$

As distance formula is:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(7-1)^2 + (11-3)^2}$$
$$|AB| = \sqrt{(6)^2 + (8)^2}$$
$$|AB| = \sqrt{36 + 64}$$
$$|AB| = \sqrt{100}$$
$$|AB| = 10$$

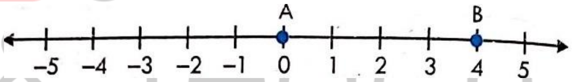
Thus, the helicopter must travel 10 miles to get the emergency site.

Ex # 9.1

Page # 216

Q1: Find the length of AB in the following figures.

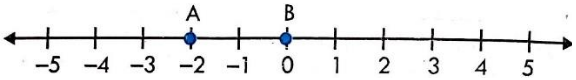
(i)



Solution:

$|AB| = |4 - 0|$
$|AB| = |4|$
$|AB| = 4$

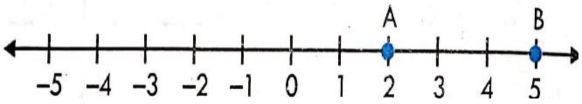
(ii)



Solution:

$|AB| = |0 - (-2)|$
$|AB| = |0 + 2|$
$|AB| = |2|$
$|AB| = 2$

(iii)

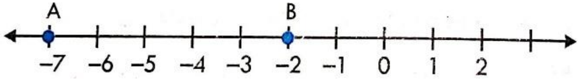


Solution:

$|AB| = |5 - 2|$
$|AB| = |3|$
$|AB| = 3$

Ex # 9.1

(iv)



Solution:

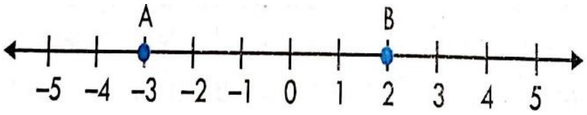
$|AB| = |-2-(-7)|$

$|AB| = |-2+7|$

$|AB| = |5|$

$|AB| = 5$

(v)



Solution:

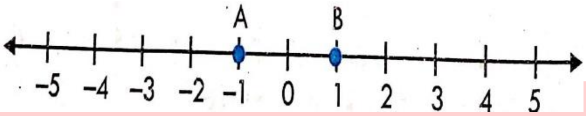
$|AB| = |2-(-3)|$

$|AB| = |2+3|$

$|AB| = |5|$

$|AB| = 5$

(vi)



Solution:

$|AB| = |1-(-1)|$

$|AB| = |1+1|$

$|AB| = |2|$

$|AB| = 2$

Q2: **Find distance between each pair of points.**

(i) $\mathbf{(1,1),(3,3)}$

Solution:

$(1,1),(3,3)$

Let $x_1 = 1,\ y_1 = 1$

And $x_2 = 3,\ y_2 = 3$

As distance formula is:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(3-1)^2+(3-1)^2}$

$d = \sqrt{(2)^2+(2)^2}$

$d = \sqrt{4+4}$

$d = \sqrt{8}$

$d = \sqrt{4\times 2}$

$d = \sqrt{4}\times\sqrt{2}$

$d = 2\sqrt{2}$

(ii) $\mathbf{(1,2),(4,5)}$

Solution:

$(1,2),(4,5)$

Let $x_1 = 1,\ y_1 = 2$

And $x_2 = 4,\ y_2 = 5$

Ex # 9.1

As distance formula is:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(4-1)^2+(5-2)^2}$

$d = \sqrt{(3)^2+(3)^2}$

$d = \sqrt{9+9}$

$d = \sqrt{18}$

$d = \sqrt{9\times 2}$

$d = \sqrt{9}\times\sqrt{2}$

$d = 3\sqrt{2}$

(iii) $\mathbf{(2,-2),(2,-3)}$

Solution:

$(2,-2),(2,-3)$

Let $x_1 = 2,\ y_1 = -2$

And $x_2 = 2,\ y_2 = -3$

As distance formula is:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(2-2)^2+\left(-3-(-2)\right)^2}$

$d = \sqrt{(0)^2+(-3+2)^2}$

$d = \sqrt{0+(-1)^2}$

$d = \sqrt{1}$

$d = 1$

(iv) $\mathbf{(3,-5),(5,-7)}$

Solution:

$(3,-5),(5,-7)$

Let $x_1 = 3,\ y_1 = -5$

And $x_2 = 5,\ y_2 = -7$

As distance formula is:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(5-3)^2+\left(-5-(-7)\right)^2}$

$d = \sqrt{(2)^2+(-5+7)^2}$

$d = \sqrt{4+(2)^2}$

$d = \sqrt{4+4}$

$d = \sqrt{8}$

$d = \sqrt{4\times 2}$

$d = \sqrt{4}\times\sqrt{2}$

$d = 2\sqrt{2}$

Ex # 9.1

Q3: **Given points $O(0,0), A(3,4), B(-5,12), C(15,-8), D(11,-3), E(-9,-4)$.**
Determine length of the following segments.

(i) $\overline{OA}$

Solution:

$\overline{OA}$

$O(0,0), A(3,4)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = 3,\ y_2 = 4$

As distance formula is:

$$|OA| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|OA| = \sqrt{(3-0)^2 + (4-0)^2}$$
$$|OA| = \sqrt{(3)^2 + (4)^2}$$
$$|OA| = \sqrt{9+16}$$
$$|OA| = \sqrt{25}$$
$$|OA| = 5$$

(ii) $\overline{OB}$

Solution:

$\overline{OB}$

$O(0,0), B(-5,12)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = -5,\ y_2 = 12$

As distance formula is:

$$|OB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|OB| = \sqrt{(-5-0)^2 + (12-0)^2}$$
$$|OB| = \sqrt{(-5)^2 + (12)^2}$$
$$|OB| = \sqrt{25+144}$$
$$|OB| = \sqrt{169}$$
$$|OB| = 13$$

(iii) $\overline{OC}$

Solution:

$\overline{OC}$

$O(0,0), C(15,-8)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = 15,\ y_2 = -8$

As distance formula is:

$$|OC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|OC| = \sqrt{(15-0)^2 + (-8-0)^2}$$
$$|OC| = \sqrt{(15)^2 + (-8)^2}$$

Ex # 9.1

$$|OC| = \sqrt{225+64}$$
$$|OC| = \sqrt{289}$$
$$|OC| = 17$$

(iv) $\overline{AD}$

Solution:

$\overline{AD}$

$A(3,4), D(11,-3)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = 11,\ y_2 = -3$

As distance formula is:

$$|AD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AD| = \sqrt{(11-3)^2 + (-3-4)^2}$$
$$|AD| = \sqrt{(8)^2 + (-7)^2}$$
$$|AD| = \sqrt{64+49}$$
$$|AD| = \sqrt{113}$$

(v) $\overline{AB}$

Solution:

$\overline{AB}$

$A(3,4), B(-5,12)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = -5,\ y_2 = 12$

As distance formula is:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(-5-3)^2 + (12-4)^2}$$
$$|AB| = \sqrt{(-8)^2 + (8)^2}$$
$$|AB| = \sqrt{64+64}$$
$$|AB| = \sqrt{128}$$
$$|AB| = \sqrt{64 \times 2}$$
$$|AB| = \sqrt{64} \times \sqrt{2}$$
$$|AB| = 8\sqrt{2}$$

(vi) $\overline{AC}$

Solution:

$\overline{AC}$

$A(3,4), C(15,-8)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = 15,\ y_2 = -8$

As distance formula is:

$$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex # 9.1

$|AC| = \sqrt{(15-3)^2 + (-8-4)^2}$

$|AC| = \sqrt{(12)^2 + (-12)^2}$

$|AC| = \sqrt{144 + 144}$

$|AC| = \sqrt{288}$

$|AC| = \sqrt{144 \times 2}$

$|AC| = \sqrt{144} \times \sqrt{2}$

$|AC| = 12\sqrt{2}$

(vii) $\overline{\boldsymbol{BE}}$

Solution:

$\overline{BE}$

$B(-5,12), E(-9,-4)$

Let $x_1 = -5,\ y_1 = 12$

And $x_2 = -9,\ y_2 = -4$

As distance formula is:

$|BE| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|BE| = \sqrt{(-9-(-5))^2 + (-4-12)^2}$

$|BE| = \sqrt{(-9+5)^2 + (-16)^2}$

$|BE| = \sqrt{(-4)^2 + 256}$

$|BE| = \sqrt{16 + 256}$

$|BE| = \sqrt{272}$

$|BE| = \sqrt{4 \times 68}$

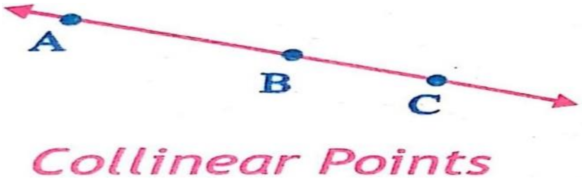
$|BE| = \sqrt{4} \times \sqrt{68}$

$|BE| = 2\sqrt{68}$

Ex # 9.2

Collinear points

Three or more points which lie on the same straight line are called collinear points.



Collinear Points

Non – collinear points

The set of points that are not lie on the same straight line is called non – collinear points.

Non-collinear Points

Ex # 9.2

Example # 6

Prove that the points

$A(5,-2), B(1,2), C(-2,5)$ are collinear.

Solution:

$A(5,-2), B(1,2), C(-2,5)$

Let $x_1 = 5,\ y_1 = -2$

And $x_2 = 1,\ y_2 = 2$

Also $x_3 = -2,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(1-5)^2 + (2-(-2))^2}$

$|AB| = \sqrt{(-4)^2 + (2+2)^2}$

$|AB| = \sqrt{16 + (4)^2}$

$|AB| = \sqrt{16 + 16}$

$|AB| = \sqrt{32}$

$|AB| = \sqrt{16 \times 2}$

$|AB| = \sqrt{16} \times \sqrt{2}$

$|AB| = 4\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(-2-1)^2 + (5-2)^2}$

$|BC| = \sqrt{(-3)^2 + (3)^2}$

$|BC| = \sqrt{9 + 9}$

$|BC| = \sqrt{18}$

$|BC| = \sqrt{9 \times 2}$

$|BC| = \sqrt{9} \times \sqrt{2}$

$|BC| = 3\sqrt{2}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(-2-5)^2 + (5-(-2))^2}$

$|AC| = \sqrt{(-7)^2 + (5+2)^2}$

$|AC| = \sqrt{49 + (7)^2}$

$|AC| = \sqrt{49 + 49}$

$|AC| = \sqrt{98}$

$|AC| = \sqrt{49 \times 2}$

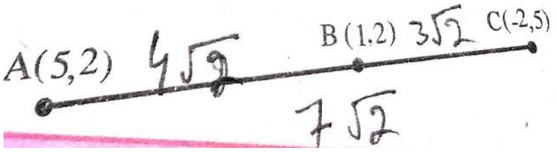
$|AC| = \sqrt{49} \times \sqrt{2}$

$|AC| = 7\sqrt{2}$

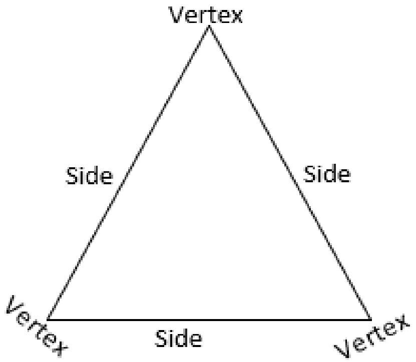
Ex # 9.2

For Colinear Points

$$|AC| = |AB| + |BC|$$
$$7\sqrt{2} = 4\sqrt{2} + 3\sqrt{2}$$



Thus the points are colinear points.



Equilateral Triangle

A triangle in which all the three sides and angles are equal is called equilateral triangle. In equilateral triangle measure of each angle is 60^0.

Example # 7

Prove that the points $A(-2,0), B(2,0)$, $C(0,\sqrt{12})$ is an equilateral triangle.

Solution:

$A(-2,0), B(2,0), C(0,\sqrt{12})$

Let $x_1 = -2,\ y_1 = 0$

And $x_2 = 2,\ y_2 = 0$

Also $x_3 = 0,\ y_3 = \sqrt{12}$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(2 - (-2))^2 + (0 - 0)^2}$$
$$|AB| = \sqrt{(2 + 2)^2 + (0)^2}$$
$$|AB| = \sqrt{(4)^2 + 0}$$
$$|AB| = \sqrt{16}$$
$$|AB| = 4$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(0 - 2)^2 + \left(\sqrt{12} - 0\right)^2}$$

Ex # 9.2

$$|BC| = \sqrt{(-2)^2 + \left(\sqrt{12}\right)^2}$$
$$|BC| = \sqrt{4 + 12}$$
$$|BC| = \sqrt{16}$$
$$|BC| = 4$$

Also distance of $\overline{AC}$:

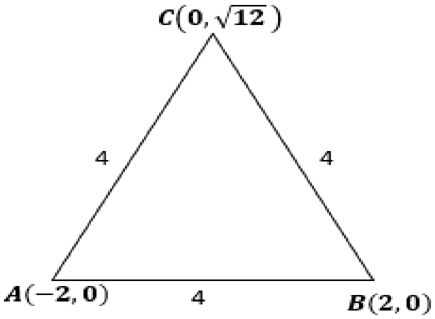
$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(0 - (-2))^2 + \left(\sqrt{12} - 0\right)^2}$$
$$|AC| = \sqrt{(0 + 2)^2 + \left(\sqrt{12}\right)^2}$$
$$|AC| = \sqrt{(2)^2 + 12}$$
$$|AC| = \sqrt{4 + 12}$$
$$|AC| = \sqrt{16}$$
$$|AC| = 4$$

For Equilateral Triangle

All three sides of a triangle are equal

$|AB| = |BC| = |AC| = 4$

Thus the points A, B and C are the vertices of an equilateral triangle.



Isosceles Triangle

A triangle in which two sides and two angles are equal is called isosceles triangle.

Note:

In isosceles triangle, two equal angles are opposite to the equal sides.

Example # 8

Show that points $A(3,2), B(9,10), C(1,16)$ are vertices of an isosceles triangle.

Solution:

$A(3,2), B(9,10), C(1,16)$

Let $x_1 = 3,\ y_1 = 2$

And $x_2 = 9,\ y_2 = 10$

Also $x_3 = 1,\ y_3 = 16$

Ex # 9.2

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(9 - 3)^2 + (10 - 2)^2}$

$|AB| = \sqrt{(6)^2 + (8)^2}$

$|AB| = \sqrt{36 + 64}$

$|AB| = \sqrt{100}$

$|AB| = 10$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(1 - 9)^2 + (16 - 10)^2}$

$|BC| = \sqrt{(-8)^2 + (6)^2}$

$|BC| = \sqrt{64 + 36}$

$|BC| = \sqrt{100}$

$|BC| = 10$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(1 - 3)^2 + (16 - 2)^2}$

$|AC| = \sqrt{(-2)^2 + (14)^2}$

$|AC| = \sqrt{4 + 196}$

$|AC| = \sqrt{200}$

$|AC| = \sqrt{100 \times 2}$

$|AC| = \sqrt{100} \times \sqrt{2}$

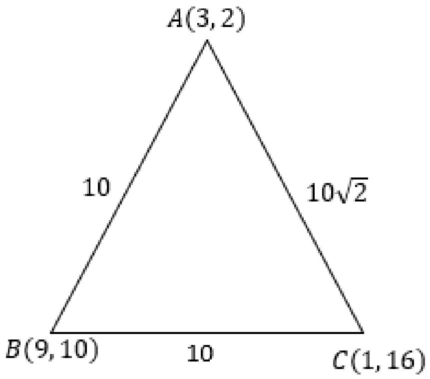
$|AC| = 10\sqrt{2}$

For Isosceles Triangle

Two sides of a triangle are equal.

$|AB| = |BC| = 10$

Thus, the points A, B and C are the vertices of isosceles triangle.



Ex # 9.2

Scalene Triangle

A triangle in which all three sides and angles are different is called scalene triangle.

Example # 9: Show that the points $A(1,2)$, $B(0,4)$, $C(3,5)$ are vertices of scalene triangle.

Solution:

$A(1,2), B(0,4), C(3,5)$

Let $x_1 = 1,\ y_1 = 2$

And $x_2 = 0,\ y_2 = 4$

Also $x_3 = 3,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(0 - 1)^2 + (4 - 2)^2}$

$|AB| = \sqrt{(-1)^2 + (2)^2}$

$|AB| = \sqrt{1 + 4}$

$|AB| = \sqrt{5}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(3 - 0)^2 + (5 - 4)^2}$

$|BC| = \sqrt{(3)^2 + (1)^2}$

$|BC| = \sqrt{9 + 1}$

$|BC| = \sqrt{10}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(3 - 1)^2 + (5 - 2)^2}$

$|AC| = \sqrt{(2)^2 + (-3)^2}$

$|AC| = \sqrt{4 + 9}$

$|AC| = \sqrt{13}$

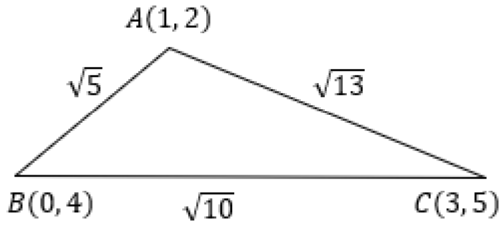
For Scalene Triangle

All the three sides of a triangle are different.

$|AB| \neq |BC| \neq |AC|$

$\sqrt{5} \neq \sqrt{10} \neq \sqrt{13}$

Thus, the points A, B and C are the vertices of scalene triangle.



Ex # 9.2

Right angled triangle

A right-angled triangle in which one angle is equal to 90^0 i.e. right angle

Pythagoras theorem

$(Base)^2 + (Prep)^2 = (Hyp)^2$

Note:

The side opposite to the 90^0 is called hypotenuse.

Hypotenuse is always greater than the other two sides.

Example # 10

Construct the triangle ABC with the help of the points $\boldsymbol{A(1,-2), B(5,1), C(2,5)}$, and prove that the triangle is a right – angled triangle.

Solution:

$A(1,-2), B(5,1), C(2,5)$

Let $x_1 = 1,\ y_1 = -2$

And $x_2 = 5,\ y_2 = 1$

Also $x_3 = 2,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(5-1)^2 + \big(1-(-2)\big)^2}$

$|AB| = \sqrt{(4)^2 + (1+2)^2}$

$|AB| = \sqrt{16 + (3)^2}$

$|AB| = \sqrt{16 + 9}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(2-5)^2 + (5-1)^2}$

$|BC| = \sqrt{(-3)^2 + (4)^2}$

$|BC| = \sqrt{9 + 6}$

$|BC| = \sqrt{25}$

$|BC| = 5$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(2-1)^2 + \big(5-(-2)\big)^2}$

$|AC| = \sqrt{(1)^2 + (5+2)^2}$

$|AC| = \sqrt{1 + (7)^2}$

Ex # 9.2

$|AC| = \sqrt{1 + 49}$

$|AC| = \sqrt{50}$

$|AC| = \sqrt{25 \times 2}$

$|AC| = \sqrt{25} \times \sqrt{2}$

$|AC| = 5\sqrt{2}$

For Right angled Triangle

$(Base)^2 + (Prep)^2 = (Hyp)^2$

So

$|AB|^2 + |BC|^2 = |AC|^2$

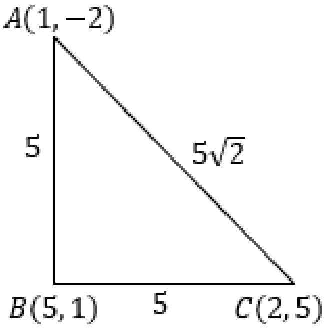
$(5\ )^2 + (5\ )^2 = \left(5\sqrt{2}\ \right)^2$

$25 + 25 = (5)^2\left(\sqrt{2}\ \right)^2$

$50 = 25(2)$

$50 = 50$

Thus, the points A, B and C are the vertices of right – angled triangle.



Square

A closed figure formed by four non – collinear points (vertices) in which the length of all sides are equal and measure of each angle is 90^0

The diagonals of square are equal in length.

Example # 11

By means of distance formula, show that the points $\boldsymbol{A(-1,4), B(1,2), C(3,4), D(1,6)}$ form a square and verify that the diagonals have equal lengths

Solution:

$A(-1,4), B(1,2), C(3,4), D(1,6)$

Let $x_1 = -1,\ y_1 = 4$

And $x_2 = 1,\ y_2 = 2$

Also $x_3 = 3,\ y_3 = 4$

Also $x_4 = 1,\ y_4 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex # 9.2

$|AB| = \sqrt{\left(1-(-1)\right)^2 + (2-4)^2}$

$|AB| = \sqrt{(1+1)^2 + (-2)^2}$

$|AB| = \sqrt{(2)^2 + 4}$

$|AB| = \sqrt{4+4}$

$|AB| = \sqrt{8}$

$|AB| = \sqrt{4 \times 2}$

$|AB| = \sqrt{4} \times \sqrt{2}$

$|AB| = 2\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3-x_2)^2 + (y_3-y_2)^2}$

$|BC| = \sqrt{(3-1)^2 + (4-2)^2}$

$|BC| = \sqrt{(2)^2 + (2)^2}$

$|BC| = \sqrt{4+4}$

$|BC| = \sqrt{8}$

$|BC| = \sqrt{4 \times 2}$

$|BC| = \sqrt{4} \times \sqrt{2}$

$|BC| = 2\sqrt{2}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4-x_3)^2 + (y_4-y_3)^2}$

$|CD| = \sqrt{(1-3)^2 + (6-4)^2}$

$|CD| = \sqrt{(-2)^2 + (2)^2}$

$|CD| = \sqrt{(2)^2 + (2)^2}$

$|CD| = \sqrt{4+4}$

$|CD| = \sqrt{8}$

$|CD| = \sqrt{4 \times 2}$

$|CD| = \sqrt{4} \times \sqrt{2}$

$|CD| = 2\sqrt{2}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4-x_1)^2 + (y_4-y_1)^2}$

$|AD| = \sqrt{\left(1-(-1)\right)^2 + (6-4)^2}$

$|AD| = \sqrt{(1+1)^2 + (2)^2}$

$|AD| = \sqrt{(2)^2 + (2)^2}$

$|AD| = \sqrt{4+4}$

$|AD| = \sqrt{8}$

$|AD| = \sqrt{4 \times 2}$

$|AD| = \sqrt{4} \times \sqrt{2}$

Ex # 9.2

$|AD| = 2\sqrt{2}$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3-x_1)^2 + (y_2-y_1)^2}$

$|AC| = \sqrt{\left(3-(-1)\right)^2 + (4-4)^2}$

$|AC| = \sqrt{(3+1)^2 + (0)^2}$

$|AC| = \sqrt{(4)^2 + 0}$

$|AC| = \sqrt{16}$

$|AC| = 4$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4-x_2)^2 + (y_4-y_2)^2}$

$|BD| = \sqrt{(1-1)^2 + (6-2)^2}$

$|BD| = \sqrt{(0)^2 + (4)^2}$

$|BD| = \sqrt{0+16}$

$|BD| = \sqrt{16}$

$|BD| = 4$

For Square

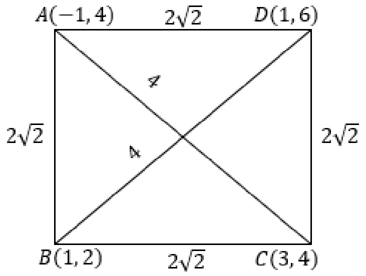
All the sides are equal.

$|AB| = |BC| = |CD| = |AD| = 2\sqrt{2}$

And also, diagonals are equal

$|AC| = |BD| = 4$

Thus, the points A, B, C and D are the vertices of Square.



Rectangle

A rectangle is a geometric shape that has four sides, four vertices and four angles.

The opposite sides of a rectangle are equal in length and measure of each angle is 90^0.

The diagonals of a rectangle are equal in length.

Ex # 9.2

Example # 12 **Show that the points $A(2,4)$, $B(4,2)$, $C(8,6)$, $D(6,8)$ are the vertices of a rectangle. Also plot the points.**

Solution:

$A(2,4), B(4,2), C(8,6), D(6,8)$

Let $x_1 = 2,\ y_1 = 4$ And $x_2 = 4,\ y_2 = 2$

Also $x_3 = 8,\ y_3 = 6$ Also $x_4 = 6,\ y_4 = 8$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(4-2)^2 + (2-4)^2}$

$|AB| = \sqrt{(2)^2 + (-2)^2}$

$|AB| = \sqrt{4+4}$

$|AB| = \sqrt{8}$

$|AB| = \sqrt{4 \times 2}$

$|AB| = \sqrt{4} \times \sqrt{2}$

$|AB| = 2\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(8-4)^2 + (6-2)^2}$

$|BC| = \sqrt{(4)^2 + (4)^2}$

$|BC| = \sqrt{16+16}$

$|BC| = \sqrt{32}$

$|BC| = \sqrt{16 \times 2}$

$|BC| = \sqrt{16} \times \sqrt{2}$

$|BC| = 4\sqrt{2}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(6-8)^2 + (8-6)^2}$

$|CD| = \sqrt{(-2)^2 + (2)^2}$

$|CD| = \sqrt{(2)^2 + (2)^2}$

$|CD| = \sqrt{4+4}$

$|CD| = \sqrt{8}$

$|CD| = \sqrt{4 \times 2}$

$|CD| = \sqrt{4} \times \sqrt{2}$

$|CD| = 2\sqrt{2}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(6-2)^2 + (8-4)^2}$

Ex # 9.2

$|AD| = \sqrt{(4)^2 + (4)^2}$

$|AD| = \sqrt{16+16}$

$|AD| = \sqrt{32}$

$|AD| = \sqrt{16 \times 2}$

$|AD| = \sqrt{16} \times \sqrt{2}$

$|AD| = 4\sqrt{2}$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_2 - y_1)^2}$

$|AC| = \sqrt{(8-2)^2 + (6-4)^2}$

$|AC| = \sqrt{(6)^2 + (2)^2}$

$|AC| = \sqrt{36+4}$

$|AC| = \sqrt{40}$

$|AC| = \sqrt{4 \times 10}$

$|AC| = \sqrt{4} \times \sqrt{10}$

$|AC| = 2\sqrt{10}$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(6-4)^2 + (8-2)^2}$

$|BD| = \sqrt{(2)^2 + (6)^2}$

$|BD| = \sqrt{4+36}$

$|BD| = \sqrt{40}$

$|BD| = \sqrt{4 \times 10}$

$|BD| = \sqrt{4} \times \sqrt{10}$

$|BD| = 2\sqrt{10}$

For Rectangle

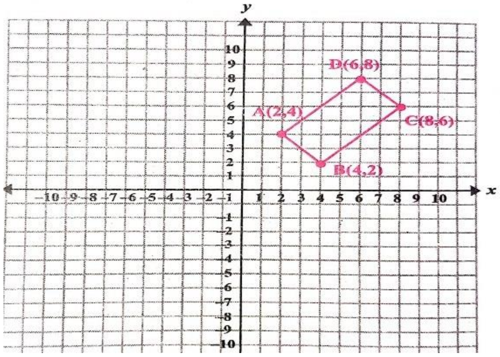
Opposite sides are equal.

$|AB| = |CD| = 2\sqrt{2}$ ***and*** $|BC| = |AD| = 4\sqrt{2}$

And also, diagonals are equal

$|AC| = |BD| = 2\sqrt{10}$

Thus, the points A, B, C and D are the vertices of Rectangle.



Ex # 9.2

Parallelogram

In a parallelogram the opposite sides are congruent and the diagonal bisect each other.

Example # 13

Show that the points $\boldsymbol{F(-1,5), G(3,3), H(6,-4)}$ and $\boldsymbol{J(2,-2)}$ are the vertices of a parallelogram. Also plot the points.

Solution:

$F(-1,5), G(3,3), H(6,-4)\ and\ J(2,-2)$

Let $x_1 = -1,\ y_1 = 5$

And $x_2 = 3,\ y_2 = 3$

Also $x_3 = 6,\ y_3 = -4$

Also $x_4 = 2,\ y_4 = -2$

As distance of $\overline{FG}$:

$$|FG| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|FG| = \sqrt{\left(3 - (-1)\right)^2 + (3 - 5)^2}$$
$$|FG| = \sqrt{(3 + 1)^2 + (-2)^2}$$
$$|FG| = \sqrt{(4)^2 + 4}$$
$$|FG| = \sqrt{16 + 4}$$
$$|FG| = \sqrt{20}$$
$$|FG| = \sqrt{4 \times 5}$$
$$|FG| = \sqrt{4} \times \sqrt{5}$$
$$|FG| = 2\sqrt{5}$$

Now distance of $\overline{GH}$:

$$|GH| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|GH| = \sqrt{(ss6 - 3)^2 + (-4 - 3)^2}$$
$$|GH| = \sqrt{(3)^2 + (-7)^2}$$
$$|GH| = \sqrt{9 + 49}$$
$$|GH| = \sqrt{58}$$

Also distance of $\overline{HJ}$:

$$|HJ| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$
$$|HJ| = \sqrt{(2 - 6)^2 + \left(-2 - (-4)\right)^2}$$
$$|HJ| = \sqrt{(-4)^2 + (-2 + 4)^2}$$
$$|HJ| = \sqrt{16 + (2)^2}$$
$$|HJ| = \sqrt{16 + 4}$$
$$|HJ| = \sqrt{20}$$
$$|HJ| = \sqrt{4 \times 5}$$

Ex # 9.2

$$|HJ| = \sqrt{4} \times \sqrt{5}$$
$$|HJ| = 2\sqrt{5}$$

Also distance of $\overline{JF}$:

$$|JF| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$$
$$|JF| = \sqrt{\left(2 - (-1)\right)^2 + (-2 - 5)^2}$$
$$|JF| = \sqrt{(2 + 1)^2 + (-7)^2}$$
$$|JF| = \sqrt{(3)^2 + 49}$$
$$|JF| = \sqrt{9 + 49}$$
$$|JF| = \sqrt{58}$$

Now to find its Diagonal

Diagonal $\overline{\boldsymbol{FH}}$:

$$|FH| = \sqrt{(x_3 - x_1)^2 + (y_2 - y_1)^2}$$
$$|FH| = \sqrt{\left(6 - (-1)\right)^2 + (-4 - 5)^2}$$
$$|FH| = \sqrt{(6 + 1)^2 + (-9)^2}$$
$$|FH| = \sqrt{(7)^2 + 81}$$
$$|FH| = \sqrt{49 + 81}$$
$$|FH| = \sqrt{130}$$

And Diagonal $\overline{\boldsymbol{GJ}}$:

$$|GJ| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$$
$$|GJ| = \sqrt{(2 - 3)^2 + (-2 - 3)^2}$$
$$|GJ| = \sqrt{(-1)^2 + (-5)^2}$$
$$|GJ| = \sqrt{1 + 25}$$
$$|GJ| = \sqrt{26}$$

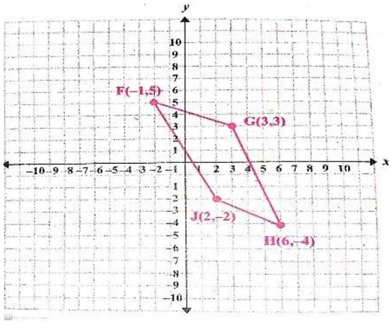
For Parallelogram

Opposite sides are equal.

$$|FG| = |HJ| = 2\sqrt{5}\ \boldsymbol{and}\ |GH| = |JF| = \sqrt{58}$$
$$|FH| \neq |GJ|$$
$$\sqrt{130} \neq \sqrt{26}$$

As diagonals are not equal

Thus, the points A, B, C and D are the vertices of Parallelogram.



Ex # 9.2

Page # 224

Q1: **Prove that $\boldsymbol{A(-4,-3), B(1,4), C(6,11)}$ are collinear.**

Solution:

$A(-4,-3), B(1,4), C(6,11)$

Let $x_1 = -4,\ y_1 = -3$

And $x_2 = 1,\ y_2 = 4$

Also $x_3 = 6,\ y_3 = 11$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(1 - (-4))^2 + (4 - (-3))^2}$$
$$|AB| = \sqrt{(1 + 4)^2 + (4 + 3)^2}$$
$$|AB| = \sqrt{(5)^2 + (7)^2}$$
$$|AB| = \sqrt{25 + 49}$$
$$|AB| = \sqrt{74}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(6 - 1)^2 + (11 - 4)^2}$$
$$|BC| = \sqrt{(5)^2 + (7)^2}$$
$$|BC| = \sqrt{25 + 49}$$
$$|BC| = \sqrt{74}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(6 - (-4))^2 + (11 - (-3))^2}$$
$$|AC| = \sqrt{(6 + 4)^2 + (11 + 3)^2}$$
$$|AC| = \sqrt{(10)^2 + (14)^2}$$
$$|AC| = \sqrt{100 + 196}$$
$$|AC| = \sqrt{296}$$
$$|AC| = \sqrt{4 \times 74}$$
$$|AC| = \sqrt{4} \times \sqrt{74}$$
$$|AC| = 2\sqrt{74}$$

For Colinear Points

$$|AC| = |AB| + |BC|$$
$$2\sqrt{74} = \sqrt{74} + \sqrt{74}$$

Thus, the points are colinear points.

Ex # 9.2

Q2: **Prove that $\boldsymbol{A(-1,3), B(-4,7), C(0,4)}$ is an isosceles triangle.**

Solution:

$A(-1,3), B(-4,7), C(0,4)$

Let $x_1 = -1,\ y_1 = 3$

And $x_2 = -4,\ y_2 = 7$

Also $x_3 = 0,\ y_3 = 4$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(-4 - (-1))^2 + (7 - 3)^2}$$
$$|AB| = \sqrt{(-4 + 1)^2 + (4)^2}$$
$$|AB| = \sqrt{(-3)^2 + 16}$$
$$|AB| = \sqrt{9 + 16}$$
$$|AB| = \sqrt{25}$$
$$|AB| = 5$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{\left(0 - (-4)\right)^2 + (4 - 7)^2}$$
$$|BC| = \sqrt{(0 + 4)^2 + (-3)^2}$$
$$|BC| = \sqrt{(4)^2 + 9}$$
$$|BC| = \sqrt{16 + 9}$$
$$|BC| = \sqrt{25}$$
$$|BC| = 5$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(0 - (-1))^2 + (4 - 3)^2}$$
$$|AC| = \sqrt{(0 + 1)^2 + (1)^2}$$
$$|AC| = \sqrt{(1)^2 + 1}$$
$$|AC| = \sqrt{1 + 1}$$
$$|AC| = \sqrt{2}$$

For Isosceles Triangle

Two sides of a triangle are equal.

$$|AB| = |BC| = 5$$

Thus, the points A, B and C are the vertices of isosceles triangle.

Ex # 9.2

Q3: **Show that points $A(2,3), B(8,11), C(0,17)$ are vertices of an isosceles triangle.**

Solution:

$A(2,3), B(8,11), C(0,17)$

Let $x_1 = 2,\ y_1 = 3$

And $x_2 = 8,\ y_2 = 11$

Also $x_3 = 0,\ y_3 = 17$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(8-2)^2 + (11-3)^2}$$
$$|AB| = \sqrt{(6)^2 + (8)^2}$$
$$|AB| = \sqrt{36 + 64}$$
$$|AB| = \sqrt{100}$$
$$|AB| = 10$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(0-8)^2 + (17-11)^2}$$
$$|BC| = \sqrt{(-8)^2 + (6)^2}$$
$$|BC| = \sqrt{64 + 36}$$
$$|BC| = \sqrt{100}$$
$$|BC| = 10$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(0-2)^2 + (17-3)^2}$$
$$|AC| = \sqrt{(-2)^2 + (14)^2}$$
$$|AC| = \sqrt{4 + 196}$$
$$|AC| = \sqrt{200}$$
$$|AC| = \sqrt{100 \times 2}$$
$$|AC| = \sqrt{100} \times \sqrt{2}$$
$$|AC| = 10\sqrt{2}$$

For Isosceles Triangle

Two sides of a triangle are equal.

$|AB| = |BC| = 10$

Thus, the points A, B and C are the vertices of isosceles triangle.

Ex # 9.2

Q4: (i) **Show that points $A(1,2), B(3,4), C(0,-1)$ are vertices of scalene triangle.**

Solution:

$A(1,2), B(3,4), C(0,-1)$

Let $x_1 = 1,\ y_1 = 2$

And $x_2 = 3,\ y_2 = 4$

Also $x_3 = 0,\ y_3 = -1$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(3-1)^2 + (4-2)^2}$$
$$|AB| = \sqrt{(2)^2 + (2)^2}$$
$$|AB| = \sqrt{4 + 4}$$
$$|AB| = \sqrt{8}$$
$$|AB| = \sqrt{4 \times 2}$$
$$|AB| = \sqrt{4} \times \sqrt{2}$$
$$|AB| = 2\sqrt{2}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(0-3)^2 + (-1-4)^2}$$
$$|BC| = \sqrt{(-3)^2 + (-5)^2}$$
$$|BC| = \sqrt{9 + 25}$$
$$|BC| = \sqrt{34}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(0-1)^2 + (-1-2)^2}$$
$$|AC| = \sqrt{(-1)^2 + (-3)^2}$$
$$|AC| = \sqrt{1 + 9}$$
$$|AC| = \sqrt{10}$$

For Scalene Triangle

All the three sides of a triangle are different.

$|AB| \neq |BC| \neq |AC|$

$2\sqrt{2} \neq \sqrt{34} \neq \sqrt{10}$

Thus, the points A, B and C are the vertices of scalene triangle.

Ex # 9.2

Q4: (ii) **Show that points $A(-4,-1), B(1,0), C(7,-3)$ are vertices of Scalene triangle.**

Solution:

$A(-4,-1), B(1,0), C(7,-3)$

Let $x_1 = -4, \ y_1 = -1$

And $x_2 = 1, \ y_2 = 0$

Also $x_3 = 7, \ y_3 = -3$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{\left(1-(-4)\right)^2 + \left(0-(-1)\right)^2}$$
$$|AB| = \sqrt{(1+4)^2 + (0+1)^2}$$
$$|AB| = \sqrt{(5)^2 + (1)^2}$$
$$|AB| = \sqrt{25+1}$$
$$|AB| = \sqrt{26}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(7-1)^2 + (-3-0)^2}$$
$$|BC| = \sqrt{(6)^2 + (-3)^2}$$
$$|BC| = \sqrt{36+9}$$
$$|BC| = \sqrt{45}$$
$$|BC| = \sqrt{9 \times 5}$$
$$|BC| = \sqrt{9} \times \sqrt{5}$$
$$|BC| = 3\sqrt{5}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{\left(7-(-4)\right)^2 + \left(-3-(-1)\right)^2}$$
$$|AC| = \sqrt{(7+4)^2 + (-3+1)^2}$$
$$|AC| = \sqrt{(11)^2 + (-2)^2}$$
$$|AC| = \sqrt{121+4}$$
$$|AC| = \sqrt{125}$$
$$|AC| = \sqrt{25 \times 5}$$
$$|AC| = \sqrt{25} \times \sqrt{5}$$
$$|AC| = 5\sqrt{5}$$

For Scalene Triangle

All the three sides of a triangle are different.

$$|AB| \neq |BC| \neq |AC|$$
$$\sqrt{26} \neq 3\sqrt{5} \neq 5\sqrt{5}$$

Thus, the points A, B and C are the vertices of scalene triangle.

Ex # 9.2

Q5: **Prove that $A(-2,-2), B(4,-2), C(4,6)$ are vertices of right – angled triangle.**

Solution:

$A(-2,-2), B(4,-2), C(4,6)$

Let $x_1 = -2, \ y_1 = -2$

And $x_2 = 4, \ y_2 = -2$

Also $x_3 = 4, \ y_3 = 6$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{\left(4-(-2)\right)^2 + \left(-2-(-2)\right)^2}$$
$$|AB| = \sqrt{(4+2)^2 + (-2+2)^2}$$
$$|AB| = \sqrt{(6)^2 + (0)^2}$$
$$|AB| = \sqrt{36+0}$$
$$|AB| = \sqrt{36}$$
$$|AB| = 6$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(4-4)^2 + \left(6-(-2)\right)^2}$$
$$|BC| = \sqrt{(0)^2 + (6+2)^2}$$
$$|BC| = \sqrt{0 + (8)^2}$$
$$|BC| = \sqrt{64}$$
$$|BC| = 8$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{\left(4-(-2)\right)^2 + \left(6-(-2)\right)^2}$$
$$|AC| = \sqrt{(4+2)^2 + (6+2)^2}$$
$$|AC| = \sqrt{(6)^2 + (8)^2}$$
$$|AC| = \sqrt{36+64}$$
$$|AC| = \sqrt{100}$$
$$|AC| = 10$$

For Right angled Triangle

$$(Base)^2 + (Prep)^2 = (Hyp)^2$$

So

$$|AB|^2 + |BC|^2 = |AC|^2$$
$$(6\,)^2 + (8\,)^2 = (10\,)^2$$
$$36 + 64 = 100$$
$$100 = 100$$

Thus, the points A, B and C are the vertices of right – angled triangle.

Q6: **Prove that $A(-2,0), B(6,0), C(6,6), D(-2,6)$ are vertices of a rectangle.**

Solution:

$A(-2,0), B(6,0), C(6,6), D(-2,6)$

Let $x_1 = -2,\ y_1 = 0$

And $x_2 = 6,\ y_2 = 0$

Also $x_3 = 6,\ y_3 = 6$

Also $x_4 = -2,\ y_4 = 6$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{\left(6 - (-2)\right)^2 + (0 - 0)^2}$$
$$|AB| = \sqrt{(6 + 2)^2 + (0)^2}$$
$$|AB| = \sqrt{(8)^2 + 0}$$
$$|AB| = \sqrt{64}$$
$$|AB| = 8$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(6 - 6)^2 + (6 - 0)^2}$$
$$|BC| = \sqrt{(0)^2 + (6)^2}$$
$$|BC| = \sqrt{0 + (6)^2}$$
$$|BC| = \sqrt{36}$$
$$|BC| = 6$$

Also distance of $\overline{CD}$:

$$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$
$$|CD| = \sqrt{(-2 - 6)^2 + (6 - 6)^2}$$
$$|CD| = \sqrt{(-8)^2 + (0)^2}$$
$$|CD| = \sqrt{64 + 0}$$
$$|CD| = \sqrt{64}$$
$$|CD| = 8$$

Also distance of $\overline{AD}$:

$$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$$
$$|AD| = \sqrt{\left(-2 - (-2)\right)^2 + (6 - 0)^2}$$
$$|AD| = \sqrt{(-2 + 2)^2 + (6)^2}$$
$$|AD| = \sqrt{(0)^2 + 36}$$
$$|AD| = \sqrt{0 + 36}$$
$$|AD| = \sqrt{36}$$
$$|AD| = 6$$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{\left(6 - (-2)\right)^2 + (6 - 0)^2}$$
$$|AC| = \sqrt{(6 + 2)^2 + (6)^2}$$
$$|AC| = \sqrt{(8)^2 + 36}$$
$$|AC| = \sqrt{64 + 36}$$
$$|AC| = \sqrt{100}$$
$$|AC| = 10$$

And Diagonal $\overline{BD}$:

$$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$$
$$|BD| = \sqrt{(-2 - 6)^2 + (6 - 0)^2}$$
$$|BD| = \sqrt{(-8)^2 + (6)^2}$$
$$|BD| = \sqrt{64 + 36}$$
$$|BD| = \sqrt{100}$$
$$|BD| = 10$$

For Rectangle

Opposite sides are equal.

$|AB| = |CD| = 8$ ***and*** $|BC| = |AD| = 6$

And also, diagonals are equal

$|AC| = |BD| = 10$

Thus, the points A, B, C and D are the vertices of Rectangle.

Q7: **The Vertices of the rectangle ABCD are $A(2,0), B(5,0), C(5,4), D(2,4)$. How long is the diagonal AC?**

Solution:

$A(2,0), B(5,0), C(5,4), D(2,4)$

As to find diagonal AC, so take vertex A and C.

Diagonal $\overline{AC}$:

$A(2,0), C(5,4)$

Let $x_1 = 2,\ y_1 = 0$

And $x_2 = 5,\ y_2 = 4$

As distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AC| = \sqrt{(5 - 2)^2 + (4 - 0)^2}$$
$$|AC| = \sqrt{(3)^2 + (4)^2}$$
$$|AC| = \sqrt{9 + 16}$$
$$|AC| = \sqrt{25}$$
$$|AC| = 5$$

Thus, diagonal AC = 5

Ex # 9.2

Q8: **Prove that $A(-4,-1), B(1,0), C(7,-3), D(2,-4)$ are vertices of a parallelogram.**

Solution:

$A(-4,-1), B(1,0), C(7,-3), D(2,-4)$

Let $x_1 = -4,\ y_1 = -1$

And $x_2 = 1,\ y_2 = 0$

Also $x_3 = 7,\ y_3 = -3$

Also $x_4 = 2,\ y_4 = -4$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left(1-(-4)\right)^2 + (0-(-1))^2}$

$|AB| = \sqrt{(1+4)^2 + (0+1)^2}$

$|AB| = \sqrt{(5)^2 + (1)^2}$

$|AB| = \sqrt{25+1}$

$|AB| = \sqrt{26}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(7-1)^2 + (-3-0)^2}$

$|BC| = \sqrt{(6)^2 + (-3)^2}$

$|BC| = \sqrt{36+9}$

$|BC| = \sqrt{45}$

$|BC| = \sqrt{9 \times 5}$

$|BC| = \sqrt{9} \times \sqrt{5}$

$|BC| = 3\sqrt{5}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(2-7)^2 + (-4-(-3))^2}$

$|CD| = \sqrt{(-5)^2 + (-4+3)^2}$

$|CD| = \sqrt{25 + (-1)^2}$

$|CD| = \sqrt{25+1}$

$|CD| = \sqrt{26}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{\left(2-(-4)\right)^2 + (-4-(-1))^2}$

$|AD| = \sqrt{(2+4)^2 + (-4+1)^2}$

$|AD| = \sqrt{(6)^2 + (-3)^2}$

Ex # 9.2

$|AD| = \sqrt{36+9}$

$|AD| = \sqrt{0+36}$

$|AD| = \sqrt{45}$

$|AD| = \sqrt{9 \times 5}$

$|AD| = \sqrt{9} \times \sqrt{5}$

$|AD| = 3\sqrt{5}$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(7-(-4)\right)^2 + (-3-(-1))^2}$

$|AC| = \sqrt{(7+4)^2 + (-3+1)^2}$

$|AC| = \sqrt{(11)^2 + (-2)^2}$

$|AC| = \sqrt{121+4}$

$|AC| = \sqrt{125}$

$|AC| = \sqrt{25 \times 5}$

$|AC| = \sqrt{25} \times \sqrt{5}$

$|AC| = 5\sqrt{5}$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(2-1)^2 + (-4-0)^2}$

$|BD| = \sqrt{(1)^2 + (-4)^2}$

$|BD| = \sqrt{1+16}$

$|BD| = \sqrt{17}$

For Parallelogram

Opposite sides are equal.

$|AB| = |CD| = \sqrt{26}\ \textbf{and}\ |BC| = |AD| = 3\sqrt{5}$

And also, diagonals are equal

$|AC| \neq |BD|$

$5\sqrt{5} \neq \sqrt{17}$

As diagonals are not equal

Thus, the points A, B, C and D are the vertices of Parallelogram.

Ex # 9.2

Q9: **Find b such that the points $A(2, b), B(5, 5)$, $C(-6, 0)$ are vertices of a right-angled triangle with $\angle BAC = 90^0$**

Solution:

$A(2, b), B(5, 5), C(-6, 0)$

Let $x_1 = 2,\ y_1 = b$

And $x_2 = 5,\ y_2 = 5$

Also $x_3 = -6,\ y_3 = 0$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$|AB| = \sqrt{(5-2)^2 + (5-b)^2}$$
$$|AB| = \sqrt{(3)^2 + (5)^2 + (b)^2 - 2(5)(b)}$$
$$|AB| = \sqrt{9 + 25 + b^2 - 10b}$$
$$|AB| = \sqrt{34 + b^2 - 10b}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$|BC| = \sqrt{(-6-5)^2 + (0-5)^2}$$
$$|BC| = \sqrt{(-11)^2 + (-5)^2}$$
$$|BC| = \sqrt{121 + 25}$$
$$|BC| = \sqrt{146}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$|AC| = \sqrt{(-6-2)^2 + (0-b)^2}$$
$$|AC| = \sqrt{(-8)^2 + (-b)^2}$$
$$|AC| = \sqrt{64 + b^2}$$

As $\angle BAC = 90^0$

Now by Pythagoras theorem

$$(Base)^2 + (Prep)^2 = (Hyp)^2$$
$$|AB|^2 + |AC|^2 = |BC|^2$$

By putting values

$$\left(\sqrt{34 + b^2 - 10b}\right)^2 + \left(\sqrt{64 + b^2}\right)^2 = \left(\sqrt{146}\right)^2$$
$$34 + b^2 - 10b + 64 + b^2 = 146$$
$$b^2 + b^2 - 10b + 34 + 64 = 146$$
$$2b^2 - 10b + 98 = 146$$
$$2b^2 - 10b + 98 - 146 = 0$$
$$2b^2 - 10b - 48 = 0$$
$$2(b^2 - 5b - 24) = 0$$

Divided B.S by 2, we get

$$b^2 - 5b - 24 = 0$$

Ex # 9.2

$$b^2 + 3b - 8b - 24 = 0$$
$$b(b+3) - 8(b+3) = 0$$
$$(b+3)(b-8) = 0$$
$$b + 3 = 0 \quad or \quad b - 8 = 0$$
$$b = -3 \quad or \quad b = 8$$

Q10: **Given $A(-4, -2), B(1, -3), C(3, 1)$, find the coordinate of D in the 2nd quadrant such that quadrilateral ABCD is a parallelogram.**

Solution:

Let the coordinate D is (x, y)

Thus the vertices of a parallelogram are

$A(-4, -2), B(1, -3), C(3, 1), D(x, y)$

As AC and BD are the diagonals

Now

$$\text{Mid – point of AC} = \left(\frac{-4+3}{2}, \frac{-2+1}{2}\right)$$
$$\text{Mid – point of AC} = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

Also

$$\text{Mid – point of BD} = \left(\frac{1+x}{2}, \frac{-3+y}{2}\right)$$

As diagonals of a parallelogram bisect each other

So

$$\frac{1+x}{2} = \frac{-1}{2} \quad and \quad \frac{-3+y}{2} = \frac{-1}{2}$$
$$1 + x = -1 \quad and \quad -3 + y = -1$$
$$x = -1 - 1 \quad and \quad y = -1 + 3$$
$$x = -2 \quad and \quad y = 2$$

Thus

Thus the coordinate D is $(-2, 2)$

Mid – Point Formula:

The mid – point of the line segment obtained by joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ and $C(x, y)$ is mid – point of AB, then

$$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex # 9.3

Example # 14

Find the coordinates of mid – point of the segment joining the points A(4, 6) and B(2, 1)

Solution:

Let $C(x\,,y)$ is the mid − point of AB, then

$$C(x\,,y)=\left(\frac{x_1+x_2}{2}\ ,\frac{y_1+y_2}{2}\right)$$

Put the values

$$C(x\,,y)=\left(\frac{4+2}{2}\ ,\frac{6+1}{2}\right)$$

$$C(x\,,y)=\left(\frac{6}{2}\ ,\frac{7}{2}\right)$$

$$C(x\,,y)=\left(3\ ,\frac{7}{2}\right)$$

Example # 15

The coordinates of the mid – point of a line segment $\overline{AB}$ are $(2,5)$ and that of A are $(-4,-6)$. Find the coordinates of point B.

Solution:

Let the midpoint is $C(2,5)$

As one end of a line segment $=A(x_1,y_1)=A(-4,-6)$

And other end of a line segment $=B(x_2,y_2)=?$

As midpoint formula is:

$$\text{Midpoint}=\left(\frac{x_1+x_2}{2}\ ,\frac{y_1+y_2}{2}\right)$$

Put the values

$$C(2,5)=\left(\frac{-4+x_2}{2}\ ,\frac{-6+y_2}{2}\right)$$

Now by comparing

$$2=\frac{-4+x_2}{2}\quad \&\quad 5=\frac{-6+y_2}{2}$$

$$2\times 2=-4+x_2\quad \&\quad 5\times 2=-6+y_2$$

$$4=-4+x_2\quad \&\quad 10=-6+y_2$$

$$4+4=x_2\quad \&\quad 10+6=y_2$$

$$8=x_2\quad \&\quad 16=y_2$$

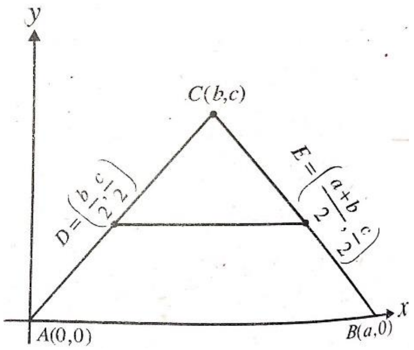
$$x_2=8\quad \&\quad y_2=16$$

Thus the other end of a line segment $=B(8,16)$

Ex # 9.3

Result # 1

Prove that the line segment joining the mid – points of two sides of a triangle is equal to half of the length of the third side.



Proof:

$A(0,0),B(a,0),C(b,c)$

As D is the midpoint of AC

$$D=\left(\frac{0+b}{2}\ ,\frac{0+c}{2}\right)$$

$$D=\left(\frac{b}{2}\ ,\frac{c}{2}\right)$$

Also E is the midpoint of BC

$$E=\left(\frac{a+b}{2}\ ,\frac{0+c}{2}\right)$$

$$E=\left(\frac{a+b}{2}\ ,\frac{c}{2}\right)$$

Now to find the distance of $\overline{AB}$

$$|AB|=\sqrt{(a-0)^2+(0-0)^2}$$

$$|AB|=\sqrt{(a)^2+(0)^2}$$

$$|AB|=\sqrt{a^2}$$

$$|AB|=a$$

Now to find the distance of $\overline{DE}$

$$|DE|=\sqrt{\left(\frac{a+b}{2}-\frac{b}{2}\right)^2+\left(\frac{c}{2}-\frac{c}{2}\right)^2}$$

$$|DE|=\sqrt{\left(\frac{a+b-b}{2}\right)^2+(0)^2}$$

$$|DE|=\sqrt{\left(\frac{a}{2}\right)^2}$$

$$|DE|=\frac{a}{2}$$

But $a=|AB|$

$$|DE|=\frac{|AB|}{2}$$

$$|DE|=\frac{1}{2}|AB|$$

Result # 2

The mid – points of the hypotenuse of a right angled triangle is equidistance from the vertices.

Proof:

In right angled triangle ABC $\overline{BC}$ is the hypotenuse and D is the mid – point

The vertices are $A(0,0), B(a,0), C(0,b)$

As D is the midpoint of BC

$$D = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$$

$$D = \left(\frac{a}{2}, \frac{b}{2}\right)$$

To Prove:

As mid – point D is equidistant from the vertices.

Thus $|AD| = |BD| = |CD|$

Now to find the distance of $\overline{AD}$

$$|AD| = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2}$$

$$|AD| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$|AD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \quad \ldots\ldots equ(i)$$

Now to find the distance of $\overline{BD}$

$$|BD| = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2}$$

$$|BD| = \sqrt{\left(\frac{a-2a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$|BD| = \sqrt{\left(\frac{-a}{2}\right)^2 + \frac{b^2}{4}}$$

$$|BD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$|BD| = \sqrt{\frac{a^2+b^2}{4}} \quad \ldots\ldots equ(ii)$$

Now to find the distance of $\overline{CD}$

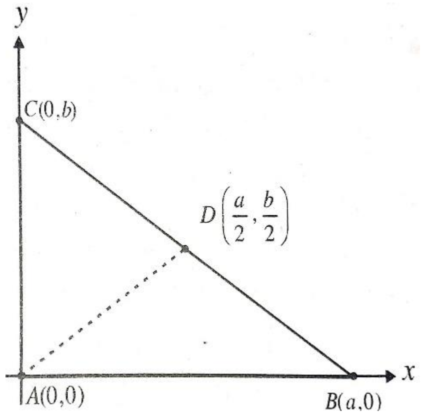
$$|CD| = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-b\right)^2}$$

$$|CD| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b-2b}{2}\right)^2}$$

$$|CD| = \sqrt{\frac{a^2}{4} + \left(\frac{-b}{2}\right)^2}$$

$$|CD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$|CD| = \sqrt{\frac{a^2+b^2}{4}} \quad \ldots\ldots equ(iii)$$



From equ(i), (ii) & (iii)

$$|AD| = |BD| = |CD|$$

Result # 3

Verify the diagonals of any rectangle are equal in length.

Proof:

In rectangle ABCD $\overline{AC}$ *and* $\overline{BD}$ are the diagonals

The vertices are $A(0,0), B(a,0), C(a,b), D(0,b)$

To Prove:

As diagonals are equal in length

Thus $|AC| = |BD|$

Now to find the distance of $\overline{AC}$

$$|AC| = \sqrt{(a-0)^2 + (b-0)^2}$$

$$|AC| = \sqrt{(a)^2 + (b)^2}$$

$$|AC| = \sqrt{a^2 + b^2} \quad \ldots\ldots equ(i)$$

Now to find the distance of $\overline{BD}$

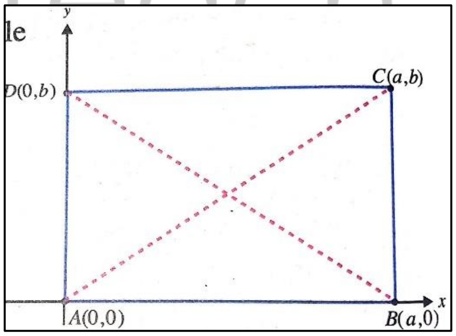
$$|BD| = \sqrt{(0-a)^2 + (b-0)^2}$$

$$|BD| = \sqrt{(-a)^2 + (b)^2}$$

$$|BD| = \sqrt{a^2 + b^2} \quad \ldots\ldots equ(ii)$$

From equ(i), (ii)

$$|AC| = |BD|$$



Result # 4

Show that diagonals of a parallelogram bisect each other.

Proof:

In parallelogram ABCD $\overline{AC}$ and $\overline{BD}$ are the diagonals

The vertices are $A(0,0), B(a,0), C(b,c), D(b-a,c)$

Let E is the midpoint of AC

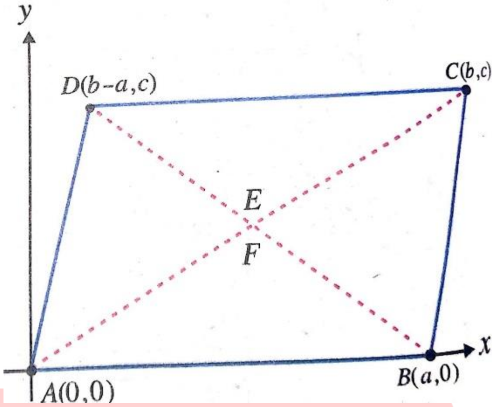
$$E = \left(\frac{0+b}{2}, \frac{0+c}{2}\right)$$

$$E = \left(\frac{b}{2}, \frac{c}{2}\right)$$

Also F is the midpoint of BD

$$F = \left(\frac{a+b-a}{2}, \frac{0+c}{2}\right)$$

$$F = \left(\frac{b}{2}, \frac{c}{2}\right)$$



As the mid – points E and F are same.

Thus

$|AE| = |EC|$ and $|BF| = |FD|$

Result # 5 :Prove that is a right angled triangle square of the length of the hypotenuse is equal to the sum of the square of the length of two legs.

Proof:

In right – angled triangle $\overline{BC}$ *hypotenuse*

The vertices are $A(0,0), B(a,0), C(0,b)$

To prove:

$(Hyp)^2 = (one\ leg)^2 + (other\ leg)^2$

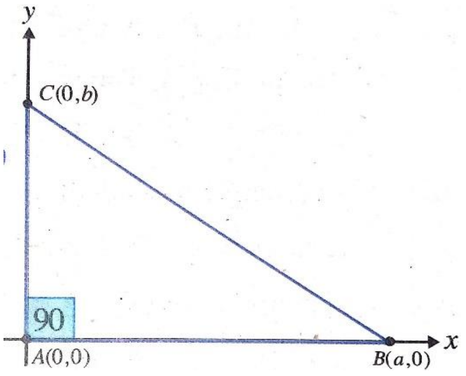
As distance of $\overline{AB}$:

$$|AB| = \sqrt{(a-0)^2 + (0-0)^2}$$

$$|AB| = \sqrt{(a)^2 + (0)^2}$$

$$|AB| = \sqrt{a^2}$$

$$|AB| = a$$



Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(0-a)^2 + (b-0)^2}$$

$$|BC| = \sqrt{(-a)^2 + (b)^2}$$

$$|BC| = \sqrt{a^2 + b^2}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(0-0)^2 + (b-0)^2}$$

$$|AC| = \sqrt{(0)^2 + (b)^2}$$

$$|AC| = \sqrt{b^2}$$

$$|AC| = b$$

Let $\angle BAC = 90^0$

$$(Hyp)^2 = (one\ leg)^2 + (other\ leg)^2$$

$$|BC|^2 = |AB|^2 + |AC|^2$$

$$\left(\sqrt{a^2 + b^2}\right)^2 = (a)^2 + (b)^2$$

$$a^2 + b^2 = a^2 + b^2$$

Hence the result is proved.

Ex # 9.3

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Q1: Find the coordinates of the midpoint of the segment with the given end points.

(i) $(8,-5)$ and $(-2,9)$

Solution:

$(8,-5)$ and $(-2,9)$

Let $x_1 = 8,\ y_1 = -5$

And $x_2 = -2,\ y_2 = 9$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{8 + (-2)}{2}, \frac{9 + (-5)}{2}\right)$$

$$\text{Midpoint} = \left(\frac{8-2}{2}, \frac{9-5}{2}\right)$$

$$\text{Midpoint} = \left(\frac{6}{2}, \frac{4}{2}\right)$$

$$\text{Midpoint} = (3, 2)$$

(ii) $(7,6)$ and $(3,2)$

Solution:

$(7,6)$ and $(3,2)$

Let $x_1 = 7,\ y_1 = 6$

And $x_2 = 3,\ y_2 = 2$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{3+7}{2}, \frac{2+6}{2}\right)$$

$$\text{Midpoint} = \left(\frac{10}{2}, \frac{8}{2}\right)$$

$$\text{Midpoint} = (5, 4)$$

Ex # 9.3

(iii) $(-2,3)$**and** $(-9,-6)$

Solution:

$(-2,3)$and $(-9,-6)$

Let $x_1 = -2,\ y_1 = -3$

And $x_2 = -9,\ y_2 = -6$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{-2 + (-9)}{2}, \frac{-3 + (-6)}{2}\right)$$

$$\text{Midpoint} = \left(\frac{-2 - 9}{2}, \frac{-3 - 6}{2}\right)$$

$$\text{Midpoint} = \left(\frac{-11}{2}, \frac{-9}{2}\right)$$

(iv) $(\boldsymbol{a+b, a-b})$ **and** $(\boldsymbol{-a, b})$

Solution:

$(a + b, a - b)$ and $(-a, b)$

Let $x_1 = a + b,\ y_1 = a - b$

And $x_2 = -a,\ y_2 = b$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

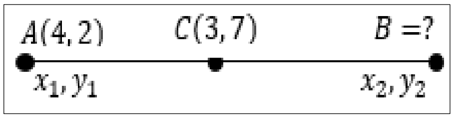
$$\text{Midpoint} = \left(\frac{a + b + (-a)}{2}, \frac{a - b + b}{2}\right)$$

$$\text{Midpoint} = \left(\frac{a + b - a}{2}, \frac{a}{2}\right)$$

$$\text{Midpoint} = \left(\frac{a - a + b}{2}, \frac{a}{2}\right)$$

$$\text{Midpoint} = \left(\frac{b}{2}, \frac{a}{2}\right)$$

Q2: The mid-point and one end of a line segment are $(3,7)$ and $(4,2)$ respectively. Find the other end point.



Solution:

Let the midpoint is $C(3,7)$

As one end of a line segment $= A(x_1, y_1) = A(4,2)$

And other end of a line segment $= B(x_2, y_2) =?$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex # 9.3

Put the values

$$C(3,7) = \left(\frac{3 + x_2}{2}, \frac{7 + y_2}{2}\right)$$

Now by comparing

$$3 = \frac{4 + x_2}{2} \quad \& \quad 7 = \frac{2 + y_2}{2}$$

$$3 \times 2 = 4 + x_2 \quad \& \quad 7 \times 2 = 2 + y_2$$

$$6 = 4 + x_2 \quad \& \quad 14 = 2 + y_2$$

$$6 - 4 = x_2 \quad \& \quad 14 - 2 = y_2$$

$$2 = x_2 \quad \& \quad 12 = y_2$$

$$x_2 = 2 \quad \& \quad y_2 = 12$$

Thus the other end of a line segment $= B(2,12)$

Q3: The midpoints of the sides of a triangle are (2, 5), (4, 2), (1, 1). Find the coordinates of the three vertices.

Solution:

As the midpoints are (2, 5), (4, 2), (1, 1)

Let the coordinates of the vertices are

$A(x_1, y_1)\ , B(x_2, y_2)\ \&\ C(x_3, y_3)$

Let $(2,5)$ be the midpoint of AB

$$(2,5) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Now by comparing

$$2 = \frac{x_1 + x_2}{2} \quad \& \quad 5 = \frac{y_1 + y_2}{2}$$

$$2 \times 2 = x_1 + x_2 \quad \& \quad 5 \times 2 = y_1 + y_2$$

$$4 = x_1 + x_2 \quad \& \quad 10 = y_1 + y_2$$

$$x_1 + x_2 = 4 \quad \& \quad y_1 + y_2 = 10$$

Let $(4,2)$ be the midpoint of BC

$$(4,2) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

Now by comparing

$$4 = \frac{x_2 + x_3}{2} \quad \& \quad 2 = \frac{y_2 + y_3}{2}$$

$$4 \times 2 = x_2 + x_3 \quad \& \quad 2 \times 2 = y_2 + y_3$$

$$8 = x_2 + x_3 \quad \& \quad 4 = y_2 + y_3$$

$$x_2 + x_3 = 8 \quad \& \quad y_2 + y_3 = 4$$

Let $(1,1)$ be the midpoint of AC

$$(1,1) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

Now by comparing

$$1 = \frac{x_1 + x_3}{2} \quad \& \quad 1 = \frac{y_1 + y_3}{2}$$

$$1 \times 2 = x_1 + x_3 \quad \& \quad 1 \times 2 = y_1 + y_3$$

$$2 = x_1 + x_3 \quad \& \quad 2 = y_1 + y_3$$

Ex # 9.3

$x_1 + x_3 = 2 \quad \& \quad y_1 + y_3 = 2$

Let

$x_1 + x_2 = 4 \ \dots\dots equ(\text{i})$

$x_2 + x_3 = 8 \ \dots\dots equ(\text{ii})$

$x_1 + x_3 = 2 \ \dots\dots equ(\text{iii})$

$y_1 + y_2 = 10 \ \dots\dots equ(\text{a})$

$y_2 + y_3 = 4 \ \dots\dots equ(\text{b})$

$y_1 + y_3 = 2 \ \dots\dots equ(\text{c})$

Now $equ(\text{i}) - equ(\text{ii})$

$(x_1 + x_2) - (x_2 + x_3) = 4 - 8$

$x_1 + x_2 - x_2 - x_3 = -4$

$x_1 - x_3 = -4 \ \dots\dots equ(\text{iv})$

Now $equ(\text{iii}) + equ(\text{iv})$

$x_1 + x_3 + x_1 - x_3 = 2 + (-4)$

$x_1 + x_1 = 2 - 4$

$2x_1 = -2$

$\frac{2x_1}{2} = \frac{-2}{2}$

$x_1 = -1$

Put $x_1 = -1$ in $\text{equ}(\text{i})$

$-1 + x_2 = 4$

$x_2 = 4 + 1$

$x_2 = 5$

Put $x_2 = 5$ in $\text{equ}(\text{ii})$

$5 + x_3 = 8$

$x_3 = 8 - 5$

$x_3 = 3$

Now $equ(\text{a}) - equ(\text{b})$

$(y_1 + y_2) - (y_2 + y_3) = 10 - 4$

$y_1 + y_2 - y_2 - y_3 = 6$

$y_1 - y_3 = 6 \ \dots\dots equ(\text{d})$

Now $equ(\text{c}) + equ(\text{d})$

$y_1 + y_3 + y_1 - y_3 = 2 + 6$

$y_1 + y_1 = 8$

$2y_1 = 8$

$\frac{2y_1}{2} = \frac{8}{2}$

$y_1 = 4$

Put $y_1 = 4$ in $\text{equ}(\text{a})$

$4 + y_2 = 10$

$y_2 = 10 - 4$

$y_2 = 6$

Ex # 9.3

Put $y_2 = 6$ in $\text{equ}(\text{b})$

$6 + y_3 = 4$

$y_3 = 4 - 6$

$y_3 = -2$

Let the coordinates of the vertices are

$A(-1, 4)\,, B(5, 6)\ \&\ C(3, -2)$

Q4: The distance between two points with coordinates (1, 1) and (4, *y*) is 5.

Solution:

As the coordinates are $(1, 1), (4, y)$

And distance $= d = 5$

Let $x_1 = 1,\ y_1 = 1$

And $x_2 = 4,\ y_2 = y$

As distance formula is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put the values

$5 = \sqrt{(4 - 1)^2 + (y - 1)^2}$

$5 = \sqrt{(3)^2 + (y)^2 - 2(y)(1) + (1)^2}$

$5 = \sqrt{9 + y^2 - 2y + 1}$

$5 = \sqrt{y^2 - 2y + 1 + 9}$

$5 = \sqrt{y^2 - 2y + 10}$

$\sqrt{y^2 - 2y + 10} = 5$

Taking square on B.S

$\left(\sqrt{y^2 - 2y + 10}\right)^2 = (5)^2$

$y^2 - 2y + 10 = 25$

$y^2 - 2y + 10 - 25 = 0$

$y^2 - 2y - 15 = 0$

$y^2 + 3y - 5y - 15 = 0$

$y(y + 3) - 5(y + 3) = 0$

$(y + 3)(y - 5) = 0$

$y + 3 = 0 \quad or \quad y - 5 = 0$

$y = -3 \quad or \quad y = 5$

Thus $y = -3 \ \ or \ \ y = 5$

Review Ex #9

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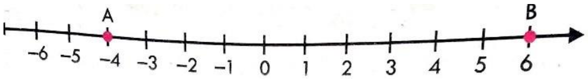
Q2: **Find the distance between A and B on the number line below.**



Solution:

$|AB| = |6-(-4)|$

$|AB| = |6+4|$

$|AB| = |10|$

$|AB| = 10$

Q3: **What is the distance between two points with coordinates of $(1,-5)$ and $(-5,7)$?**

Solution:

$(1,-5)$ and $(-5,7)$

Let $x_1 = 1,\ y_1 = -5$

And $x_2 = -5,\ y_2 = 7$

As distance formula is:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(-5-1)^2+(7-(-5))^2}$

$d = \sqrt{(-6)^2+(7+5)^2}$

$d = \sqrt{36+(12)^2}$

$d = \sqrt{36+144}$

$d = \sqrt{180}$

$d = \sqrt{36\times 5}$

$d = \sqrt{36}\times\sqrt{5}$

$d = 6\sqrt{5}$

Q4: **Using distance formula, show that the points $(4,-3), B(2,0), C(-2,6)$ are collinear.**

Solution:

$(4,-3), B(2,0), C(-2,6)$

Let $x_1 = 4,\ y_1 = -3$

And $x_2 = 2,\ y_2 = 0$

Also $x_3 = -2,\ y_3 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB| = \sqrt{(2-4))^2+(0-(-3))^2}$

Review # 9

$|AB| = \sqrt{(-2)^2+(0+3)^2}$

$|AB| = \sqrt{4+(3)^2}$

$|AB| = \sqrt{4+9}$

$|AB| = \sqrt{13}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$

$|BC| = \sqrt{(-2-2)^2+(6-0)^2}$

$|BC| = \sqrt{(-4)^2+(6)^2}$

$|BC| = \sqrt{16+36}$

$|BC| = \sqrt{52}$

$|BC| = \sqrt{4\times 13}$

$|BC| = \sqrt{4}\times\sqrt{13}$

$|BC| = 2\sqrt{13}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$

$|AC| = \sqrt{(-2-4)^2+(6-(-3))^2}$

$|AC| = \sqrt{(-6)^2+(6+3)^2}$

$|AC| = \sqrt{36+(9)^2}$

$|AC| = \sqrt{36+81}$

$|AC| = \sqrt{117}$

$|AC| = \sqrt{9\times 13}$

$|AC| = \sqrt{9}\times\sqrt{13}$

$|AC| = 3\sqrt{13}$

For Colinear Points

$|AC| = |AB|+|BC|$

$2\sqrt{74} = \sqrt{74}+\sqrt{74}$

Thus, the points are colinear points.

Q5: **Find the point on the $x-$ *axis* which is equidistant from $(0,1)$ *and* $(3,3)$.**

Solution:

As the given points are $A(0,1)$ *and* $B(3,3)$

Let P be the point on x − axis

So $P(x,0)$

As point P is equidistant from A and B

$|AP| = |BP|$

$\sqrt{(x-0)^2+(0-1)^2} = \sqrt{(x-3)^2+(0-3)^2}$

$\sqrt{(x)^2+(-1)^2} = \sqrt{x^2-2(x)(3)+(3)^2+(-3)^2}$

$\sqrt{x^2+1} = \sqrt{x^2-6x+9+9}$

$\sqrt{x^2+1} = \sqrt{x^2-6x+18}$

Taking square on B.S

$$\left(\sqrt{x^2+1}\right)^2 = \left(\sqrt{x^2-6x+18}\right)^2$$

$$x^2+1 = x^2-6x+18$$

$$x^2-x^2+6x = 18-1$$

$$6x = 17$$

$$x = \frac{17}{6}$$

$$Thus, the\ point\ on\ x-axis\ is\ \left(\frac{17}{6},0\right)$$

Q6: **A segment has one endpoint at (15, 22) and a midpoint at (5, 18), what are the coordinates of the other endpoint?**

Solution:

Let the midpoint is $C(5,18)$

As one end of a line segment $= A(x_1,y_1) = A(15,22)$

And other end of a line segment $= B(x_2,y_2)$ =?

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Put the values

$$C(5,18) = \left(\frac{15+x_2}{2},\frac{22+y_2}{2}\right)$$

Now by comparing

$$5 = \frac{15+x_2}{2} \quad \& \quad 18 = \frac{22+y_2}{2}$$

$$5\times 2 = 15+x_2 \quad \& \quad 18\times 2 = 22+y_2$$

$$10 = 15+x_2 \quad \& \quad 36 = 22+y_2$$

$$10-15 = x_2 \quad \& \quad 36-22 = y_2$$

$$-5 = x_2 \quad \& \quad 14 = y_2$$

$$x_2 = -5 \quad \& \quad y_2 = 14$$

Thus the other end of a line segment $= B(-5,14)$

Q7: **Prove that $(2,1),(0,0),(-1,2),(1,3)$ are vertices of a rectangle.**

Solution:

$Let\ A(2,1), B(0,0), C(-1,2), D(1,3)$

Let $x_1 = 2,\ y_1 = 1$

And $x_2 = 0,\ y_2 = 0$

Also $x_3 = -1,\ y_3 = 2$

Also $x_4 = 1,\ y_4 = 3$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$|AB| = \sqrt{(0-2)^2+(0-1)^2}$$

$$|AB| = \sqrt{(-2)^2+(-1)^2}$$

$$|AB| = \sqrt{4+1}$$

$$|AB| = \sqrt{5}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$

$$|BC| = \sqrt{(-1-0)^2+(2-0)^2}$$

$$|BC| = \sqrt{(-1)^2+(2)^2}$$

$$|BC| = \sqrt{1+4}$$

$$|BC| = \sqrt{5}$$

Also distance of $\overline{CD}$:

$$|CD| = \sqrt{(x_4-x_3)^2+(y_4-y_3)^2}$$

$$|CD| = \sqrt{(1-(-1))^2+(3-2)^2}$$

$$|CD| = \sqrt{(1+1)^2+(1)^2}$$

$$|CD| = \sqrt{(2)^2+1}$$

$$|CD| = \sqrt{4+1}$$

$$|CD| = \sqrt{5}$$

Also distance of $\overline{AD}$:

$$|AD| = \sqrt{(x_4-x_1)^2+(y_4-y_1)^2}$$

$$|AD| = \sqrt{(1-2)^2+(3-1)^2}$$

$$|AD| = \sqrt{(-1)^2+(2)^2}$$

$$|AD| = \sqrt{1+4}$$

$$|AD| = \sqrt{5}$$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$$|AC| = \sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$$

$$|AC| = \sqrt{(-1-2)^2+(2-1)^2}$$

$$|AC| = \sqrt{(-3)^2+(1)^2}$$

$$|AC| = \sqrt{9+1}$$

$$|AC| = \sqrt{10}$$

And Diagonal $\overline{BD}$:

$$|BD| = \sqrt{(x_4-x_2)^2+(y_4-y_2)^2}$$

$$|BD| = \sqrt{(1-0)^2+(3-0)^2}$$

$$|BD| = \sqrt{(1)^2+(3)^2}$$

$$|BD| = \sqrt{1+9}$$

$$|BD| = \sqrt{10}$$

For Rectangle

Opposite sides are equal.

$$|AB| = |CD| = \sqrt{5} \ \ \textbf{and} \ |BC| = |AD| = \sqrt{5}$$

And also, diagonals are equal

$|AC| = |BD| = \sqrt{10}$

Thus, the points A, B, C and D are the vertices of Rectangle.

Also

$|AB| = |BC| = |CD| = |AD| = \sqrt{5}$

So it is also a square

Q8: Prove that $\boldsymbol{A(-1,0), B(3,3), C(6,-1),}$ $\boldsymbol{D(2,-4)}$ are vertices of a square.

Solution:

$$A(-1,0), B(3,3), C(6,-1), D(2,-4)$$

Let $x_1 = -1, \ y_1 = 0$

And $x_2 = 3, \ y_2 = 3$

Also $x_3 = 6, \ y_3 = -1$

Also $x_4 = 2, \ y_4 = -4$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(3-(-1))^2 + (3-0)^2}$

$|AB| = \sqrt{(3+1)^2 + (3)^2}$

$|AB| = \sqrt{(4)^2 + 9}$

$|AB| = \sqrt{16+9}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(6-3)^2 + (-1-3)^2}$

$|BC| = \sqrt{(3)^2 + (-4)^2}$

$|BC| = \sqrt{9+16}$

$|BC| = \sqrt{25}$

$|BC| = 5$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(2-6)^2 + (-4-(-1))^2}$

$|CD| = \sqrt{(-4)^2 + (-4+1)^2}$

$|CD| = \sqrt{16 + (-3)^2}$

$|CD| = \sqrt{16+9}$

$|CD| = \sqrt{25}$

$|CD| = 5$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(2-(-1))^2 + (-4-0)^2}$

$|AD| = \sqrt{(2+1)^2 + (-4)^2}$

$|AD| = \sqrt{(3)^2 + (4)^2}$

$|AD| = \sqrt{9+16}$

$|AD| = \sqrt{25}$

$|AD| = 5$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(6-(-1)\right)^2 + (-1-0)^2}$

$|AC| = \sqrt{(6+1)^2 + (-1)^2}$

$|AC| = \sqrt{(7)^2 + 1}$

$|AC| = \sqrt{49+1}$

$|AC| = \sqrt{50}$

$|AC| = \sqrt{25 \times 2}$

$|AC| = \sqrt{25} \times \sqrt{2}$

$|AC| = 5\sqrt{2}$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(2-3)^2 + (-4-3)^2}$

$|BD| = \sqrt{(-1)^2 + (-7)^2}$

$|BD| = \sqrt{1+49}$

$|BD| = \sqrt{50}$

$|BD| = \sqrt{25 \times 2}$

$|BD| = \sqrt{25} \times \sqrt{2}$

$|BD| = 5\sqrt{2}$

For Square

All the sides are equal.

$|AB| = |BC| = |CD| = |AD| = 5$

And also, diagonals are equal

$|AC| = |BD| = 5\sqrt{2}$

Thus, the points A, B, C and D are the vertices of Square.

Review # 9

Q9: **Show that $(6,5),(2,-4),\text{and}(5,-1)$ is an isosceles triangle.**

Solution:

Let $A(6,5), B(2,-4), C(5,-1)$

Let $x_1 = 6,\ y_1 = 5$

And $x_2 = 2,\ y_2 = -4$

Also $x_3 = 5,\ y_3 = -1$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(2-6)^2 + (-4-5)^2}$$

$$|AB| = \sqrt{(-4)^2 + (-9)^2}$$

$$|AB| = \sqrt{16 + 81}$$

$$|AB| = \sqrt{97}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

$$|BC| = \sqrt{(5-2)^2 + (-1-(-4))^2}$$

$$|BC| = \sqrt{(-3)^2 + (-1+4)^2}$$

$$|BC| = \sqrt{9 + (3)^2}$$

$$|BC| = \sqrt{9 + 9}$$

$$|BC| = \sqrt{18}$$

$$|BC| = \sqrt{9 \times 2}$$

$$|BC| = \sqrt{9} \times \sqrt{2}$$

$$|BC| = 3\sqrt{2}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$|AC| = \sqrt{(5-6)^2 + (-1-5)^2}$$

$$|AC| = \sqrt{(-1)^2 + (-6)^2}$$

$$|AC| = \sqrt{1 + 36}$$

$$|AC| = \sqrt{37}$$

Here

$$|AB| \neq |BC| \neq |CD| \neq |AD|$$

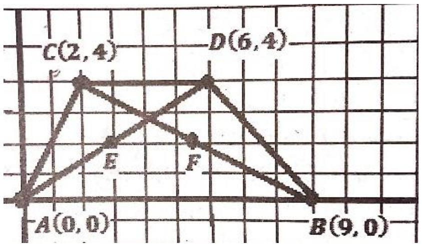
So these are not the vertices of an isosceles triangle.

Review # 9

Activity

You have a quadrilateral with vertices A(0, 0), B(9, 0), C(2, 4), D(6, 4). Find the mid – points of their diagonals. Does diagonals cut at the midpoint. Show it on graph paper.

Solution:



Let $x_1 = 0,$

$y_1 = 0$

And $x_2 = 9,$

$y_2 = 0$

Also $x_3 = 2,$

$y_3 = 4$

Also $x_4 = 6,\ y_4 = 4$

Here the diagonals are AD and BC

Now

$$\text{Midpoint of AD} = \left(\frac{x_1 + x_4}{2}\ , \frac{y_1 + y_4}{2}\right)$$

Put the values

$$\text{Midpoint of AD} = \left(\frac{0+6}{2}\ , \frac{0+4}{2}\right)$$

$$\text{Midpoint of AD} = \left(\frac{6}{2}\ , \frac{4}{2}\right)$$

$$\text{Midpoint of AD} = (3\ , 2)$$

$$\text{Midpoint of BC} = \left(\frac{x_2 + x_3}{2}\ , \frac{y_2 + y_3}{2}\right)$$

Put the values

$$\text{Midpoint of BC} = \left(\frac{9+2}{2}\ , \frac{0+4}{2}\right)$$

$$\text{Midpoint of BC} = \left(\frac{11}{2}\ , \frac{4}{2}\right)$$

$$\text{Midpoint of BC} = (5.5\ , 2)$$

As the mid – points of diagonals are not same.

So, the diagonals do not cut at mid – point.

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MATHEMATICS

Class 9th (KPK)

Unit # 17 Practical Geometry Triangles

NAME: ______________________________

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CLASS:_____________ SECTION: _________

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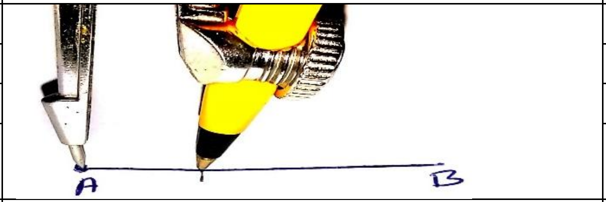
UNIT # 17

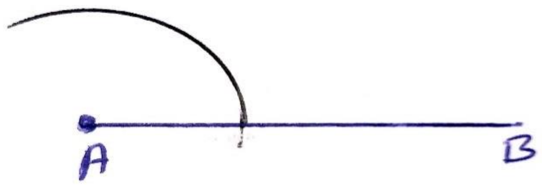
PRACTICAL GEOMETRY

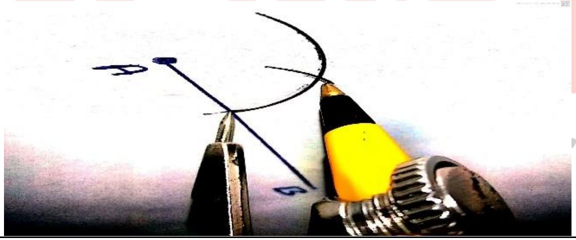
Ex # 17.1

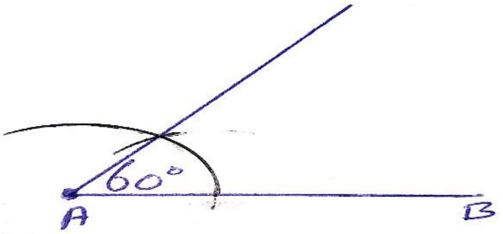
How to draw different angles with the help of compass

First we draw an angle of 60^0

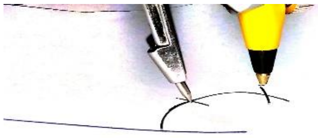


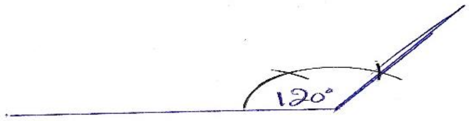






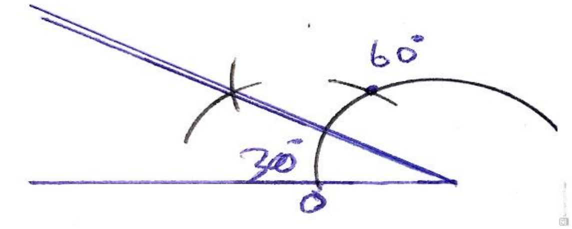
Angle of 120^0



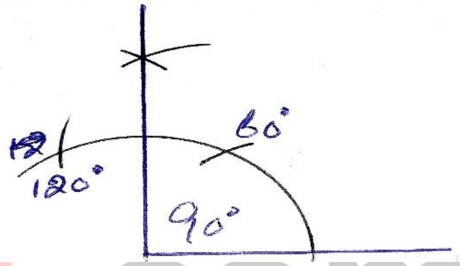


Ex # 17.1

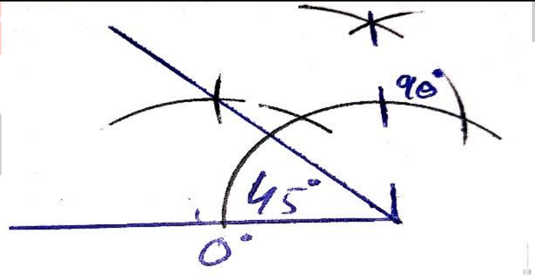
$30^0 = 0$ & 60^0



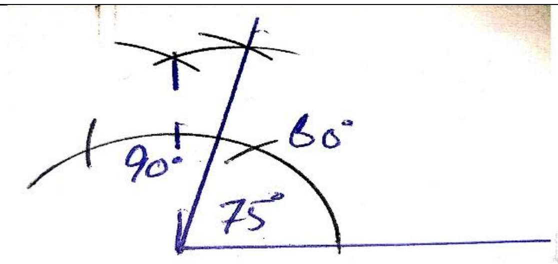
$90^0 = 60^0$ & 120^0



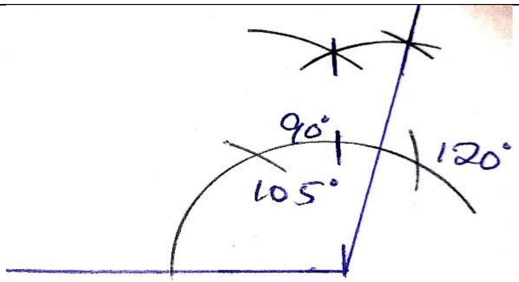
$45^0 = 0$ & 90^0



$75^0 = 60^0$ & 90^0

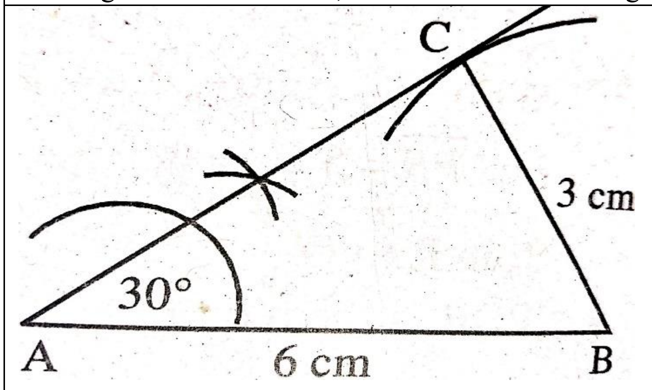


$105^0 = 90^0$ & 120^0



Elements of Triangle

A triangle has six elements, three sides and three angles.



In the above example, sides are

$\overline{AB}, \overline{BC}, \overline{AC}$ **OR** Side AB, Side BC , Side AC

The three angles are

$\angle A, \angle B, \angle C$ OR $\angle BAC, \angle ABC, \angle ACB$ OR
Angle A, Angle B, Angle C respectively

Construction of a Triangle

Case 1:

When two sides and the included angle are given

Example # 1

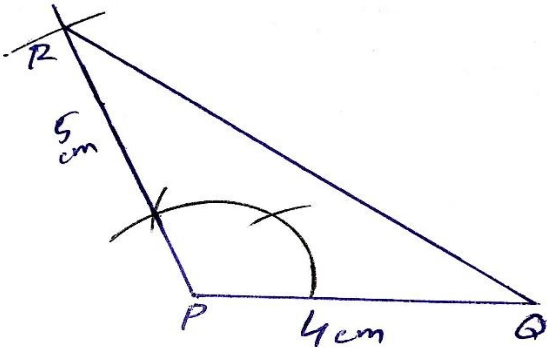
Construct a triangle PQR given that, $m\overline{PQ} = 4$ cm, $m\overline{PR} = 5$ cm, and $m\angle P = 120^0$

Given

$\overline{PQ} = 4\,cm,\ m\overline{PR} = 5\,cm,$ and $m\angle R = 120^0$

Required

Construct ΔPQR



Steps of Construction

I. Draw a line $m\overline{PQ} = 4\ cm$
II. At point R, draw an angle of 120^0
III. With P as centre, draw an arc of radius 5 cm which cuts angle 120^0 at point R.
IV. Join Q to R.
V. Thus $PQR$is the required triangle.

Case 2:

Whwn one side and two of the angles are given

Example # 2

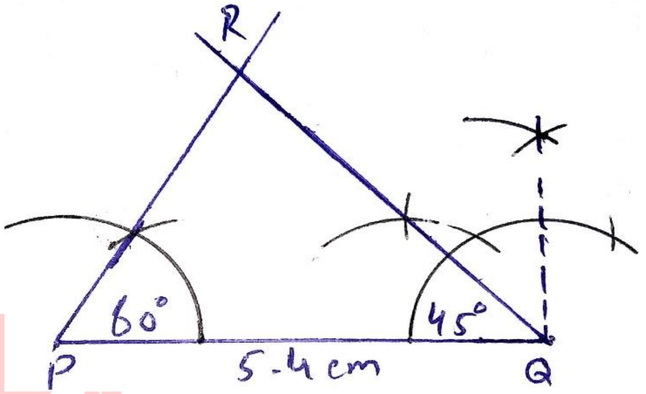
Construct ΔPQR such that, m $\overline{PQ} = 5.4$ cm, m $\angle$PQR $= 45^0$, and m$\angle$RPQ $= 60^0$

Given

$m\ \overline{PQ} = 5.4\ cm,\ m\ \angle Q = 45^0$, and $m\angle P = 60^0$

Required

Construct ΔPQR



Steps of Construction

I. Draw a line $m\ \overline{PQ} = 5.4\ cm$
II. At point P, draw an angle of 60^0
III. At point Q, draw another angle of 45^0
IV. Both the angles meet at point R
V. Thus PQR is the required triangle.

Case 3:

When two of its sides and the angle opposite to one of them are given.

In this case, there are three possibilities.

1. When only one triangle can be constructed with the help of given data. (Example # 3)
2. When two triangles can be constructed with the help of given data (Example # 4)
3. When construction of a trjiangle is not possible with the help of given datat. (Example # 5)

Unit # 17

Example # 3

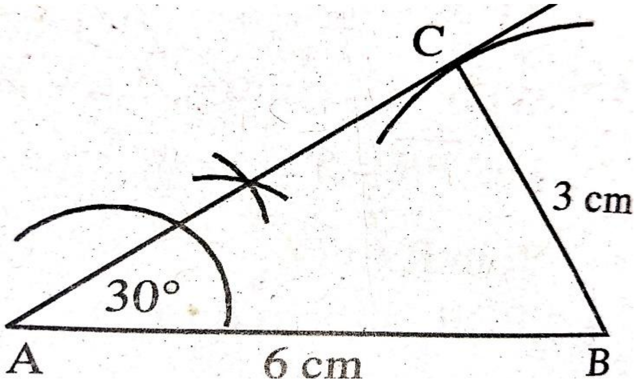
Construct a ΔABC, such that $m\overline{BC} = 3\ cm$, $m\overline{AB} = 6\ cm$, and $m\angle A = 30^0$

Given

$m\overline{BC} = 3\ cm,\ m\overline{AB} = 6\ cm,$ and $m\angle A = 30^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 6\ cm$
2. At point A, draw an angle of 30^0
3. With B as centre, draw an arc of radius 3 cm which cuts angle 30^0 at point C
4. Join B to C
5. Thus ABC is the required triangle.

Example # 4

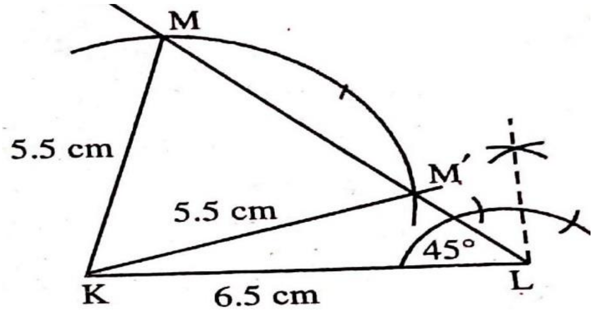
Construct a ΔKLM, such that $m\overline{KL} = 6.5\ cm$, $m\overline{KM} = 5.5\ cm$, and $m\angle L = 45^0$

Given

$m\overline{KL} = 6.5\ cm,\ m\overline{KM} = 5.5\ cm,$ and $m\angle L = 45^0$

Required

Construct a ΔKLM



Steps of Construction

1. Draw a line $m\overline{KL} = 6.5\ cm$
2. At point L, draw an angle of 45^0
3. With K as centre, draw an arc of radius 5.5 cm which cuts angle 45^0 at points M and M'
4. Join K to M and M'.
5. Thus KLM and KLM' are the required triangles.

Example # 5

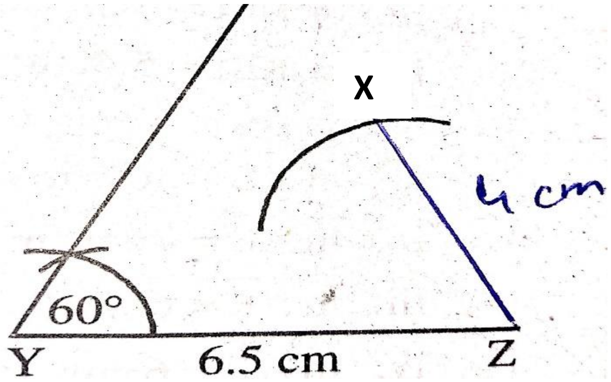
Construct a ΔXYZ, when $m\overline{ZX} = 4\ cm$, $m\overline{YZ} = 6.5\ cm$, and $m\angle Y = 60^0$

Given

$m\overline{ZX} = 4\ cm,\ m\overline{YZ} = 6.5\ cm,$ and $m\angle Y = 60^0$

Required

Construct a ΔXYZ



Steps of Construction

1. Draw a line $m\overline{YZ} = 6.5\ cm$
2. At point Y, draw an angle of 60^0
3. With Z as centre, draw an arc of radius 4 cm which does not cut angle 60^0
4. Join Z to X
5. So no triangle is constructed according to the given data.

Ex # 17.1

Page # 302

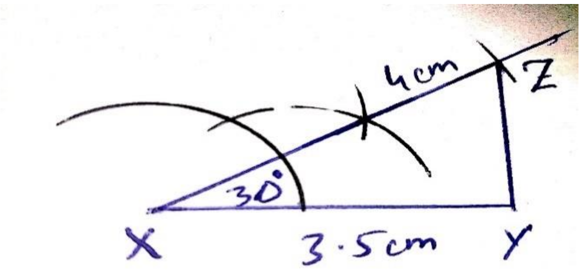
Q1 (i) Construct a ΔXYZ, when $m\angle X = 30^0$, $m\overline{XY} = 3.5\ cm$, and $m\overline{XZ} = 4\ cm$

Given

$m\angle X = 30^0,\ m\overline{XY} = 3.5\ cm,$ and $m\overline{XZ} = 4\ cm$

Required

Construct a ΔXYZ



Steps of Construction

1. Draw a line $m\overline{XY} = 3.5\ cm$
2. At point X, draw an angle of 30^0
3. With X as centre, draw an arc of radius 4 cm which cuts angle 30^0 at point Z.
4. Join Y to Z.
5. Thus XYZ is the required triangle.

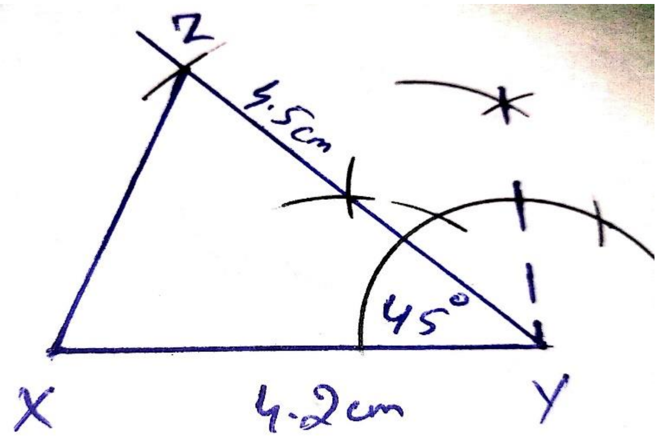
Q1 (ii) Construct a ΔXYZ, when $m\angle Y = 45^0$, $m\overline{XY} = 4.2\ cm$, and $m\overline{YZ} = 4.5\ cm$

Given

$m\angle Y = 45^0$, $m\overline{XY} = 4.2\ cm$, and $m\overline{YZ} = 4.5\ cm$

Required

Construct a ΔXYZ



Steps of Construction

1. Draw a line $m\overline{XY} = 4.2\ cm$
2. At point Y, draw an angle of 45^0
3. With Y as centre, draw an arc of radius 4.5 cm which cuts angle 45^0 at point Z.
4. Join X to Z.
5. Thus XYZ is the required triangle.

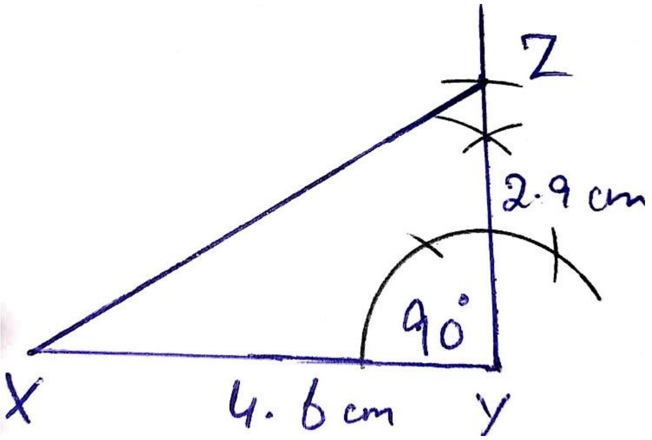
Q1 (iv) Construct a ΔXYZ, when $m\angle Y = 90^0$, $m\overline{XY} = 4.6\ cm$, and $m\overline{YZ} = 2.9\ cm$

Given

$m\angle Y = 90^0$, $m\overline{XY} = 4.6\ cm$, and $m\overline{YZ} = 2.9\ cm$

Required

Construct a ΔXYZ



Steps of Construction

1. Draw a line $m\overline{XY} = 4.6\ cm$
2. At point Y, draw an angle of 90^0
3. With Y as centre, draw an arc of radius 2.9 cm which cuts angle 90^0 at point Z.
4. Join X to Z.
5. Thus XYZ is the required triangle.

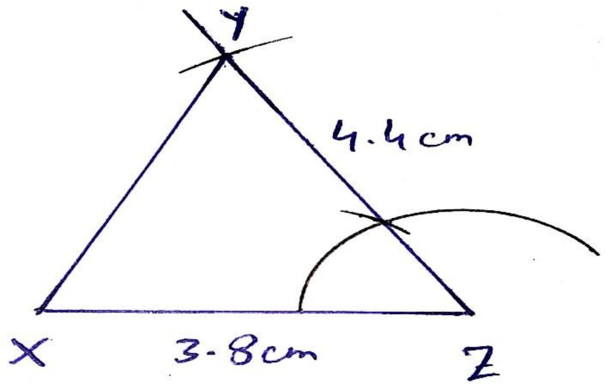
Q1 (iii) Construct a ΔXYZ, when $m\angle Z = 60^0$, $m\overline{XZ} = 3.8\ cm$, and $m\overline{YZ} = 4.4\ cm$

Given

$m\angle Z = 60^0$, $m\overline{XZ} = 3.8\ cm$, and $m\overline{YZ} = 4.4\ cm$

Required

Construct a ΔXYZ



Steps of Construction

1. Draw a line $m\overline{XZ} = 3.8\ cm$
2. At point Z, draw an angle of 60^0
3. With Z as centre, draw an arc of radius 4.4 cm which cuts angle 60^0 at point Y.
4. Join X to Y.
5. Thus XYZ is the required triangle.

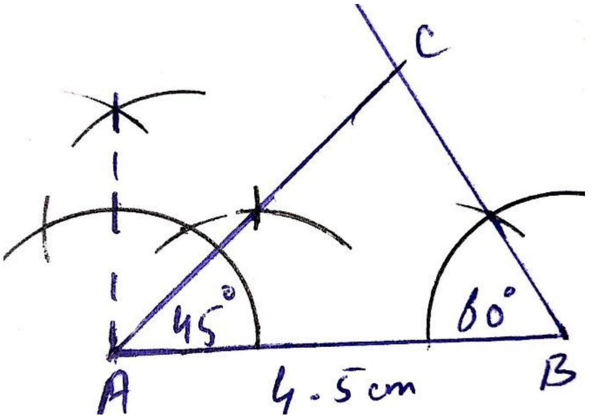
Q2 (i) Construct a ΔABC, when $m\ \overline{AB} = 4.5\ cm$, $m\ \angle A = 45^0$, and $m\angle B = 60^0$

Given

$m\ \overline{AB} = 4.5\ cm,\ m\ \angle A = 45^0,\ and\ m\angle B = 60^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 4.5\ cm$
2. At point A, draw an angle of 45^0
3. At point B, draw another angle of 60^0
4. Both the angles meet at point C
5. Thus ABC is the required triangle.

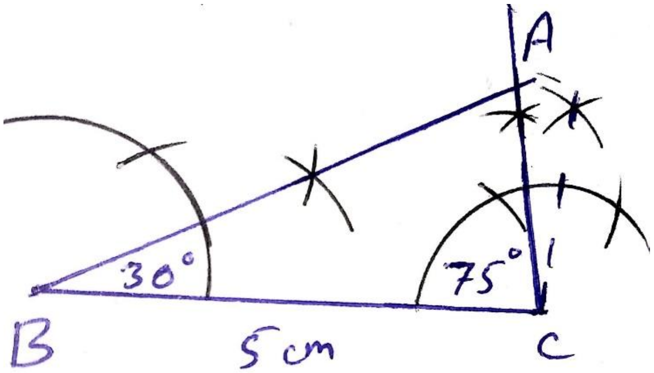
Q2 (ii) Construct a ΔABC, when $\boldsymbol{m\,\overline{BC} = 5\ cm}$, $\boldsymbol{m\,\angle B = 30^0}$, and $\boldsymbol{m\angle C = 75^0}$

Given

$m\,\overline{BC} = 5\ cm,\ m\,\angle B = 30^0,\ and\ m\angle C = 75^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{BC} = 5\ cm$
2. At point B, draw an angle of 30^0
3. At point C, draw another angle of 75^0
4. Both the angles meet at point A
5. Thus ABC is the required triangle.

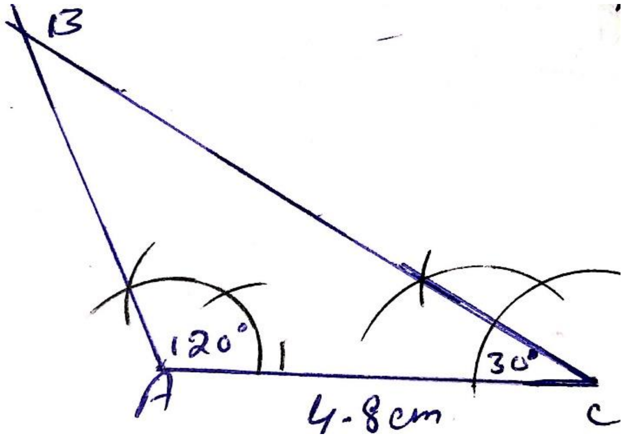
Q2 (iii) Construct a ΔABC, when $\boldsymbol{m\,\overline{AC} = 4.8\ cm}$, $\boldsymbol{m\,\angle A = 120^0}$, and $\boldsymbol{m\angle C = 30^0}$

Given

$m\,\overline{AC} = 4.8\ cm,\ m\,\angle A = 120^0,$ and $m\angle C = 30^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AC} = 4.8\ cm$
2. At point A, draw an angle of 120^0
3. At point C, draw another angle of 30^0
4. Both the angles meet at point B
5. Thus ABC is the required triangle.

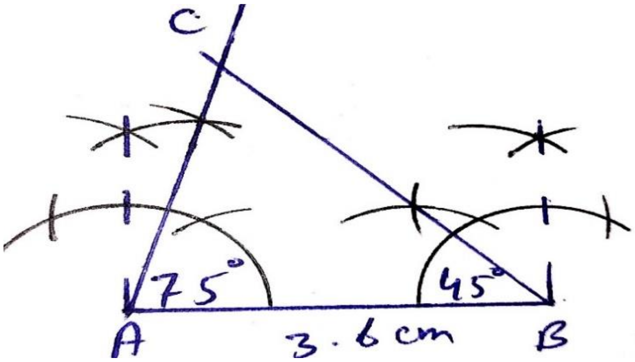
Q2 (iv) Construct a ΔABC, when $\boldsymbol{m\,\overline{AB} = 3.6\ cm}$, $\boldsymbol{m\,\angle A = 75^0}$, and $\boldsymbol{m\angle B = 45^0}$

Given

$m\,\overline{AB} = 3.6\ cm,\ m\,\angle A = 75^0,$ and $m\angle B = 45^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 3.6\ cm$
2. At point A, draw an angle of 75^0
3. At point B, draw another angle of 45^0
4. Both the angles meet at point C
5. Thus ABC is the required triangle.

Q3 (i) Construct a ΔKLM, when $m\,\overline{KL} = 4.8\ cm$, $\boldsymbol{m\,\angle K = 45^0}$, **and** $\boldsymbol{m\angle M = 60^0}$

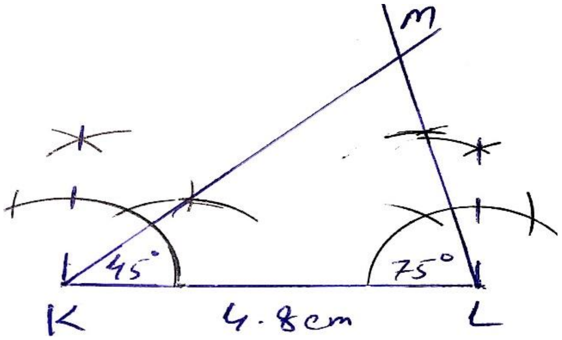
Given

$\boldsymbol{m}\,\overline{KL} = 4.8\ cm,\ m\,\angle K = 45^0,$ and $m\angle M = 60^0$

Required

Construct a ΔKLM

$$m\angle K + m\angle L + m\angle M = 180^0$$
$$45^0 + m\angle L + 60^0 = 180^0$$
$$105^0 + m\angle L = 180^0$$
$$m\angle L = 180^0 - 105^0$$
$$m\angle L = 75^0$$



Steps of Construction

1. Draw a line $m\,\overline{KL} = 4.8\ cm$
2. At point K, draw an angle of 45^0
3. At point L, draw another angle of 75^0
4. Both the angles meet at point M
5. Thus KLM is the required triangle.

Q3 (ii) Construct a ΔKLM, when $m\,\overline{LM} = 3.8\ cm$, $m\ \angle K = 30^0$, and $m\angle M = 75^0$

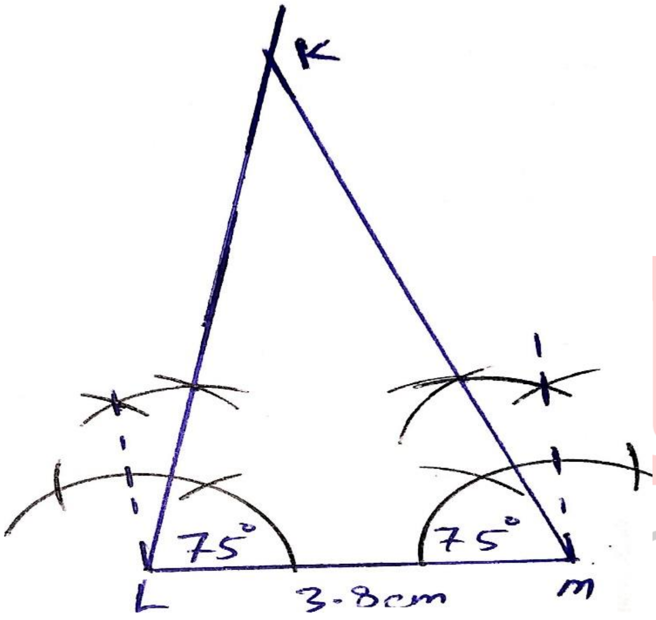
Given

$m\ \overline{LM} = 3.8\ cm,\ m\ \angle K = 30^0,$ and $m\angle M = 75^0$

Required

Construct a ΔKLM

$$m\angle K + m\angle L + m\angle M = 180^0$$
$$30^0 + m\angle L + 75^0 = 180^0$$
$$105^0 + m\angle L = 180^0$$
$$m\angle L = 180^0 - 105^0$$
$$m\angle L = 75^0$$



Steps of construction

1. Draw a line $m\ \overline{LM} = 3.8\ cm$
2. At point L, draw an angle of 75^0
3. At point M, draw another angle of 75^0
4. Both the angles meet at point K
5. Thus KLM is the required triangle.

Q3 (iii) Construct a ΔKLM, when $m\ \overline{KM} = 5\ cm$, $m\ \angle K = 105^0$, and $m\angle L = 45^0$

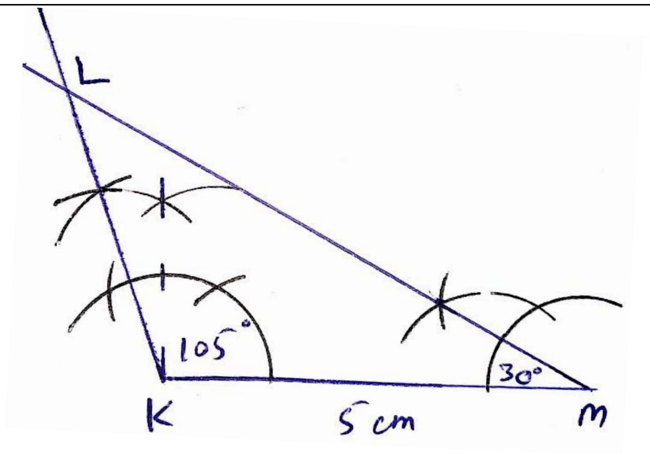
Given

$m\ \overline{KM} = 5\ cm,\ m\ \angle K = 105^0,$ and $m\angle L = 45^0$

Required

Construct a ΔKLM

$$m\angle K + m\angle L + m\angle M = 180^0$$
$$105^0 + 45^0 + m\angle M = 180^0$$
$$150^0 + m\angle M = 180^0$$
$$m\angle M = 180^0 - 150^0$$
$$m\angle M = 30^0$$



Steps of construction

1. Draw a line $m\ \overline{KM} = 5\ cm$
2. At point K, draw an angle of 105^0
3. At point M, draw another angle of 30^0
4. Both the angles meet at point L
5. Thus KLM is the required triangle.

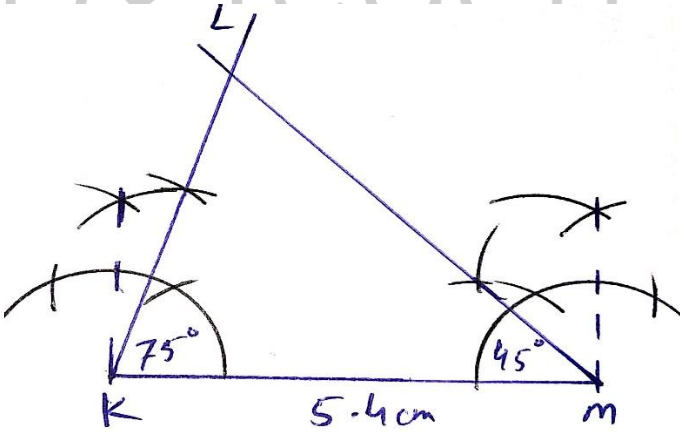
Q3 (iv) Construct a ΔKLM, when $m\overline{KM} = 5.4\ cm$, $m\ \angle K = 75^0$, and $m\angle M = 45^0$

Given

$m\ \overline{KM} = 5.4\ cm,\ m\ \angle K = 75^0,$ and $m\angle M = 45^0$

Required

Construct a ΔKLM



Steps of construction

1. Draw a line $m\ \overline{KM} = 5.4\ cm$
2. At point K, draw an angle of 75^0
3. At point M, draw another angle of 60^0
4. Both the angles meet at point L
5. Thus KLM is the required triangle.

Q4: Construct a ΔABC (Whenever possible), for the following assumptions:

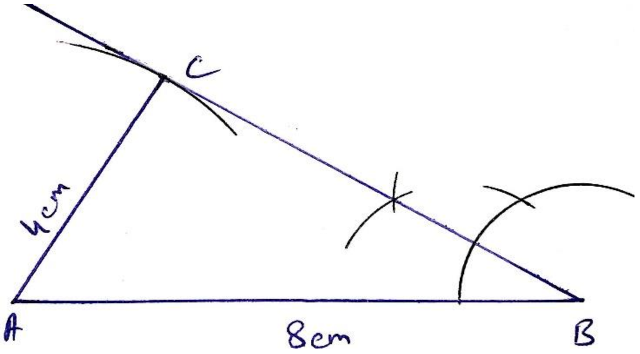
Q4 (i) Construct a ΔABC, when $m\angle B = 30^0$, $m\overline{AB} = 8\ cm$, and $m\overline{AC} = 4\ cm$

Given

$m\angle B = 30^0$, $m\overline{AB} = 8\ cm$, and $m\overline{AC} = 4\ cm$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 8\ cm$
2. At point B, draw an angle of 30^0
3. With A as centre, draw an arc of radius 4 cm which cuts angle 30^0 at point C.
4. Join A to C.
5. Thus ABC is the required triangle.

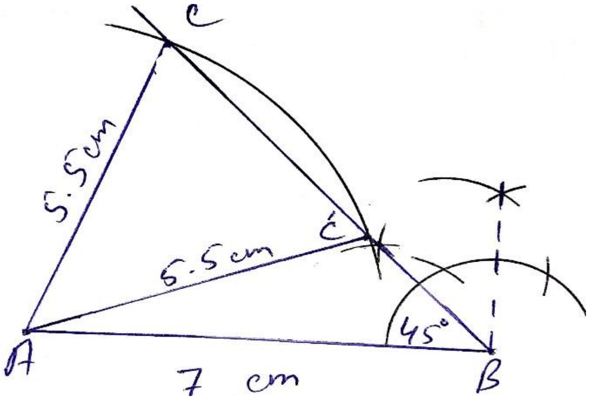
Q4 (ii) Construct a ΔABC, when $m\overline{AB} = 7\ cm$, $m\overline{AC} = 5.5\ cm$, and $m\angle B = 45^0$

Given

$m\overline{AB} = 7\ cm$, $m\overline{AC} = 5.5\ cm$, and $m\angle B = 45^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 7\ cm$
2. At point B, draw an angle of 45^0
3. With A as centre, draw an arc of radius 5.5 cm which cuts angle 45^0 at points C and C'
4. Join A to C and C'.
5. Thus ABC and ABC' are the required triangles.

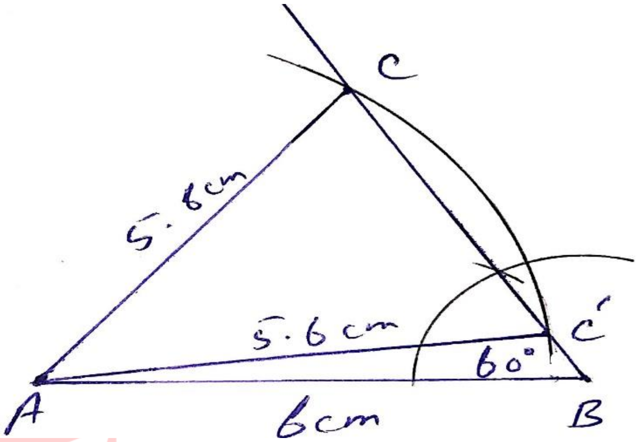
Q4 (iii) Construct a ΔABC, when $m\overline{AB} = 6\ cm$, $m\overline{AC} = 5.6\ cm$, and $m\angle B = 60^0$

Given

$m\overline{AB} = 6\ cm$, $m\overline{AC} = 5.6\ cm$, and $m\angle B = 60^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 6\ cm$
2. At point B, draw an angle of 60^0
3. With A as centre, draw an arc of radius 5.6 cm which cuts angle 60^0 at points C and C'
4. Join A to C and C'.
5. Thus ABC and ABC' are the required triangles.

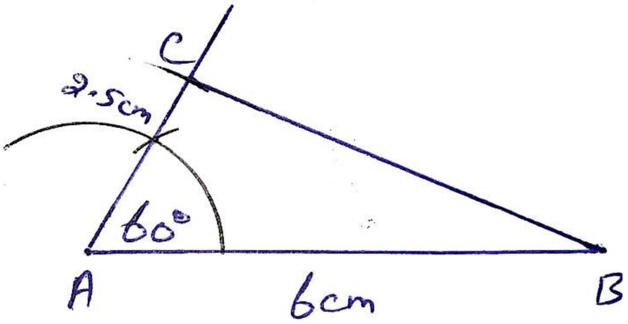
Q4 (iv) Construct a ΔABC, when $m\overline{AB} = 6\ cm$, $m\overline{AC} = 2.5\ cm$, and $m\angle A = 60^0$

Given

$m\overline{AB} = 6\ cm$, $m\overline{AC} = 2.5\ cm$, and $m\angle A = 60^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 6\ cm$
2. At point A, draw an angle of 60^0
3. With A as centre, draw an arc of radius 2.5 cm which cuts angle 60^0 at points C
4. Join B to C
5. Thus ABC is the required triangle.

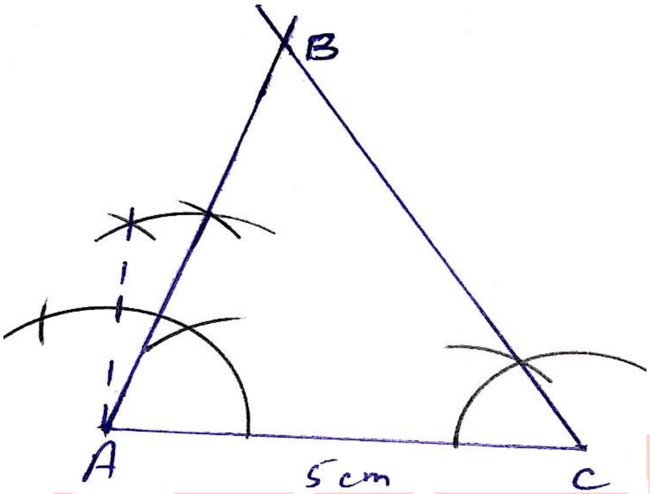
Q4 (v) Construct a ΔABC, when $m\ \overline{AC} = 5\ cm$, $m\ \angle A = 75^0$, and $m\angle C = 60^0$

Given

$m\ \overline{AC} = 5\ cm,\ m\ \angle A = 75^0,$ and $m\angle C = 60^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AC} = 5\ cm$
2. At point A, draw an angle of 75^0
3. At point C, draw another angle of 60^0
4. Both the angles meet at point B
5. Thus ABC is the required triangle.

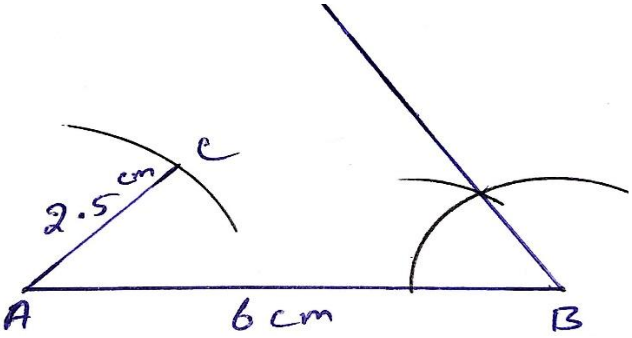
Q4 (vi) Construct a ΔABC, when $m\overline{AB} = 6\ cm$, $m\overline{AC} = 2.5\ cm$, and $m\angle B = 60^0$

Given

$m\overline{AB} = 6\ cm,\ m\overline{AC} = 2.5\ cm,$ and $m\angle B = 60^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 6\ cm$
2. At point B, draw an angle of 60^0
3. With A as centre, draw an arc of radius 2.5 cm which does not cut angle 60^0
4. Join B to C
5. So no triangle is constructed according to the given data.

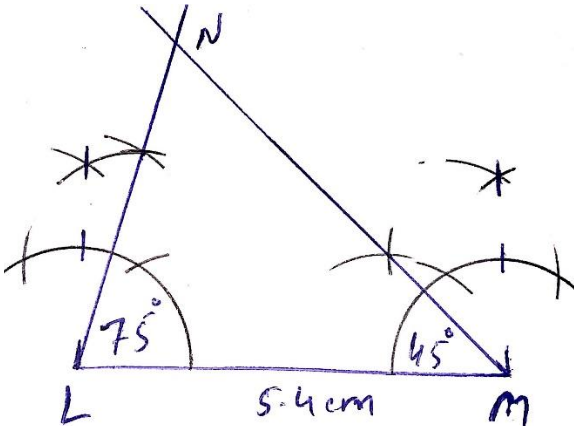
Q#5 Construct ΔLMN such that, $m\ \overline{LM} = 5.4\ cm$, $m\ \angle L = 75^0$, and $m\angle M = 45^0$

Given

$m\ \overline{LM} = 5.4\ cm,\ m\ \angle L = 75^0,$ and $m\angle M = 45^0$

Required

Construct a ΔLMN



Steps of Construction

1. Draw a line $m\overline{LM} = 5.4\ cm$
2. At point L, draw an angle of 75^0
3. At point M, draw another angle of 45^0
4. Both the angles meet at point N
5. Thus LMN is the required triangle.

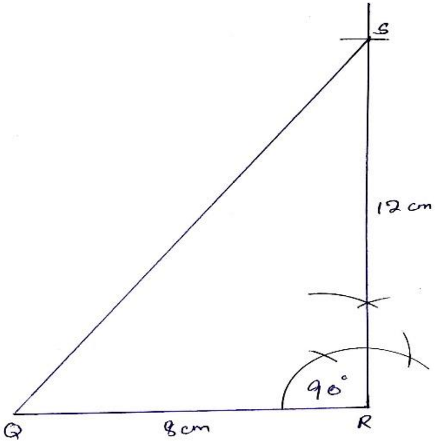
Q#6 Construct ΔQRS such that, $m\overline{QR} = 8\ cm$, $m\overline{RS} = 12\ cm$, and $m\angle R = 90^0$

Given

$\overline{QR} = 8\ cm,\ m\overline{RS} = 12\ cm,$ and $m\angle R = 90^0$

Required

Construct ΔPQR



Steps of Construction

1. Draw a line $m\overline{QR} = 8\ cm$
2. At point R, draw an angle of 90^0
3. With R as centre, draw an arc of radius 12 cm which cuts angle 90^0 at point S.
4. Join Q to S.
5. Thus $PQR$is the required triangle.

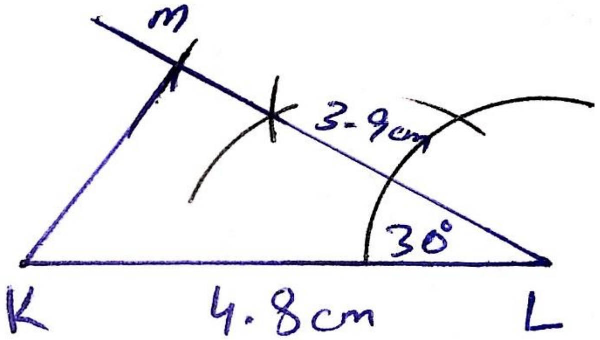
Q#7 Construct ΔKLM such that, $m\overline{KL} = 4.8$ cm, $m\overline{LM} = 3.9$ cm, and $m\angle L = 30^0$

Given

$m\overline{KL} = 4.8\ cm,\ m\overline{LM} = 3.9\ cm,$ and $m\angle L = 30^0$

Required

Construct ΔKLM



Steps of Construction

1. Draw a line $m\overline{KL} = 4.8\ cm$
2. At point L, draw an angle of 30^0
3. With L as centre, draw an arc of radius 3.9 cm which cuts angle 30^0 at point M.
4. Join K to M.
5. Thus KLM is the required triangle.

Q8: Construct ΔPQR such that, m $\overline{QR} = 6.5$ cm, m $\angle P = 30^0$, and $m\angle Q = 60^0$

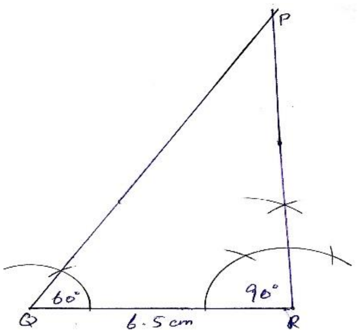
Given

$m\ \overline{QR} = 6.5\ cm,\ m\ \angle P = 30^0$, and $m\angle Q = 60^0$

Required

Construct ΔPQR

$$m\angle P + m\angle Q + m\angle R = 180^0$$
$$30^0 + 60^0 + m\angle R = 180^0$$
$$90^0 + m\angle R = 180^0$$
$$m\angle R = 180^0 - 90^0$$
$$m\angle R = 90^0$$



Steps of Construction

1. Draw a line $m\ \overline{QR} = 6.5\ cm$
2. At point Q, draw an angle of 60^0
3. At point R, draw another angle of 90^0
4. Both the angles meet at point P
5. Thus PQR is the required triangle.

Coplanar lines

Two or more lines lying in the same plane are called coplanar lines.

Note:

Two coplanar lines are either parallel or intersecting.

Parallel lines

Two lines that are at equal distance from each other and never meet is called parallel lines.

Intersecting lines

If two lines intersect each other at one and only one point is called intersecting lines.

Note:

The point where two lines intersect is called point of intersection.

Concurrent lines

Three or more lines pass through the same/single point is called concurrent lines.

Note:

The point where three lines pass is called point of concurrency.

Angle Bisector

A line which divides an angle into two equal parts is called angle bisector.

Example # 6

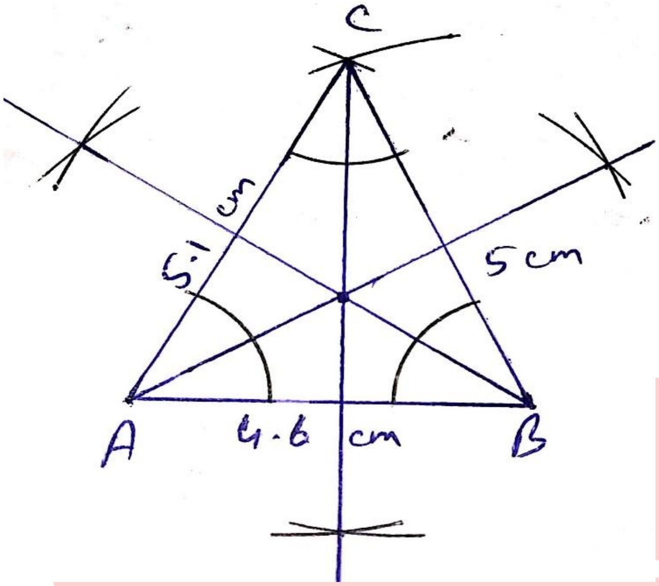
Construct ΔABC, where $m\overline{AB} = 4.6\ cm,\ m\overline{BC} = 5\ cm,$ and $m\overline{CA} = 5.1\ cm$. Draw angle bisectors of the triangle, and verify their concurrency

Given

$m\ \overline{AB} = 4.6\ cm,\ m\overline{BC} = 5\ cm,$ and $m\overline{CA} = 5.1\ cm$

Required

Angle bisectors of a ΔABC and their concurrency



Steps of construction

1. Draw a line $m\ \overline{AB} = 4.6\ cm$
2. With B as centre, draw an arc of radius 5 cm.
3. With A as centre, draw another arc of radius 5.1 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. Thus angle bisectors of a triangle are concurrent.

Altitude of the triangle

A perpendicular drawn from the verex of a triangle to its opposite side is called altitude of the triangle.

Example # 7

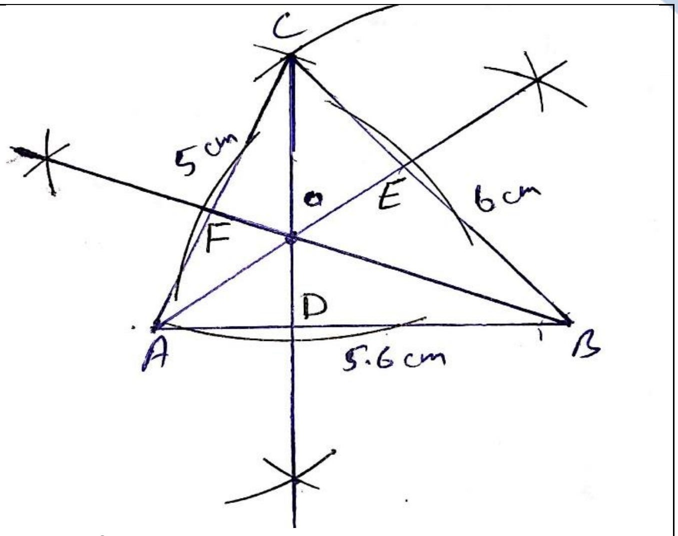
Construct ΔABC, where $m\overline{AB} = 5.6\ cm,\ m\overline{BC} = 6\ cm,$ and $m\overline{CA} = 5\ cm$. Draw its altitude and verify their concurrency

Given

$m\ \overline{AB} = 5.6\ cm,\ m\overline{BC} = 6\ cm,$ and $m\overline{CA} = 5\ cm$

Required

Altitudes of a ΔPQR and their concurrency



Steps of construction

1. Draw a line $m\ \overline{AB} = 5.6\ cm$
2. With B as centre, draw an arc of radius 6 cm.
3. With A as centre, draw another arc of radius 5 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Draw perpendiculars from A to $\overline{BC}$, B to $\overline{AC}$ and C to $\overline{AB}$ at point A, B and C respectively.
7. Thus $\overline{PA}$, $\overline{QB}$ and $\overline{RS}$ are the required altitudes
8. These altitudes pass through the same point O i.e. altitudes are concurrent.

Perpendicular Bisector

A line which is perpendicular to a line segment and divides it into two equal parts, is called its Perpendicular bisectors or Right Bisectors

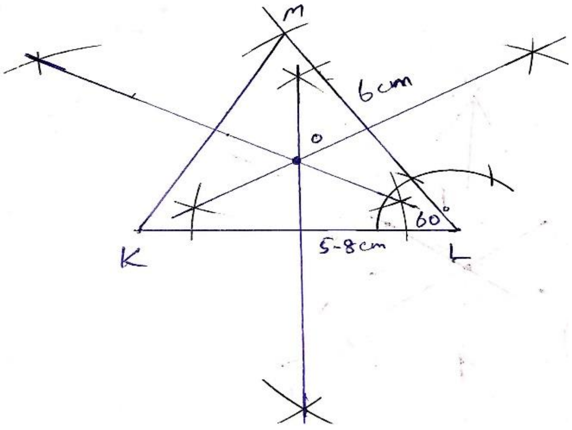
Example # 8 Construct ΔKLM, where $m\overline{KL} = 5.8\ cm,$ and $m\overline{LM} = 6\ cm,$ and $m\angle L = 60^0$ Draw its perpendicular bisectors and verify their concurrency

Given

$m\overline{KL} = 5.8\ cm,$ and $m\overline{LM} = 6\ cm,$ and $m\angle L = 60^0$

Required

Perpendicular bisectors of a ΔKLM and their concurrency



Steps of construction

1. Draw a line $m\overline{KL} = 5.8\ cm$
2. At point L, draw an angle of 60^0
3. With L as centre, draw an arc of radius 6 cm which cuts angle 60^0 at point M.
4. Join K to M.
5. Thus KLM is the triangle according to data.
6. Draw the perpendicular bisectors of $\overline{KL},\ \overline{LM}$ and $m\overline{MK}$
7. These perpendicular bisectors pass through point O i.e. perpendicular bisectors are concurrent.

Median

Line segment joining the midpoint of one side of a triangle to its opposite vertex is called Median.

Example # 9

Construct ΔABC, where $m\ \overline{AB} = 6\ cm,\ m\ \angle A = 70^0$, and $m\angle C = 50^0$ draw its medians and verify their concurrency

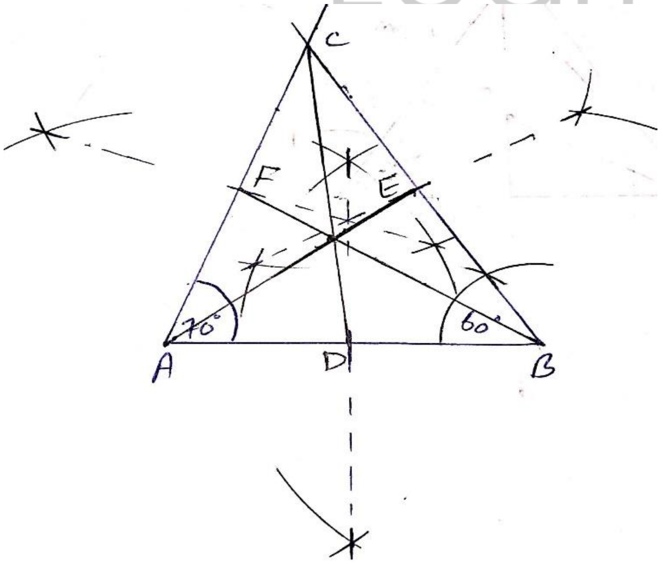
Given

$m\ \overline{AB} = 6\ cm,\ m\ \angle A = 70^0$, and $m\angle C = 50^0$

Required

Medians of a ΔABC and their concurrency

$$m\angle A + m\angle B + m\angle C = 180^0$$
$$70^0 + m\angle B + 50^0 = 180^0$$
$$70^0 + 50^0 + m\angle B = 180^0$$
$$120^0 + m\angle B = 180^0$$
$$m\angle B = 180^0 - 120^0$$
$$m\angle B = 60^0$$



Steps of construction

1. Draw a line $m\ \overline{AB} = 6\ cm$
2. At point A, draw an angle of 70^0
3. At point B, draw another angle of 60^0
4. Both the angles meet at point C.
5. Thus ABC is the triangle according to data.
6. Find the mid points D, E and F of $\overline{AB}, \overline{BC}$ and $\overline{AC}$ respectively by right bisectors.
7. Draw the medians $\overline{AE},\ \overline{BF}$ and $m\overline{CD}$
8. These medians pass through point O i.e. the medians of a triangle are concurrent.

Exercise # 17.2
Page # 306

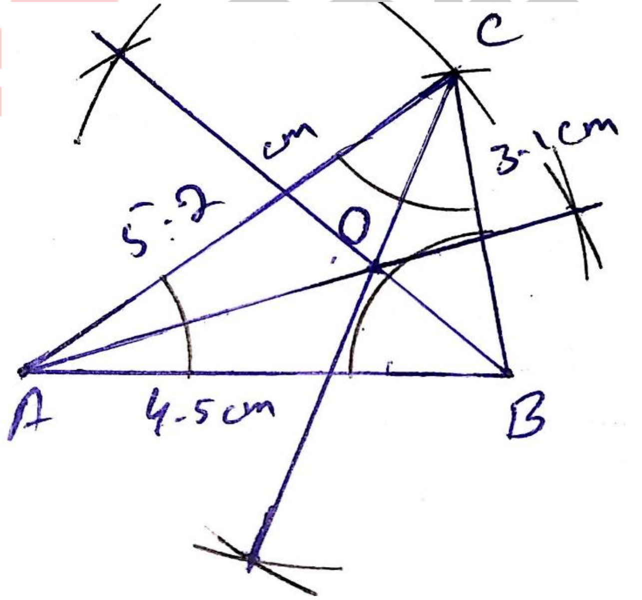
Q1 (i) Construct ΔABC, where $m\overline{AB} = 4.5\ cm$, $m\overline{BC} = 3.1\ cm$, and $m\overline{CA} = 5.2\ cm$ draw their angle bisectors and verify their concurrency

Given

$m\overline{AB} = 4.5\ cm,\ m\overline{BC} = 3.1\ cm$, and $m\overline{CA} = 5.2\ cm$

Required

Angle bisectors of a ΔABC and their concurrency



Steps of construction

1. Draw a line $m\ \overline{AB} = 4.5\ cm$
2. With B as centre, draw an arc of radius 3.1 cm.
3. With A as centre, draw another arc of radius 5.2 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. Thus angle bisectors of a triangle are concurrent.

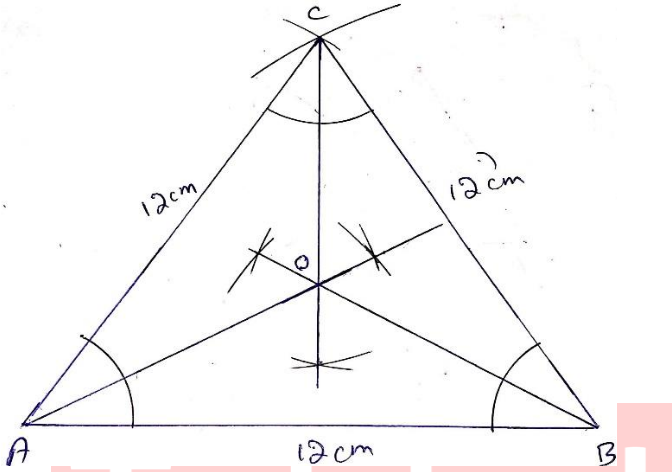
Q1 (ii) Construct ΔABC, where $m\overline{AB} = m\overline{BC} = m\overline{CA} = 12\ cm$ draw their angle bisectors and verify their concurrency

Given

$m\ \overline{AB} = m\overline{BC} = m\overline{CA} = 12\ cm$

Required

Angle bisectors of a ΔABC and their concurrency



Steps of construction

1. Draw a line $m\ \overline{AB} = 12\ cm$
2. With B as centre, draw an arc of radius 12 cm.
3. With A as centre, draw another arc of radius 12 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. Thus angle bisectors of a triangle are concurrent.

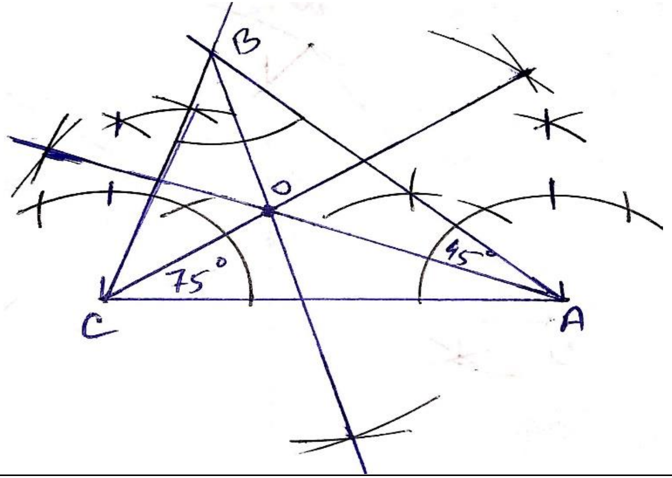
Q1 (iii) Construct ΔABC, where $m\ \overline{CA} = 5.8\ cm$, $m\ \angle A = 45^0$, and $m\angle C = 75^0$ draw their angle bisectors and verify their concurrency

Given

$m\ \overline{CA} = 5.8\ cm,\ m\ \angle A = 45^0,$ and $m\angle C = 75^0$

Required

Angle bisectors of a ΔABC and their concurrency



Steps of construction

1. Draw a line $m\overline{CA} = 5.8\ cm$
2. At point C, draw an angle of 75^0
3. At point A, draw another angle of 45^0
4. Both the angles meet at point B.
5. Thus ABC is the triangle according to data
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. Thus angle bisectors of a triangle are concurrent.

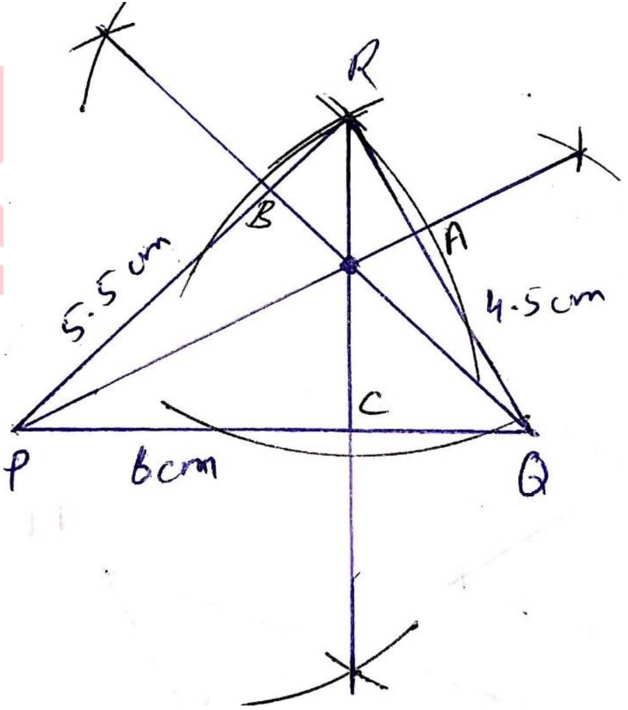
Q2 (i) Construct ΔPQR, where $m\overline{PQ} = 6\ cm$, $m\overline{QR} = 4.5\ cm$, and $m\overline{PR} = 5.5\ cm$ draw their altitudes and verify their concurrency

Given

$m\overline{PQ} = 6\ cm,\ m\overline{QR} = 4.5\ cm,$ and $m\overline{PR} = 5.5\ cm$

Required

Altitudes of a ΔPQR and their concurrency



Steps of construction

1. Draw a line $m\ \overline{PQ} = 6\ cm$
2. With Q as centre, draw an arc of radius 4.5 cm.
3. With P as centre, draw another arc of radius 5.5 cm.
4. Both the arcs meet at point R.
5. Thus PQR is the triangle according to data
6. Draw perpendiculars from P to $\overline{QR}$, Q to $\overline{PR}$ and R to $\overline{PQ}$ at point A, B and C respectively.
7. Thus $\overline{PA}$, $\overline{QB}$ and $\overline{RS}$ are the required altitudes
8. These altitudes pass through the same point O i.e. altitudes are concurrent.

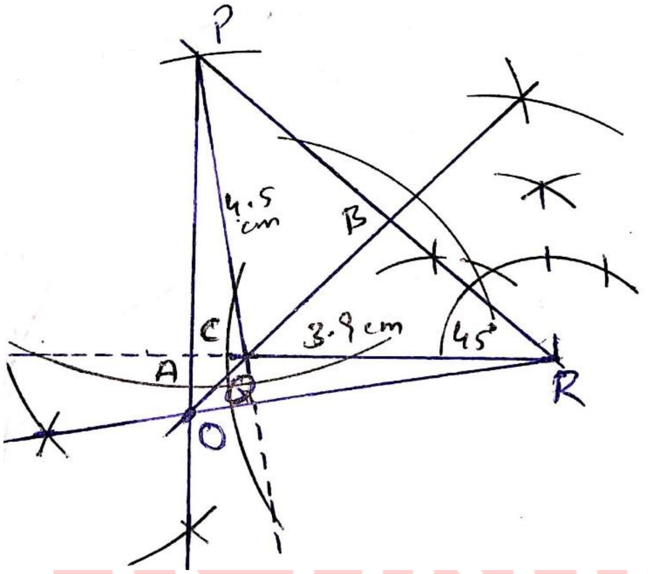
Q2 (ii) Construct ΔPQR, where $m\overline{PQ} = 4.5\ cm$, $m\overline{QR} = 3.9\ cm$, and $m\angle R = 45^0$ draw their altitudes and verify their concurrency

Given

$m\overline{PQ} = 4.5\ cm,\ m\overline{QR} = 3.9\ cm,$ and $m\angle R = 45^0$

Required

Altitudes of a ΔPQR and their concurrency



Steps of construction

1. Draw a line $m\overline{QR} = 3.9\ cm$
2. At point R, draw an angle of 45^0
3. With Q as centre, draw an arc of radius 4.5 cm which cuts angle 45^0 at point P.
4. Join Q to P.
5. Thus PQR is the triangle according to data
6. Draw perpendiculars from P to $\overline{QR}$, Q to $\overline{PR}$ and R to $\overline{PQ}$ at point A, B and C respectively.
7. Thus $\overline{PA}$, $\overline{QB}$ and $\overline{RS}$ are the required altitudes.
8. These altitudes pass through the same point O i.e. altitudes are concurrent.

Q2 (iii) Construct ΔPQR, where $m\overline{PQ} = 6\ cm$, $m\ \angle P = 70^0$, and $m\angle Q = 65^0$ draw their altitudes and verify their concurrency

Given

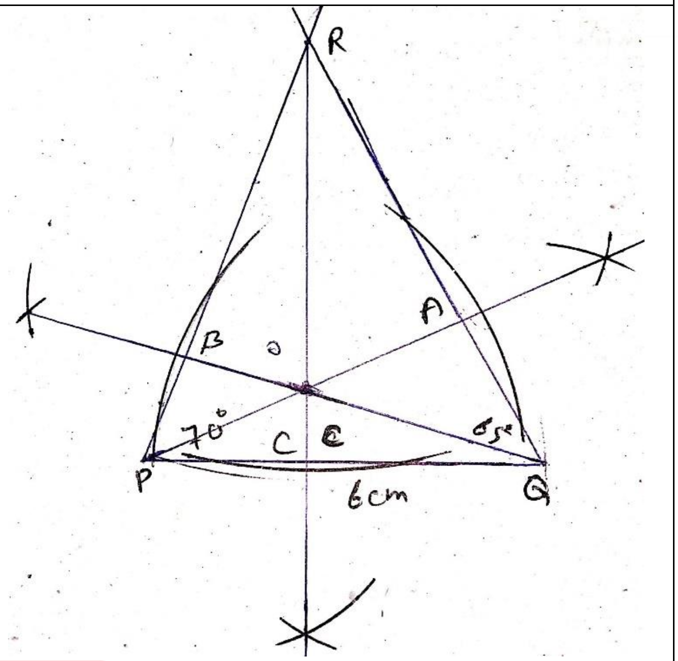
$m\overline{PQ} = 6\ cm,\ m\ \angle P = 70^0,$ and $m\angle Q = 65^0$

Required

Altitudes of a ΔPQR and their concurrency

Steps of construction

1. Draw a line $m\ \overline{PQ} = 6\ cm$
2. At point P, draw an angle of 70^0
3. At point Q, draw another angle of 65^0



4. Both the angles meet at point R
5. Thus PQR is the triangle according to data
6. Draw perpendiculars from P to $\overline{QR}$, Q to $\overline{PR}$ and R to $\overline{PQ}$ at point A, B and C respectively.
7. Thus $\overline{PA}$, $\overline{QB}$ and $\overline{RS}$ are the required altitudes.
8. These altitudes pass through the same point O i.e. altitudes are concurrent

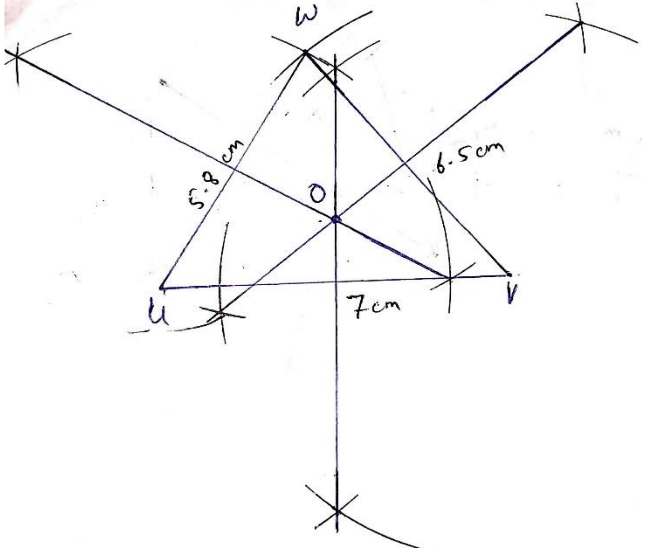
Q3 (i) Construct ΔUVW, where $m\overline{UV} = 7\ cm$, $m\overline{VW} = 6.5\ cm$, and $m\overline{WU} = 5.8\ cm$ draw their perpendicular bisectors and verify their concurrency

Given

$\overline{UV} = 7\ cm,\ m\overline{VW} = 6.5\ cm,$ and $m\overline{WU} = 5.8\ cm$

Required

Perpendicular bisectors of a ΔUVW and their concurrency



Steps of construction

1. Draw a line $m\,\overline{UV} = 7\,cm$
2. With V as centre, draw an arc of radius 6.5 cm.
3. With U as centre, draw another arc of radius 5.8 cm.
4. Both the arcs meet at point W.
5. Thus UVW is the triangle according to data.
6. Draw the perpendicular bisectors of $\overline{UV}$, $\overline{VW}$ and $m\overline{WU}$
7. These perpendicular bisectors pass through point O i.e. perpendicular bisectors are concurrent.

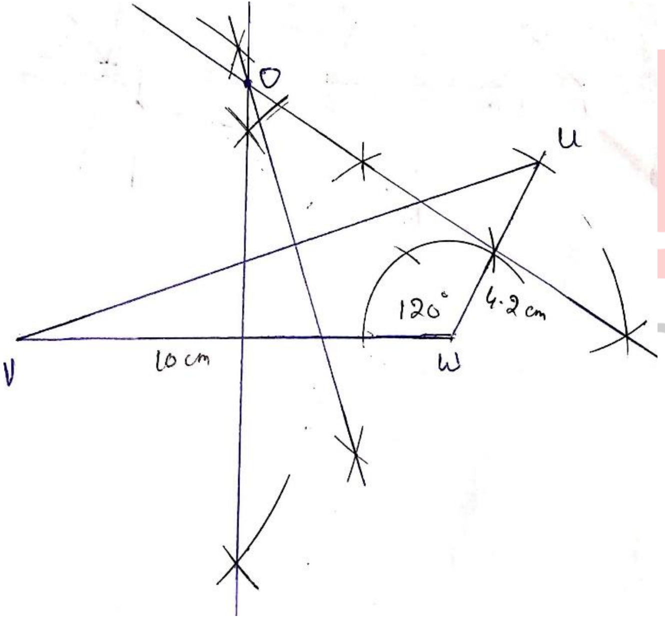
Q3 (ii) Construct ΔUVW, where $m\overline{VW} = 10\ cm,$ and $m\overline{WU} = 4.2\ cm,$ and $m\angle W = 120^0$ draw their perpendicular bisectors and verify their concurrency

Given

$m\overline{VW} = 10\,cm,$ and $m\overline{WU} = 4.2\,cm,$ and $m\angle W = 120^0$

Required

Perpendicular bisectors of a ΔUVW and their concurrency



Steps of construction

1. Draw a line $m\overline{VW} = 10\,cm$
2. At point W, draw an angle of 120^0
3. With W as centre, draw an arc of radius 4.2 cm which cuts angle 120^0 at point U.
4. Join V to U.
5. Thus UVW is the triangle according to data.
6. Draw the perpendicular bisectors of $\overline{UV}$, $\overline{VW}$ and $m\overline{WU}$
7. These perpendicular bisectors pass through point O i.e. perpendicular bisectors are concurrent.

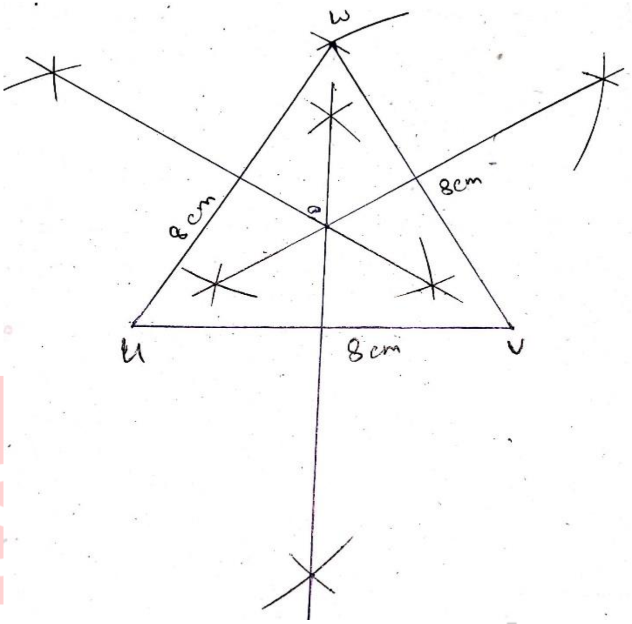
Q3 (iii) Construct ΔUVW, where $m\overline{UV} = m\overline{VW} = m\overline{WU} = 0.8\ dm$ draw their perpendicular bisectors and verify their concurrency

Given

$m\overline{UV} = m\overline{VW} = m\overline{WU} = 0.8\,dm = 8\,cm$

Required

Perpendicular bisectors of a ΔUVW and their concurrency



Steps of construction

1. Draw a line $m\,\overline{UV} = 8\,cm$
2. With V as centre, draw an arc of radius 8 cm.
3. With U as centre, draw another arc of radius 8 cm.
4. Both the arcs meet at point W.
5. Thus UVW is the triangle according to data.
6. Draw the perpendicular bisectors of $\overline{UV}$, $\overline{VW}$ and $m\overline{WU}$
7. These perpendicular bisectors pass through point O i.e. perpendicular bisectors are concurrent.

Q4 (i) Construct ΔXYZ, where $m\,\overline{YZ} = 4.1\ cm$, $m\,\angle Y = 60^0$, and $m\angle X = 75^0$ draw their medians and verify their concurrency

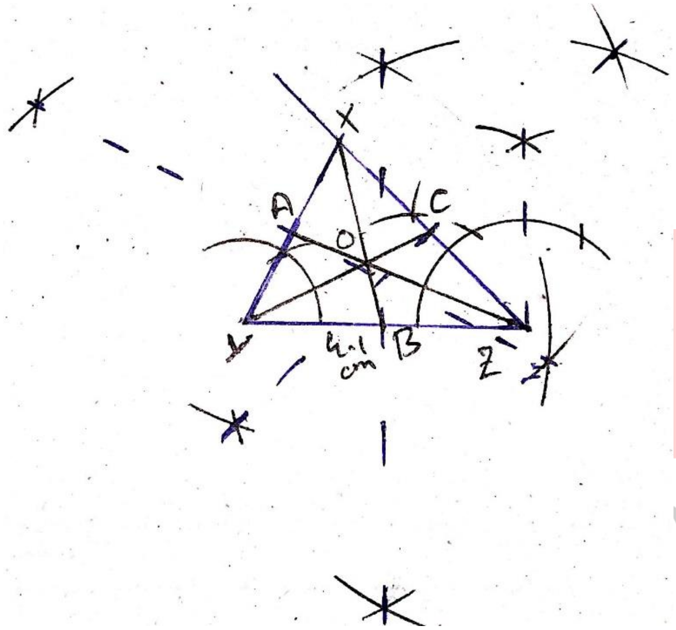
Given

$m\,\overline{YZ} = 4.1\ cm,\ m\,\angle Y = 60^0,$ and $m\angle X = 75^0$

Required

Medians of a ΔXYZ and their concurrency

$$m\angle X + m\angle Y + m\angle Z = 180^0$$
$$75^0 + 60^0 + m\angle Z = 180^0$$
$$135^0 + m\angle Z = 180^0$$
$$m\angle Z = 180^0 - 135^0$$
$$m\angle Z = 45^0$$



Steps of construction

1. Draw a line $m\,\overline{YZ} = 4.1\ cm$
2. At point Y, draw an angle of 60^0
3. At point Z, draw another angle of 45^0
4. Both the angles meet at point X
5. Thus XYZ is the triangle according to data.
6. Find the mid points A, B and C of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively by right bisectors.
7. Draw the medians $\overline{ZA}$, $\overline{XB}$ and $m\overline{YC}$
8. These medians pass through point O i.e. the medians of a triangle are concurrent.

Q4 (ii) Construct ΔXYZ, where $m\,\overline{ZX} = 4.3\ cm$, $m\,\angle X = 75^0$, and $m\angle Y = 45^0$ draw their medians and verify their concurrency

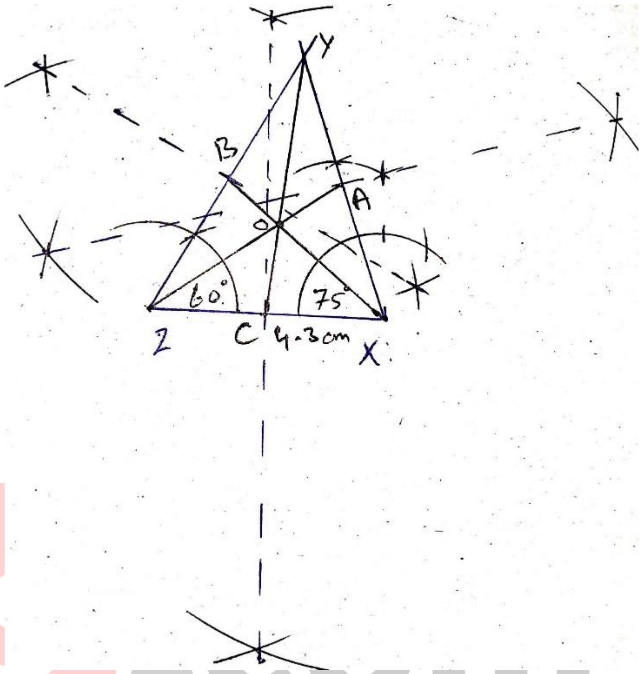
Given

$m\,\overline{ZX} = 4.3\ cm,\ m\,\angle X = 75^0,$ and $m\angle Y = 45^0$

Required

Medians of a ΔXYZ and their concurrency

$$m\angle X + m\angle Y + m\angle Z = 180^0$$
$$75^0 + 45^0 + m\angle Z = 180^0$$
$$120^0 + m\angle Z = 180^0$$
$$m\angle Z = 180^0 - 120^0$$
$$m\angle Z = 60^0$$



Steps of construction

1. Draw a line $m\,\overline{ZX} = 4.3\ cm$
2. At point Z, draw an angle of 60^0
3. At point X, draw another angle of 75^0
4. Both the angles meet at point Y.
5. Thus XYZ is the triangle according to data.
6. Find the mid points A, B and C of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively by right bisectors.
7. Draw the medians $\overline{ZA}$, $\overline{XB}$ and $m\overline{YC}$
8. These medians pass through point O i.e. the medians of a triangle are concurrent.

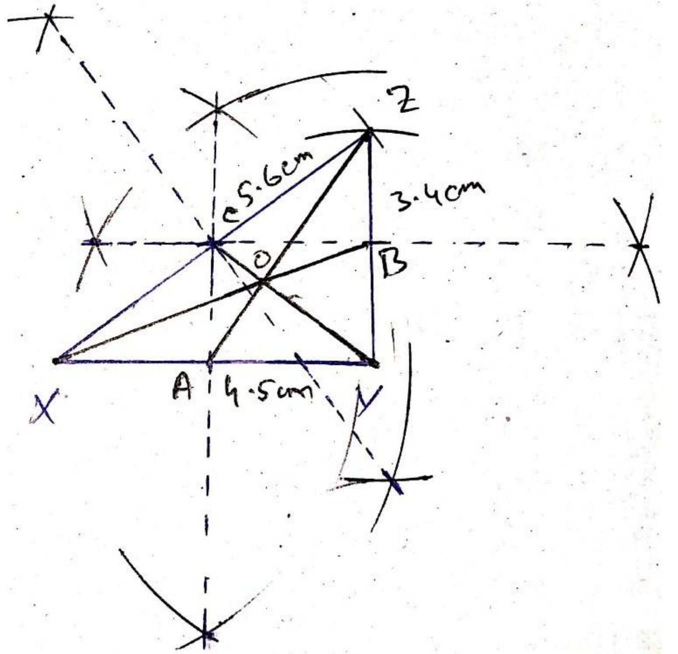
Q4 (iii) Construct ΔXYZ, where $m\overline{XY} = 4.5\ cm$, $m\overline{YZ} = 3.4\ cm$, and $m\overline{ZX} = 5.6\ cm$ draw their medians and verify their concurrency

Given

$m\overline{XY} = 4.5\ cm,\ m\overline{YZ} = 3.4\ cm,$ and $m\overline{ZX} = 5.6\ cm$

Required

Medians of a ΔXYZ and their concurrency



Steps of construction

1. Draw a line $m\,\overline{XY} = 4.5\ cm$
2. With Y as centre, draw an arc of radius 3.4 cm.
3. With X as centre, draw another arc of radius 5.6 cm.
4. Both the arcs meet at point Z.
5. Thus XYZ is the triangle according to data.
6. Find the mid points A, B and C of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively by right bisectors.
7. Draw the medians $\overline{ZA}$, $\overline{XB}$ and $m\overline{YC}$
8. These medians pass through point O i.e. the medians of a triangle are concurrent.

Exercise # 17.3

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Q # 1 Draw a quadrilateral ABCD such that $m\overline{AB} = 3\ cm$, $m\angle B = 60^0$, $m\angle A = 110^0$, $m\overline{BC} = 3.5\ cm$ and $m\overline{AD} = 4\ cm$. Construct a triangle equal in area to the quadrilateral ABCD.

Given

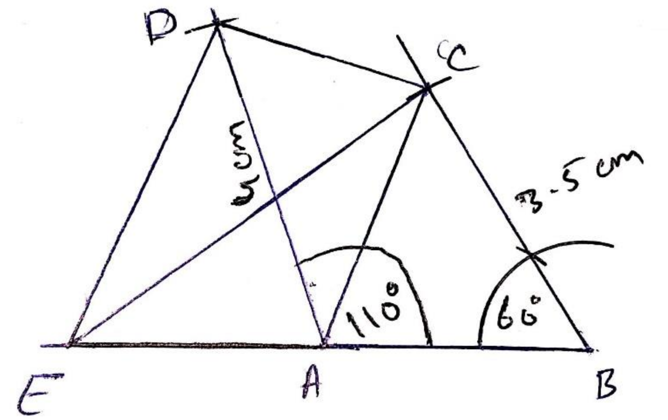
$\overline{AB} = 3\ cm$, $m\angle B = 60^0$, $m\angle A = 110^0$, $m\overline{BC} = 3.5\ cm$ and $m\overline{AD} = 4\ cm$

Required

Area of Triangle= Area of Quadrilateral

Steps of construction

1. Draw a line $m\overline{AB} = 3\ cm$
2. At point B, draw an angle of 60^0
3. At point A, draw an angle of 110^0
4. At point B, draw an arc of 3.5 cm which cuts 60^0 at point C.



5. At point A, draw another arc of 4 cm which cuts 110^0 at point D.
6. Join C to D.
7. Thus a quadrilateral ABCD is constructed.
8. Join C to A.
9. Through D, draw $\overline{DE}$ || $\overline{CA}$ meeting $\overline{BA}$ produced at E.
10. Join C to E.

As Area of ΔACE = Area of ΔACD

Area of $\Delta ACE + \Delta ABC$ = Area of $\Delta ACD + \Delta ABC$

Thus Area of ΔBCE = Area of quadrilateral ABCD so ΔBCE is the required triangle.

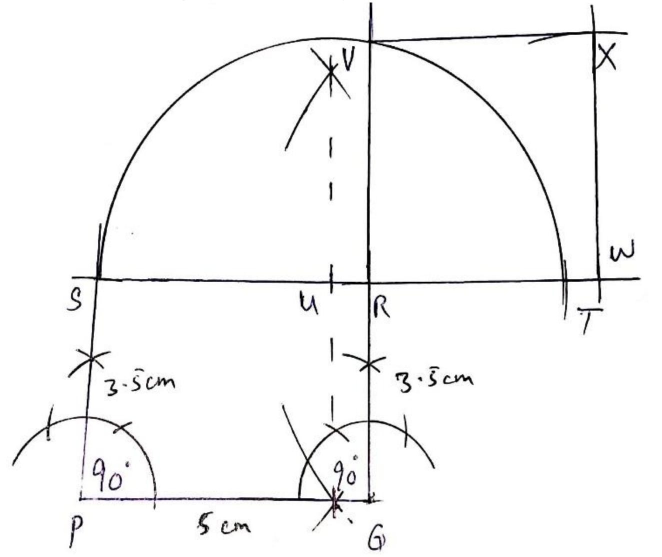
Q # 2 Draw a rectangle PQRS such that $m\overline{PQ} = 5\ cm$ and $m\overline{QR} = 3.5\ cm$. Construct a square equal in area to the rectangle PQRS.

Given

$m\overline{PQ} = 5\ cm$ and $m\overline{QR} = 3.5\ cm$

Required

Area of rectangle=Area of square



Steps of construction

1. Draw a line $m\overline{PQ} = 5\ cm$
2. At points P and Q, draw angle of 90^0.
3. At points P and Q, draw arc of 3.5 cm which cut both 90^0 at points S and R respectively.
4. Join S to R.
5. Thus rectangle PQRS is constructed.
6. Produce $\overrightarrow{SR}$ to T such that $m\overline{RT} = m\overline{RQ}$
7. Bisect $m\overline{ST}$ at point U.
8. At center U and radius $m\overline{US}$, draw a semicircle.
9. Produce $\overrightarrow{QR}$ to meet the semicircle at point V.
10. On $\overline{RV}$ as one side of the square, draw other three sides $\overline{RW}$, $\overline{VX}$ and $\overline{WX}$.
11. Thus RWXV is the required square.

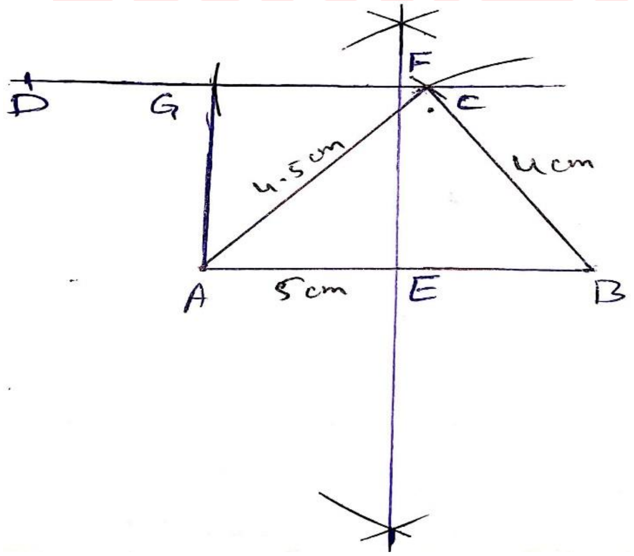
Q # 3 Draw a triangle ABC such that $m\overline{AB} = 5$ cm, $m\overline{BC} = 4$ cm and $m\overline{CA} = 4.5$ cm. Construct a rectangle equal in area to the given triangle.

Given

$\overline{AB} = 5\ cm$, $m\overline{BC} = 4\ cm$ and $m\overline{CA} = 4.5\ cm$

Required

Area of triangle = Area of rectangle



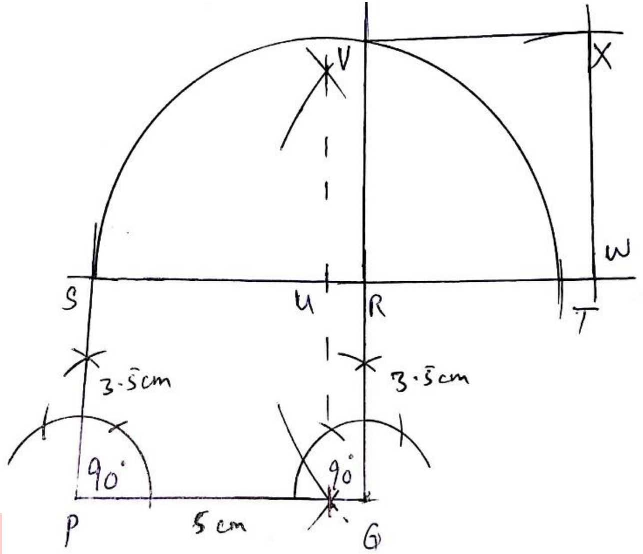
Steps of construction

1. Draw a line $m\ \overline{AB} = 5\ cm$
2. With B as centre, draw an arc of radius 4 cm.
3. With A as centre, draw another arc of radius 4.5 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Through C, draw $\overrightarrow{CD}\ ||\overline{BA}$
7. Bisect $\overline{AB}$ at E and cutting $\overrightarrow{CD}$ at F.
8. With F as center, draw an arc of radius $\overline{AE}$ which cuts CD at G
9. Join A to G
10. Thus AEFG is the required rectangle.

Q # 4 Construct a square having area equal to the given rectangle.

Required

Area of rectangle=Area of square



1. Let a rectangle PQRS is constructed.
2. Produce $\overrightarrow{SR}$ to T such that $m\overline{RT} = m\overline{RQ}$
3. Bisect $m\overline{ST}$ at point U.
4. At center U and radius $m\overline{US}$, draw a semicircle.
5. Produce $\overrightarrow{QR}$ to meet the semicircle at point V.
6. On $\overline{RV}$ as one side of the square, draw other three sides $\overline{RW}$, $\overline{VX}$ and $\overline{WX}$.
7. Thus RWXV is the required square.

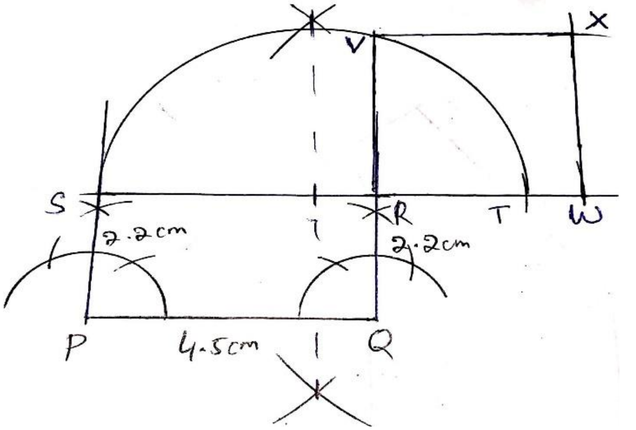
Q # 5 Construct a square equal in area to a rectangle whose adjacent sides are 4.5 cm and 2.2 cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.

Given

Let $m\overline{PQ} = 4.5\ cm$ and $m\overline{QR} = 2.2\ cm$

Required

Area of rectangle=Area of square



Steps of construction

1. Draw a line $m\overline{PQ} = 4.5\ cm$
2. At points P and Q, draw angle of 90^0.
3. At points P and Q, draw arc of 2.2 cm which cut both 90^0 at points S and R respectively.
4. Join S to R.
5. Thus rectangle PQRS is constructed.
6. Produce $\overline{SR}$ to T such that $m\overline{RT} = m\overline{RQ}$
7. Bisect $m\overline{ST}$ at point U.
8. At center U and radius $m\overline{US}$, draw a semicircle.
9. Produce $\overline{QR}$ to meet the semicircle at point V.
10. On $\overline{RV}$ as one side, draw other three sides $\overline{RW}$, $\overline{VX}$ and $\overline{WX}$.
11. Thus RWXV is the required square.

Area of square

As side of square is almost 3.1 cm.

So area of square = 3.1 X 3.1 = 9.61 cm^2

Area of rectangle

Area of rectangle = 4.5 X 2.2 = 9.9 cm^2

Comparison

Both values are almost equal.

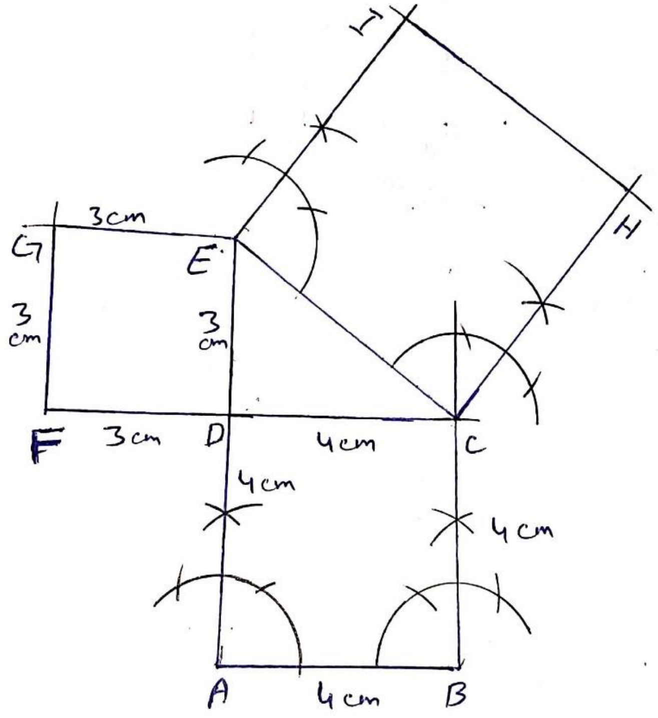
Q # 6 Construct a square equal in area to the sum of two squares having sides 3 cm and 4 cm respectively.

Given

Sides of two squares of 3 cm and 4 cm.

Required

Construct a square equal in area to the sum of two squares.



Steps of construction

1. Draw a line $m\overline{AB} = 4\ cm$
2. At points A and B, draw angles of 90^0
3. At points A and B, draw arcs of 4 cm which cut both 90^0 at points D and C respectively.
4. Join C to D.
5. Thus a square ABCD is formed.
6. Produce $\overrightarrow{AD}$ to E and $\overrightarrow{CD}$ to F such that $\overline{DE}$ and $\overline{DF}$ equal to 3cm.
7. At point E and F, draw two arcs of radius 3cm which intersect at point G.
8. Join G to E and F.
9. Thus another square DEGF is formed.
10. Join E to C.
11. At points C and E, draw angles of 90^0
12. At points C and E, draw arcs of radius $\overline{CE}$ which cut both 90^0 at points H and I respectively.
13. Join I to H.
14. Thus the required rectangle ECHI is constructed.

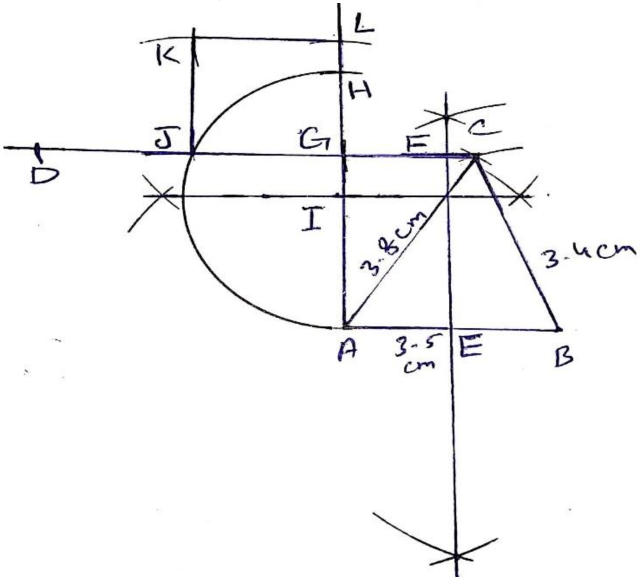
<u>Now to find the side of the third square</u>

By Pythagoras Theorem,

$$(EC)^2 = (DC)^2 + (DE)^2$$
$$(EC)^2 = (4)^2 + (3)^2$$
$$(EC)^2 = 16 + 9$$
$$(EC)^2 = 25$$
$$\sqrt{(EC)^2} = \sqrt{25}$$
$$\text{EC} = 5$$

Q # 7 Construct a triangle having base 3.5 cm and other two sides equal to 3.4 cm and 3.8 cm respectively. Transform it into an equal square.

Let $m\ \overline{AB} = 3.5\ cm$, $m\ \overline{BC} = 3.4\ cm$ and $m\ \overline{AC} = 3.8\ cm$



Steps of construction

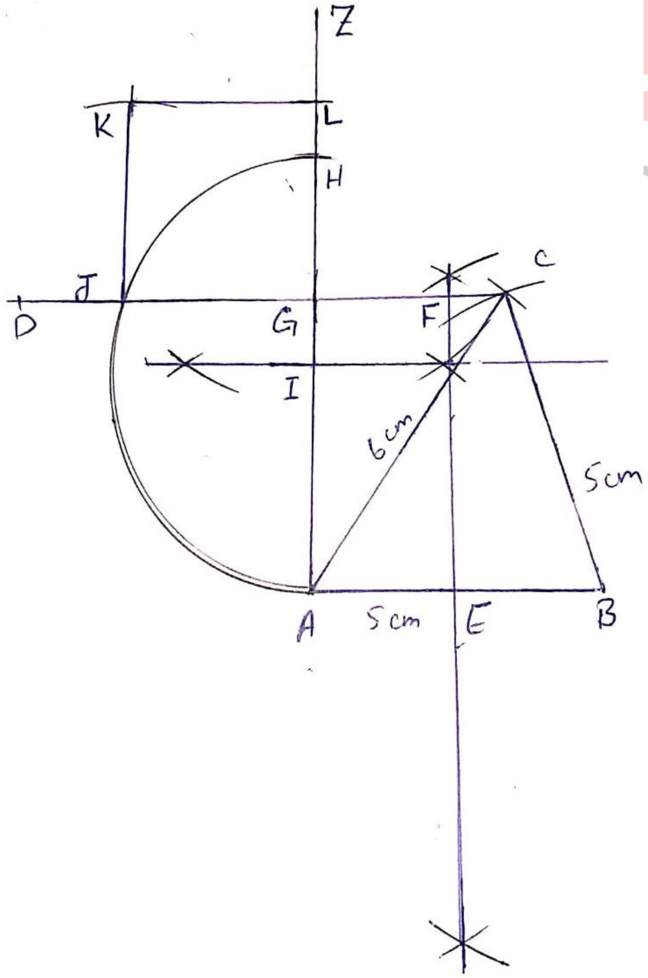
1. Draw a line $m\,\overline{AB} = 3.5\ cm$
2. With B as centre, draw an arc of radius 3.4 cm.
3. With A as centre, draw another arc of radius 3.8 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Through C, draw $\overrightarrow{CD}\ ||\overline{BA}$
7. Bisect $\overline{AB}$ at E and cutting $\overrightarrow{CD}$ at F.
8. Through A, draw $\overrightarrow{AZ}\ ||\overline{EF}$, meeting $\overrightarrow{CD}$ at G.
9. At point G, draw an arc of radius $m\overline{GF}$ which cuts $\overrightarrow{AZ}$ at point H.
10. Bisect $\overline{AH}$ at Point I.
11. With I as centre and radius $m\overline{IA}$, draw a semicircle which cuts $\overrightarrow{CD}$ at point J.
12. With $\overline{GJ}$ as side, complete the square GHKL.

Steps of construction

1. Draw a line $m\,\overline{AB} = 5\ cm$
2. With B as centre, draw an arc of radius 5 cm.
3. With A as centre, draw another arc of radius 6 cm.
4. Both the arcs meet at point C.
5. Thus ABC is the triangle according to data
6. Through C, draw $\overrightarrow{CD}\ ||\overline{BA}$
7. Bisect $\overline{AB}$ at E and cutting $\overrightarrow{CD}$ at F.
8. Through A, draw $\overrightarrow{AZ}\ ||\overline{EF}$, meeting $\overrightarrow{CD}$ at G.
9. At point G, draw an arc of radius $m\overline{GF}$ which cuts $\overrightarrow{AZ}$ at point H.
10. Bisect $\overline{AH}$ at Point I.
11. With I as centre and radius $m\overline{IA}$, draw a semicircle which cuts $\overrightarrow{CD}$ at point J.
12. With $\overline{GJ}$ as side, complete the square GHKL.

Q # 8 Construct a triangle having base 5 cm and other sides equal to 5 cm and 6 cm. Also construct a square equal in area to the given triangle.

Let $m\,\overline{AB} = 5\ cm, m\,\overline{BC} = 5\ cm$ and $m\,\overline{AC} = 6\ cm$



Review Ex # 17

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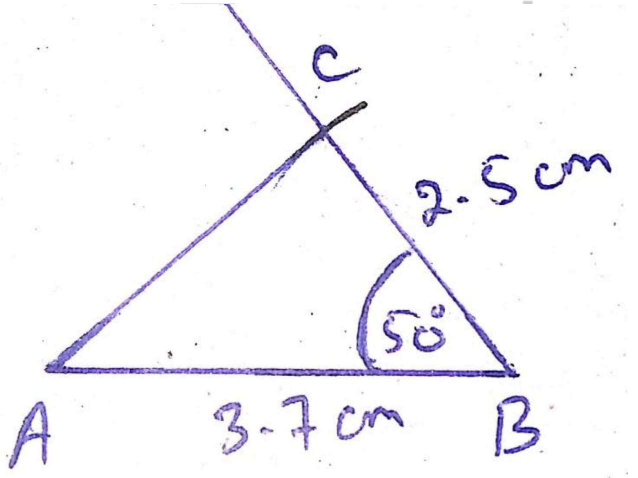
Q # 2 Construct a ΔABC, such that $m\overline{AB} = 3.7\ cm$, $m\overline{BC} = 2.5\ cm$, and $m\angle B = 50^0$

Given

$m\overline{AB} = 3.7\ cm,\ m\overline{BC} = 2.5\ cm,$ and $m\angle B = 50^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{AB} = 3.7\ cm$
2. At point B, draw an angle of 50^0
3. With B as centre, draw an arc of radius 2.5 cm which cuts angle 50^0 at point C
4. Join A to C
5. Thus ABC is the required triangle.

Q # 3 Construct ΔABC, where $m\,\overline{BC} = 5.8\ cm$, $m\,\angle A = 30^0$, and $m\angle B = 45^0$

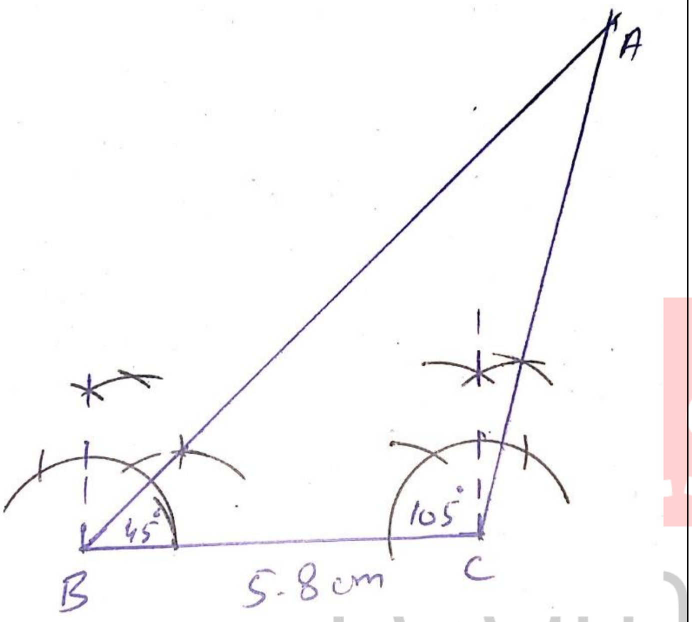
Given

$m\,\overline{BC} = 5.8\ cm,\ m\,\angle A = 30^0, m\angle B = 45^0$

Required

Construct a ΔABC

$$m\angle A + m\angle B + m\angle C = 180^0$$
$$30^0 + 45^0 + m\angle C = 180^0$$
$$75^0 + m\angle C = 180^0$$
$$m\angle C = 180^0 - 75^0$$
$$m\angle C = 105^0$$



Steps of construction

1. Draw a line $m\,\overline{BC} = 5.8\ cm$
2. At point B, draw an angle of 45^0
3. At point C, draw another angle of 105^0
4. Both the angles meet at point A.
5. Thus ABC is the triangle according to data.

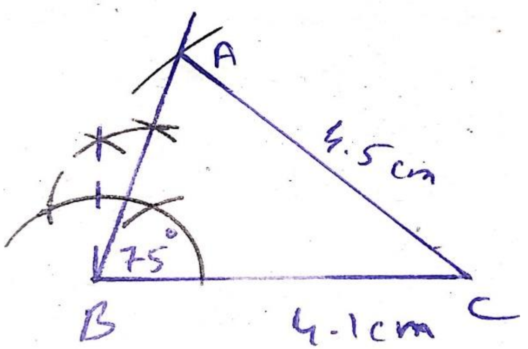
Q # 4 Construct a ΔABC, such that $m\overline{AC} = 4.5\ cm$, $m\overline{BC} = 4.1\ cm$, and $m\angle B = 75^0$

Given

$m\overline{AC} = 4.5\ cm,\ m\overline{BC} = 4.1\ cm,$ and $m\angle B = 75^0$

Required

Construct a ΔABC



Steps of Construction

1. Draw a line $m\overline{BC} = 4.1\ cm$
2. At point B, draw an angle of 75^0
3. With C as centre, draw an arc of radius 4.5 cm which cuts angle 75^0 at point A
4. Join A to C
5. Thus ABC is the required triangle.

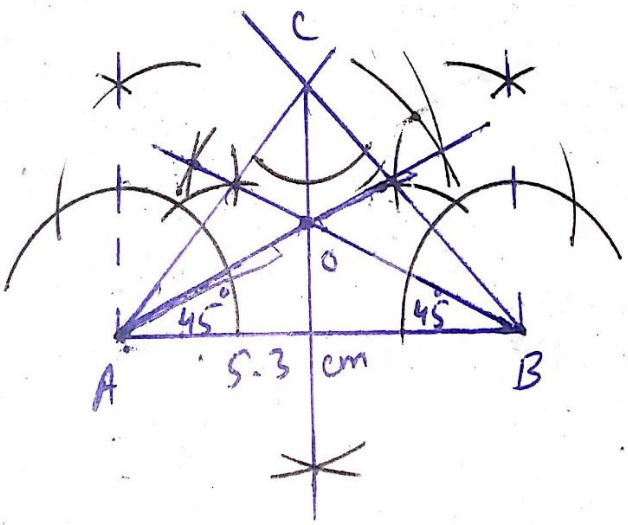
Q # 5 Construct ΔABC, draw their angle bisectors and verify their concurrency: $m\,\overline{AB} = 5.3\ cm$, $m\,\angle A = 45^0$, and $m\angle B = 45^0$

Given

$m\,\overline{AB} = 5.3\ cm,\ m\,\angle A = 45^0,$ and $m\angle B = 45^0$

Required

Angle bisectors of a ΔABC and their concurrency



Steps of construction

1. Draw a line $m\overline{AB} = 5.3\ cm$
2. At point A, draw an angle of 45^0
3. At point B, draw another angle of 45^0
4. Both the angles meet at point C.
5. Thus ABC is the triangle according to data
6. Draw the bisectors of angles A, B and C which pass through same point O.
7. Thus angle bisectors of a triangle are concurrent.

Q # 6 Construct ΔPQR, draw their altitudes and verify their concurrency: $m\overline{PR} = 5.8\ cm,\ m\angle P = 45^0,$ and $m\angle Q = 105^0$

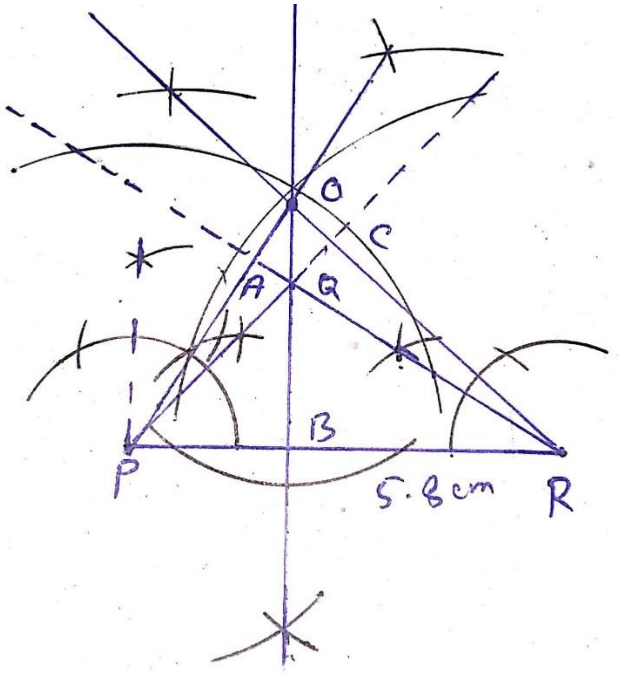
Given

$m\overline{PR} = 5.8\ cm,\ m\angle P = 45^0,$ and $m\angle Q = 105^0$

Required

Altitudes of a ΔPQR and their concurrency

$$m\angle P + m\angle Q + m\angle R = 180^0$$
$$45^0 + 105^0 + m\angle R = 180^0$$
$$150^0 + m\angle R = 180^0$$
$$m\angle R = 180^0 - 150^0$$
$$m\angle R = 30^0$$



Steps of construction

1. Draw a line $m\ \overline{PR} = 5.8\ cm$
2. At point P, draw an angle of 45^0
3. At point R, draw another angle of 30^0
4. Both the angles meet at point Q
5. Thus PQR is the triangle according to data
6. Draw perpendiculars from P to $\overline{QR}$, Q to $\overline{PR}$ and R to $\overline{PQ}$ at point A, B and C respectively.
7. Thus $\overline{PA}$, $\overline{QB}$ and $\overline{RS}$ are the required altitudes.
8. These altitudes pass through the same point O i.e. altitudes are concurrent

Q # 7 Construct ΔUVW, where $m\overline{UW} = 5.8\ cm,$ $m\angle U = 45^0,$ and $m\angle V = 105^0$ draw their perpendicular bisectors and verify their concurrency

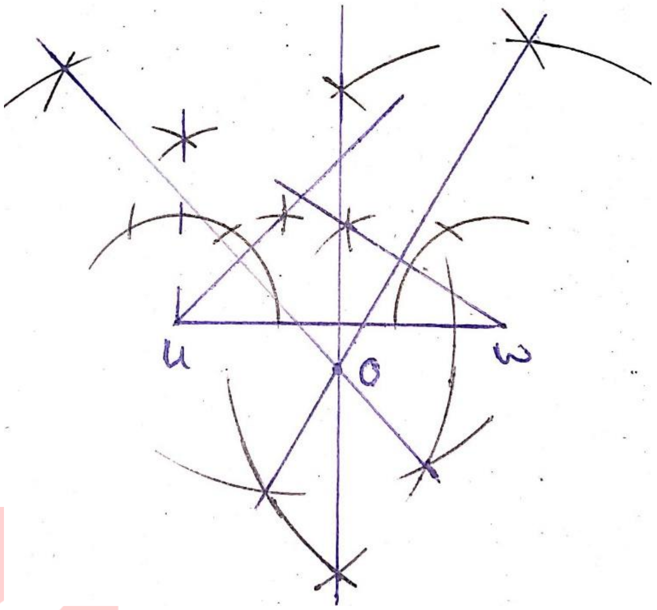
Given

$m\overline{UW} = 5.8\ cm,\ m\angle U = 45^0,$ and $m\angle V = 105^0$

Required

Perpendicular bisectors of a ΔUVW and their concurrency

$$m\angle U + m\angle V + m\angle W = 180^0$$
$$45^0 + 105^0 + m\angle W = 180^0$$
$$150^0 + m\angle W = 180^0$$
$$m\angle W = 180^0 - 150^0$$
$$m\angle W = 30^0$$



Steps of construction

1. Draw a line $m\overline{UW} = 5.8\ cm$
2. At point U, draw an angle of 45^0
3. At point W, draw another angle of 30^0
4. Both the angles meet at point V
5. Thus UVW is the triangle according to data.
6. Draw the perpendicular bisectors of $\overline{UV}$, $\overline{VW}$ and $m\overline{WU}$
7. These perpendicular bisectors pass through point O i.e. perpendicular bisectors are concurrent.

Q # 8 Construct ΔXYZ, where $m\ \overline{ZX} = 6cm,$ $m\angle Y = 60^0,$ and $m\angle Z = 75^0$ draw their medians and verify their concurrency

Given

$m\ \overline{ZX} = 6cm,\ m\angle Y = 60^0,$ and $m\angle Z = 75^0$

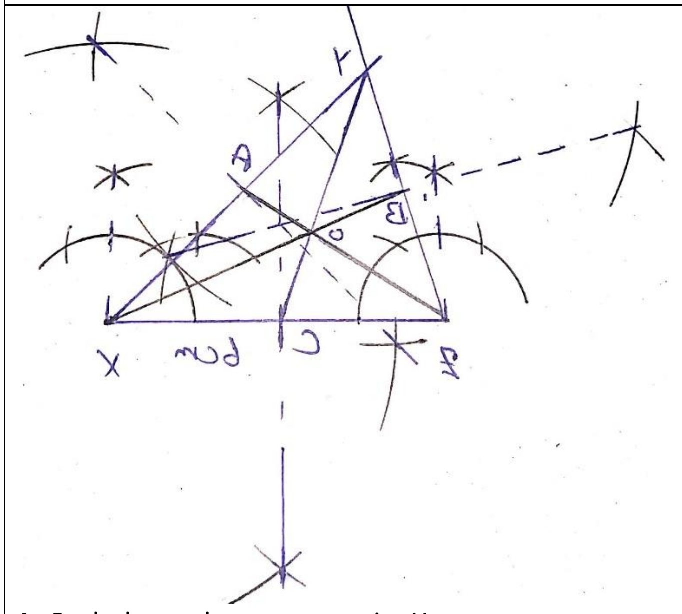
Required

Medians of a ΔXYZ and their concurrency

$$m\angle X + m\angle Y + m\angle Z = 180^0$$
$$m\angle X + 60^0 + 75^0 = 180^0$$
$$m\angle X + 135^0 = 180^0$$
$$m\angle X = 180^0 - 135^0$$
$$m\angle X = 45^0$$

Steps of construction

1. Draw a line $m\ \overline{ZX} = 6\ cm$
2. At point Z, draw an angle of 75^0
3. At point X, draw another angle of 45^0



4. Both the angles meet at point Y.
5. Thus XYZ is the triangle according to data.
6. Find the mid points A, B and C of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively by right bisectors.
7. Draw the medians $\overline{ZA}$, $\overline{XB}$ and $m\overline{YC}$
8. These medians pass through point O i.e. the medians of a triangle are concurrent.

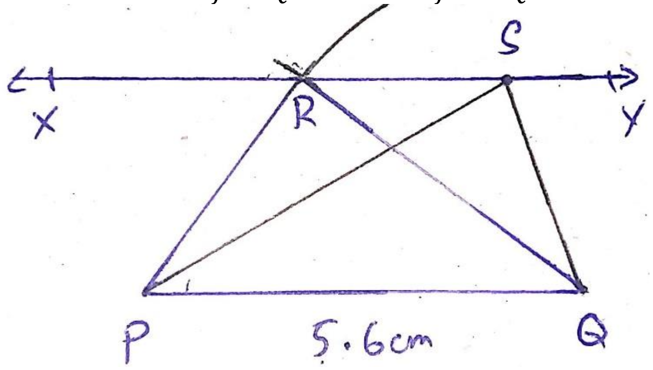
Q # 9 Draw a triangle PQR, such that $m\overline{PQ} = 5.6\ cm$, $m\overline{QR} = 4.5\ cm$, and $m\overline{RP} = 3.4\ cm$. Construct a triangle SPQ equivalent in area of the triangle PQR.

Given

$m\overline{PQ} = 5.6\ cm,\ m\overline{QR} = 4.5\ cm,$ and $m\overline{RP} = 3.4\ cm$

Required

$$Area\ of\ \Delta PQR = Area\ of\ \Delta SPQ$$



Steps of construction

1. Draw a line $m\ \overline{PQ} = 5.6\ cm$
2. With Q as centre, draw an arc of radius 4.5 cm.
3. With P as centre, draw another arc of radius 3.4 cm.
4. Both the arcs meet at point R.
5. Thus PQR is the triangle according to data
6. Through R, draw $\overline{XY}\ |\ \ |\overline{PQ}$
7. Take any point S on $\overline{XY}$
8. Join S to P and Q
9. Thus a triangle SPQ is formed which is equivalent in area to PQR.

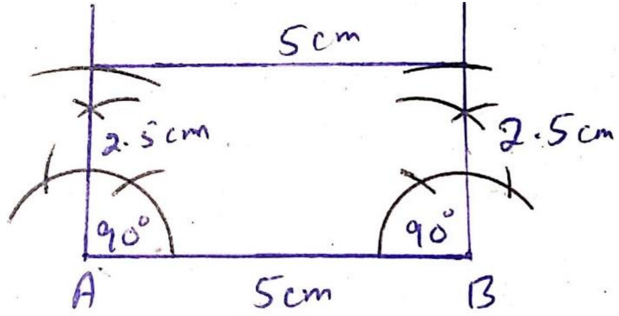
Q # 10 Construct a rectangle whose adjacent sides are 2.5 cm and 5 cm respectively.

Given

$$adjacent\ sides\ of\ rectangle\ 2.5\ and\ 5\ cm$$

Required

Draw a rectangle



Steps of construction

1. Draw a line $m\overline{AB} = 5\ cm$
2. At points A and B, draw angles of 90^0
3. At points A and B, draw arcs of 2.5 cm which cut both 90^0 at points D and C respectively.
4. Join C to D.
5. Thus a rectangle ABCD is formed.